Brief 2:

Debunking Myths about Mathematics Teaching and Learning

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Background

Mathematics teaching and learning have been the focus of debate for decades, beginning long before the "math wars" of the late 20th century. In 2006, the National Mathematics Advisory Panel was created to examine the body of scientific evidence related to mathematics teaching and learning, releasing a culminating report *Foundations of Success: The Final Report of the National Mathematics Advisory Panel* in 2008. In the nearly 20 years since the creation of that panel, the gathering of scientific evidence continues, but myths about math teaching and learning endure.

Underlying the myths and misunderstandings are persistent disagreements about the purposes and goals of mathematics education, and about how to support young people's mathematics learning. False dichotomies characterize these disagreements. For example, is the goal to develop procedural skill and fluency OR to build conceptual understanding and number sense? Is the purpose for students to solve problems efficiently and quickly OR to develop fluency, flexibility, and communication skills? Should instruction always begin with direct instruction and then move to application OR does robust mathematics learning also require opportunities for investigation and inquiry? Research demonstrates that all of these are necessary and comprise intersecting, not dichotomous, goals of mathematics education. Below we identify and debunk common myths that impede progress in mathematics education.

Common Myths

Myth #1: "Ability" grouping improves success.

Separating students into categories to organize tailored instruction is often referred to as "ability grouping." Although students may have reached different levels of skill and understanding, this does not mean that they are more or less capable. And because students can be skillful with and understand different aspects of mathematics, it is important not to apply fixed labels. Labeling often also has negative effects on children as they come to see themselves as "smart" or not.

The research on grouping students by levels of achievement is mixed: some studies show gains for students in all groups; some show increasing gaps between groups; and some show no changes. Meanwhile, other studies show gains for all students in mixed-ability classrooms when compared to ability-grouped classrooms. So, what is happening here?

In mixed ability classrooms, students can have the opportunity to learn from a wide variety of classmates and develop skill with mathematical argumentation while engaging with grade-level content. These same things can happen in ability-grouped classrooms, but they often do not. In a large study of ability grouping, teachers noted that the "lower" groups are often seen as less capable and are excluded from exposure to higher-level work or all grade-level standards (Mazenod et al., 2018). This can lead to a widening of achievement gaps between the groups. This has also been shown to have a negative effect on students' mathematical identities, leading them to see themselves as "not good at math."

However, studies have also shown that flexible grouping, where students can move into groups to target their instructional needs, can be designed in ways that increase student learning. The key is that these groups are both focused on a student's learning of a particular concept or process and that the groupings are truly flexible, meaning that groups will change depending on the learning target and on a student's need.

Myth #2: Most students struggle in mathematics.

Unfortunately, many adults complete school with a negative view of mathematics. It is commonplace to say one is "not a math person." Because of this, a false assumption is that young children, too, are scared of math and regularly struggle to learn it. This is further reinforced when students are labeled "below" or "behind" as a result of poor performance on timed tests. Responding by reducing cognitive demand when a student assumes they "are not a math person" or are labeled as "below" or "behind" results in the constraint of deeper learning. Remedial help can prevent students from using and retaining important content and skills resulting in students falling further behind. Research shows that when students are instead supported to solve challenging problems that enable them to learn and use the concepts and skills they need, they are more likely to thrive mathematically.

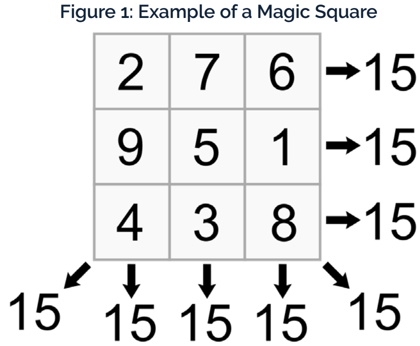
Myth #3: Mathematical competence means being efficient and accurate.

There is a long-held belief that being "good" at mathematics means being fast and correct. However, professional mathematicians do not work fast and often take false paths as they solve problems. Researchers agree that mathematical competence is broader than speed and accuracy alone. Procedural fluency (efficiency, flexibility, and accuracy) is interwoven with conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition (National Research Council, 2001) (see Brief #1). This means that students should be supported to demonstrate mathematical competence in a wide variety of ways including: (1) choosing and justifying a strategy to solve a particular task; (2) providing the reasoning or proving a result; (3) asking questions about core mathematical ideas; and (4) explaining concepts.

Myth #4: Developing competence requires isolated repetition and drills.

Procedural fluency supports student success in mathematics by reducing students' cognitive load in the course of problem solving. However, myths persist that the sole way to build such fluency is through repeated speed drills and timed tests. While these may provide repeated opportunity to build fact fluency, they also frequently cause students stress and to see themselves as not "good at math." Practice for fluency can also be accomplished using problems that require repeated use of facts to solve a more complex problem—for example, magic square (see Figure 1) or magic

Figure 1: Example of a Magic Square



triangle tasks. These tasks require repeated practice with facts, but also require reasoning and strategic thinking, accomplishing multiple mathematical goals simultaneously. The student-facing goal isn't to practice their facts, but students have repeated practice opportunities in the course of solving the problem.

Myth #5: The most effective teaching method is providing step-by-step procedures for solving problems.

