New York State
EDUCATION DEPARTMENT
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Brief 1:

The Research Base for Mathematics Teaching and Learning

Supporting Evidence-Based P–12 Mathematics Teaching Practice

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Background

Across decades, scholars, researchers, and practitioners from multiple fields have produced substantial peer-reviewed, evidence-based research to ground high-quality mathematics teaching and learning. No single field can provide all the knowledge needed to inform mathematics teaching and learning. Instead, researchers from various disciplines—mathematics, teacher education, curriculum design and use, sociology, ethnic studies, developmental psychology, cognitive science, education policy, and psychometrics—have all contributed to these important resources.

This research, which draws on different kinds and methods of inquiry (e.g., controlled experiments, descriptive and narrative inquiry, conceptual analysis, and ethnography), offers valuable insight and information to inform practice and policy. It includes information on particular topics and skills and explores what is involved for young people to develop proficiency with fundamental mathematical ideas, procedures, reasoning, and problem solving. It has contributed to understanding the role of representations and manipulatives, how particular instructional practices and classroom environments impact learning, and how to support growth in students' mathematical competence and positive mathematical identities. Some research also reveals taken-for-granted practices or curricular designs that harm or impede students' development and suggests strategies for replacing these with approaches that support positive growth in students' mathematical competence and identity.

Key Ideas

Key Idea #1: Skillful mathematics teaching makes use of instructional practices that develop all strands of mathematical proficiency.

In 2001, the National Research Council brought together top education researchers to form the Mathematics Learning Study Committee, which produced a seminal report synthesizing research on K–8* mathematics teaching and learning, *Adding It Up: Helping Children Learn Mathematics.* The report lays out five interwoven mathematical proficiencies necessary for student success in mathematics:

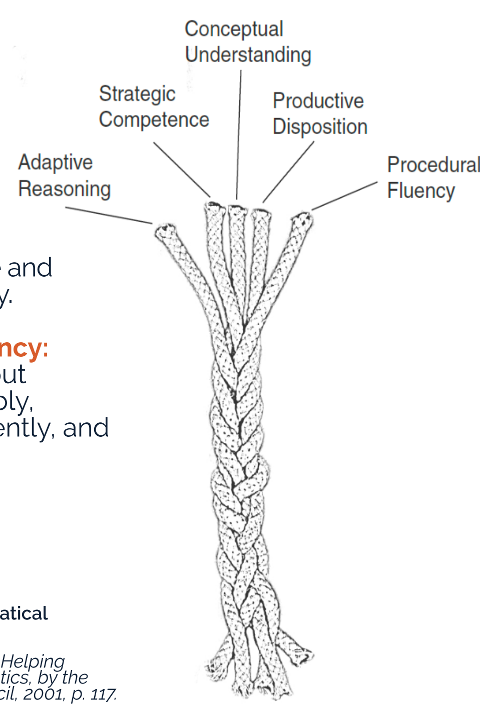
Adaptive Reasoning: Capacity for logical thought, reflection, explanation, and justification.

Strategic Competence: Ability to formulate, represent, and solve mathematical problems.

Conceptual Understanding: Comprehension of mathematical concepts, operations, and relations.

Productive Disposition: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Procedural Fluency: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.



The Strands of Mathematical Proficiency

Note: From Adding It Up: Helping Children Learn Mathematics, by the National Research Council, 2001, p. 117.

*Although this report specifically addressed K–8 mathematics, extensive research also exists around secondary mathematics teaching and learning, particularly around the teaching of functions, geometry, and statistics.

For a more detailed description of the strands, along with examples of mathematical tasks that call on each strand, see "Chapter 4: The Strands of Mathematical Proficiency" from *Adding It Up: Helping Children Learn Mathematics* (2001).

Skillful teaching that supports student learning and development makes use of instructional practices that support the building of these strands of mathematical proficiency. In addition to designing instruction around curriculum materials that provide opportunity for students to build these proficiencies (see Brief #7), teachers should utilize practices that are specifically supportive of developing these strands and are linked to student learning in mathematics (e.g., eliciting student thinking, leading discussions, setting up and managing small group work, and modeling and explaining content). Each of these instructional practices has a strong research base showing improved student outcomes with consistent use (see Brief #4). These instructional practices, when enacted skillfully, make space for students to engage in ways that develop the strands of mathematical proficiency and the Standards for Mathematical Practice (New York State Education Department, 2019).

