Algebra I Overview

The content standards associated with Algebra I are based on the <u>New York State Common Core</u> <u>Learning Standards for Mathematics</u> and the <u>PARCC Model Content Framework for Algebra I</u>. The content standards define what students should understand and be able to do at the high school level; the Model Content Framework describes which content is included and emphasized within the Algebra I course, specifically. More information about the relationship between the NYS CCLS and the PARCC Model Content Frameworks can be found in <u>this memo</u>.

For high school mathematics, the standards are organized at three levels: *conceptual categories*, *domains* and *clusters*.

Algebra I is associated with high school content standards within four *conceptual categories*: **Number & Quantity, Algebra, Functions,** and **Statistics & Probability**. Each conceptual category contains *domains* of related *clusters* of standards. This chart shows the high school mathematics domains included in Algebra I, as well as the corresponding percent of credits on the Algebra I Regents Exam:

Conceptual Category	Percent of Algebra I Regents Exam Credits	High School Mathematics Domains Included in Algebra I
Number & Quantity	2% - 8%	The Real Number System (N-RN) Quantities (N-Q)
Algebra	50% - 56%	Seeing Structure in Expressions (A-SSE) Arithmetic with Polynomials and Rational Expressions (A-APR) Creating Equations (A-CED) Reasoning with Equations and Inequalities (A-REI)
Functions	32% - 38%	Interpreting Functions (F-IF) Building Functions (F-BF) Linear, Quadratic, and Exponential Models (F-LE)
Statistics & Probability	5% - 10%	Interpreting categorical and quantitative data (S-ID)

The conceptual category of **Modeling** is also included in Algebra I, but is best interpreted not as a collection of isolated topics but rather in relation to other standards:

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For more information about modeling at the high school level, please consult the <u>High School</u> <u>Progression on Modeling</u>.

Content Emphasis

Not all of the content in a given grade is emphasized equally in the standards. The list of content standards for each grade is not a flat, one-dimensional checklist; this is by design. There are sometimes strong differences of emphasis even within a single domain. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice. Without such focus, attention to the practices would be difficult and unrealistic, as would best practices like formative assessment.

The Regents Examination in Algebra I test will mirror the organization of the standards: Major Clusters will account for a majority (58% - 73%) of the credits on the test, while Supporting Clusters (18% - 30%) and Additional Clusters (5% - 17%) will together constitute less than half the possible credits.

The chart below shows:

- Clusters and standards associated with Algebra I within each domain. Clusters are identified as
 Major Content, Supporting Content, or Additional Content, to indicate emphasis within the course.
- NYSED standards clarifications and PARCC assessment limits, following the applicable standards. More information about the assessment limits, which offer clarification for standards shared with other high school courses, can be found in the Model Content Framework linked above. Additionally, more information about the NYSED standards clarifications can be found here. The PARCC assessment limits are denoted by, "PARCC," the NYSED standards clarifications are denoted by, "NYSED".

Standards for Mathematical Practice

The Standards for Mathematical Practice form an important part of the Algebra I course, as well:

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Mathematics – High School Number & Quantity

The Real Number System (N-RN)

B. Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities (N-Q)

A. Reason quantitatively and use units to solve problems. *

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

PARCC: In Algebra I, this standard will be assessed by ensuring that some modeling tasks(involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described. *For example, a quantity of interest is not selected for the student by the task. For example, In a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.*)

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

NYSED: In Algebra I, the greatest precision for a result is only at the level of the least precise data point (example: if units are tenths and hundredths, then the appropriate preciseness is tenths). Calculation of relative error is not included in this standard.

For more information about the concepts and terms introduced in the Number and Quantity Conceptual Category, consult the <u>Progression on The Number System; High School, Number</u>.

Mathematics – High School Algebra

Seeing Structure in Expressions (A-SSE)

A. Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. \star

a. Interpret parts of an expression, such as terms, factors, and coefficients.

NYSED: The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.