Research in mathematics learning, particularly in psychology and special education, demonstrates that detailed, step-by-step instruction can support students to learn mathematical procedures. Much of this research is focused specifically on students with disabilities and utilizes assessments of procedural knowledge to measure change. Although explicit instruction can be helpful for the teaching and learning of procedures, it is important that it be grounded in meaning and understanding of why the procedures work. It is also not the "best" way to teach or learn mathematical thinking and reasoning. This requires supporting students to grapple with mathematical tasks that demand reasoning and justification, clarify and refine their reasoning with others through discourse, and demonstrate justification of their reasoning. Standards for mathematical competence include constructing, analyzing, and critiquing the completeness of mathematical arguments.

Myth #6: "Inquiry" approaches leave students to learn on their own instead of teaching them mathematics.

Misconceptions about inquiry and discovery approaches to teaching and learning mathematics persist, framing inquiry-based instruction as posing problems and leaving students to create mathematics on their own. In reality, effective inquiry approaches are highly designed and scaffolded to support both student investigation of mathematical ideas and teacher facilitated and directed summarization and instruction. Well-designed and implemented inquiry or problem-based lessons begin with an accessible mathematics task that requires students to engage in conjectures related to foundational grade-level standards. The teacher facilitates student understanding of the problem or task, structures opportunities for students to engage in conjecture, justification, and argumentation, and supports students to make sense of the conceptual and procedural ideas raised by the task, often through questioning and sometimes through explicit instruction. Studies have found that inquiry methods enhance mathematical problem solving and reasoning without reducing procedural knowledge. Inquiry-based teaching and learning has also been found to have positive longitudinal impacts on student learning (Hmelo-Silver et al., 2007). Thus, inquiry approaches support the teaching and learning of mathematical thinking and reasoning.

Myth #7: Success in calculus is the ultimate goal of school mathematics

Many high schools have taken the view that student success in calculus is the goal for all students. Two paths of reasoning have pushed schools in this direction. First, schools reasoned that the goal of schooling was to prepare students for all possible future pathways, including college. Second, analyses had shown that about two-thirds of community college students and more than one-third of four-year college students were required to take remedial coursework and that the majority of these students dropped out without degrees. Many schools reasoned that if they could get students through calculus, the required course for many undergraduate degrees, students would be prepared for college if they chose that pathway, would no longer need remediation, and would complete desired degrees. However, a study at the City University of New York (CUNY) showed that the issue may not be in students' high school pathway, but rather in college pathways. Across the seven-year study, students without challenges in basic arithmetic bypassed remedial college algebra courses, taking statistics instead. These students were more likely to earn their college mathematics credit and complete their degree. Many universities are now reconsidering their mathematics pathways, identifying the appropriate mathematics courses for degrees rather than requiring calculus for all.

How does this shift in college pathways impact schools? Schools may still hold preparing students for all future pathways as a goal, but that no longer means that calculus is the target for all students. Courses in statistics, quantitative reasoning, and using and understanding data may be more beneficial targets for many students. Regardless of the course pathways, these courses should focus on mathematical thinking, justification, fluency, and communication in order to best prepare students for all future pathways.

Key Take-Aways

1. Meeting students where they are at is important, but static ability-grouped classrooms often do not support the learning of students.

2. Inquiry and explicit instruction can be effective for student learning, particularly when used in combination with careful thinking about the learning goals and when each method is most appropriate.

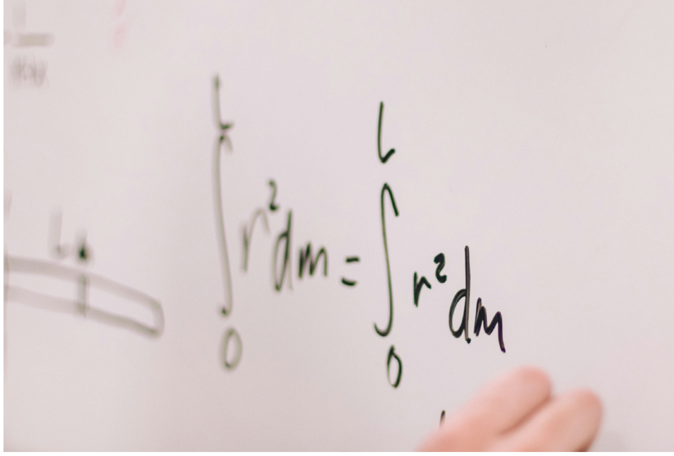
3. There are many competencies that are critical for learners of mathematics, not just fluency. Additionally, fluency can be built in tandem with problem-solving skills.

4. Supporting students to do challenging mathematical work can help to avoid cycles of failure and the development of negative math identities.

Reflect & Analyze:

1. Consider which of these myths exist in your district, school, classroom, or community. How will you work to be a mathematics myth-buster?

2. Are there any myths about which you remain unconvinced? How might you learn more to help you build a nuanced view of the issue?



Key References

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99–107.

Mazenod, A., Francis, B., Archer, L., Hodgen, J., Taylor, B., Tereshchenko, A., & Pepper, D. (2018). Nurturing learning or encouraging dependency? Teacher constructions of students in lower attainment groups in English secondary schools. *Cambridge Journal of Education, 49*(1), 53–68.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* U.S. Department of Education.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* The National Academies Press.

NRICH Project. (2011, January 2). *An introduction to magic squares*. https://nrich.maths.org/articles/introduction-magic-squares