Standards for Mathematical Practice

Make sense of problems and persevere in solving them	Reason abstractly and quantitatively
Construct viable arguments and critique the reasoning of others	Model with mathematics
Use appropriate tools strategically	Attend to precision
Look for and make use of structure	Look for and express regularity in repeated reasoning

Key Idea #2: Research-based learning trajectories provide teachers with information about students' progress toward fluency and conceptual understanding.

Research on student learning of different concepts, topics, and skills has highlighted the core elements of trajectories that develop fluency with mathematics. Conceptual understanding of high-leverage mathematical ideas (see Brief #3) and structures must be intertwined with deep sensemaking and practice in their use and application. Take, for example, the fundamental system of place value numeration. Developing competence with computation includes being able to calculate, interpret the meaning of a result, determine the reasonableness of an answer, and judge the extent of precision required for a particular situation (e.g., finding the number of vehicles needed for a field trip or determining the profit margin for a product). This goal requires that students have experience with making sense of numbers of different magnitudes. They must be able to understand, reason with, and use the structure of the base ten place value system, both with concrete and pictorial representations and in expanded and compressed symbolic form, and to calculate efficiently and flexibly with combinations of single digits. A sequence of instruction must balance and connect these different understandings and skills for students to develop comprehensive command of the place value system's structure and its application across a wide range of problems and situations. If students can name that 5 is in the ten-thousands place in 254,378, or in the thousandths place in 0.345, but do not have a sense of how big or small these quantities are, then the sheer naming of places lacks the robust knowledge needed for mathematical competence. Research-informed trajectories are crucial for ensuring that the different aspects of understanding, skill, and reasoning are connected so that students' competence has the depth needed for sustained proficiency.

Key Idea #3: Research on mathematics teaching and learning highlights the importance of culturally responsive and sustaining learning environments.

Studies over the last 30 years make plain the importance of attending to and using students' cultural and community knowledge to connect their mathematical development meaningfully with resources they have (Paris & Alim, 2017; Zavala & Aguirre, 2024). These resources are in experiences they have in their families and communities, in the languages they speak, and in their common practices. Three interlocking domains of experience and resources are vital to authentic connections.

Domains of Experience and Resources Vital to Authentic Connections

Content
Cultural knowledge, activities, practices, histories, people

Practices
Discourse patterns, ways of knowing what there is to know, values

Language
Multilingualism, translanguaging, mathematical assets of different languages

Content

The domain labeled "content" supplies resources for situating and understanding the areas in which mathematics is used and the kind of mathematical practice used and known across cultural and community contexts. For example, learning about the highly developed base 20 place value system used in Indigenous mathematics allows students to connect ways of counting and numerating used by elders in their community. Students also learn to appreciate what a place value system of numeration does and that there are reasons to structure such systems based on different needs. It is also crucial to broaden students' encounters with who makes and uses mathematical knowledge by systematically including members of non-dominant groups. One example of this is the series *Journeys of Black Mathematicians.* Exposing students to mathematical activity and thinking engaged in different cultures may highlight that there is mathematical knowledge that is both similar to and distinct from the Western canon. There are many important reasons to broaden the sources and resources for what and who students learn to label as "mathematical," including for members of historically marginalized groups to see people who look like them and to honor and preserve knowledge too often ignored, unknown, or treated as "different" or "exotic." The use of broader cultural and community knowledge enables all students to encounter and learn mathematics as a wide domain of human thought, inquiry, and development.

Practices

The domain labeled "practices" highlights the ways of interacting and knowing—the discourse patterns and what is valued—that provide guidance for the construction of inclusive learning environments. So much of the way we focus on mathematical topics in school neglects the central role of mathematical reasoning and representing. Engaging students in mathematical practices provides resources for developing norms and ways of working that draw on students' experiences in their communities. It also can support all students to develop a wide range of skills of interaction, discourse, and collaboration that are specific to mathematical activity but also extend beyond it. Drawing on, for example, the practices of working collectively that are common in many minoritized communities can enable students to use ways of working with others with which they are experienced, and contributes to enriching the learning environment and the skills for all students. Attending to a broader set of cultural practices and ways of knowing—embracing norms and patterns of non-dominant communities—also enlarges the resources for learning by expanding who contributes to the class work. Relationships with students' families and communities are vital for learning more about students' cultural resources, practices, and knowledge. Joining in community activities and learning more about traditions and common activities afford educators many opportunities for learning about how mathematics and mathematical reasoning are used, how people respect and work together, and what are valued ways of knowing.