PARCC: Tasks limited to numerical and polynomial expressions in one variable. Recognize 53^2-47^2 as a difference of squares and see an opportunity to rewrite it in the easier-to -evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2+9a+14$ as (a+7)(a+2).

NYSED: Does not include factoring by grouping and factoring the sum and difference of cubes.

B. Write expressions in equivalent forms to solve problems. *

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor quadratic expression to reveal the zeros of the function it defines.

NYSED: Includes trinomials with leading coefficients other than 1.

- **b.** Complete the square in a quadratic expression to reveal the max and min value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^{t} can be rewritten as $(1.15^{1/12})^{12t} = 1..012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

PARCC: Tasks are limited to exponential expressions with integer exponents. Tasks have a realworld context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.

Arithmetic with Polynomials and Rational Expressions (A-APR)

A. Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials.

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

PARCC: Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. *For example, find the zeros of (x-2) (x^2-9).*

Creating Equations (A-CED)

A. Create equations that describe numbers or relationships. *

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include* equations arising from linear and quadratic functions, and simple radical and exponential functions.

PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*

Reasoning with Equations and Inequalities (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

PARCC: Tasks are limited to quadratic equations.

B. Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.B.4 Solve quadratic equations in one variable.

NYSED: Solutions may include simplifying radicals.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for x^2 =49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a + bi, a - bi for real numbers a and b.

PARCC: Tasks do not require students to write solutions for quadratic equations that have roots with non-zero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.

• C. Solve systems of equations.

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

PARCC: Tasks have a real-world context. Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

D. Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

PARCC: Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

For more information about the concepts and terms introduced in the Algebra Conceptual Category, please consult the <u>High School Progression on Algebra</u>.

Mathematics - High School Functions

Interpreting Functions (F-IF)

A. Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for $n \ge 1$.

PARCC: This standard is part of the Major work in Algebra I and will be assessed accordingly.

B. Interpret functions that arise in applications in terms of the context. \star

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.

C. Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **★**

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

PARCC: Tasks are limited to linear functions, quadratic functions, square root, cube root, piecewise defined (including step functions and absolute value functions), and exponential functions with domains in the integers.

Building Functions (F-BF)

A. Build a function that models a relationship between two quantities.

F-BF.A.1 Write a function that describes a relationship between two quantities. \star

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

PARCC: Tasks have a real-world context. Tasks are limited to linear, quadratic and exponential functions with domains in the integers.

• B. Build new functions from existing functions.

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

PARCC: Identifying the effect on the graph of replacing f(x) by f(x) + k, kf(x), and f(x+k) for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.

Linear, Quadratic, and Exponential Models (F-LE) *

A. Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

- **a.** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

PARCC: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

B. Interpret expressions for functions in terms of the situation they model.

F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

PARCC: Tasks have a real-world context. Exponential functions are limited to those with domains in the integers.

For more information about the concepts and terms introduced in the Functions Conceptual Category, please consult the <u>High School Progression on Functions</u>.

High School - Statistics & Probability

Interpreting categorical and quantitative data (S-ID)

• A. Summarize, represent, and interpret data on a single count or measurement variable.

S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (inter-quartile range, standard deviation) of two or more different data sets.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

PARCC: Tasks have real world context. Exponential functions are limited to those with domains in the integers.

NYSED: Includes the regression capabilities of the calculator.

b. Informally assess the fit of a function by plotting and analyzing residuals.

NYSED: Includes creating residual plots using the capabilities of the calculator (not manually).

c. Fit a linear function for a scatter plot that suggests a linear association.

NYSED: Both correlation coefficient and residuals will be addressed in this standard.

C. Interpret linear models.

S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.9 Distinguish between correlation and causation.

For more information about the concepts and terms introduced in the Statistics & Probability Conceptual Category, please consult the <u>High School Progression on Statistics and Probability</u>.

Fluency Recommendations

The PARCC Model Content Frameworks recommend the following fluencies for Algebra I students:

A/G

Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1

Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b

Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.