Language

The domain labeled "language" highlights the importance of attending to the languages spoken by students. Students who are multilingual bring skills and resources from across the language systems and vocabulary that they know (Baker-Bell, 2020). Many will engage in translanguaging—or using the vocabulary, grammar, and linguistic structures from more than one language to communicate. Teachers support multilingual learners when they understand the assets of multilingualism as well as the challenges that learners may face. Teachers can draw on these assets and address these challenges by using instructional practices that support multilingual learners' development (e.g., using multiple representations, building clear correspondences between language and pictorial or physical representations), by encouraging students to work with peers, and by helping students draw on strengths across their languages as they develop mathematically.

Key Idea #4: Research on mathematics teaching and learning shows that teachers' own relationship to and understanding of mathematics is crucial for supporting positive student development.

All of these key areas depend on teachers' understanding of and ability to hear, notice, and communicate mathematically. This requires that teachers, pre-K through high school, are rooted in deep mathematical understanding in order to set them up for the mathematical flexibility and imagination needed to explain mathematical ideas. Teachers that have such specialized understanding and a sense of themselves as people who know and enjoy mathematics can guide young learners to do the same (see Briefs #3 and #6). This means that professional learning must take this need seriously, offering both curriculum-based professional development and ongoing opportunities for teachers to develop as mathematical knowers, situated for the purposes of lighting up mathematics for their students (see Brief #8).

Key Take-Aways

1. Different fields contribute to the research base for mathematics teaching and learning. These include research on teaching, curriculum and its use, mathematics as a discipline, sociology, ethnic studies, developmental psychology, cognitive science, education policy, psychometrics, and teacher education. No single field can provide all the knowledge needed to inform mathematics teaching and learning.

2. Skillful teaching that supports student learning and development makes use of instructional practices that support the building of five strands of mathematical proficiency: adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.

3. It is important to attend to and use students' cultural and community knowledge to connect their mathematical development meaningfully with resources they have. Three interlocking domains of experience and resources are vital to authentic connections: content, practices, and language.

4. Teachers are crucial for students' development; investing in their professional learning through curriculum-based professional development, ongoing opportunities to develop as mathematical knowers, and job-embedded professional learning that focus on mathematics instruction is crucial. These opportunities should be tailored to teachers' career stage.

Reflect & Analyze:

1. Is there research you have found particularly useful or compelling and that has informed your work? Is there research that you are curious to learn more about?

2. Are there questions you have or challenges you face that you wonder whether there is research that could provide insight or guidance?

Key References

Baker-Bell, A. (2020). *Linguistic justice: Black language, literacy, identity, and pedagogy.* Routledge.

Csicsery, G. (Director). (2024). *Journeys of black mathematicians: Forging resilience* [Film]. Zala Films and SLMath.

Csicsery, G. (Director). (2025). *Journeys of black mathematicians: Creating pathways* [Film]. Zala Films and SLMath.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* The National Academies Press.

Nasir, N. S., Lee, C. D., Pea, R., & de Royston, M. M. (2020). *Handbook of the cultural foundations of learning*. Routledge.

New York State Education Department. (2019). *New York State Next Generation Mathematics Learning Standards.* New York State Education Department.

Paris, D. (2012). Culturally sustaining pedagogy: A needed change in stance, terminology, and practice. *Educational Researcher, 41*(3), 91–97.

Paris, D., & Alim, H. S. (2017). *Culturally sustaining pedagogies: Teaching and learning for justice in a changing world.* Teachers College Press.

Zavala, M., & Aguirre, J. (2024). *Cultivating mathematical hearts: Culturally responsive mathematics teaching in elementary classrooms.* Corwin Press.