Aligning Local Curricula: Understanding the Guide for Aligning Local Curricula to the Next Generation Mathematics Standards (2017)

NYSED Office of Curriculum & Instruction

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Aligning Local Curricula
The Goal and Objectives

Goal:

- To facilitate the use of the guide titled, "*Aligning Local Curricula to the Next Generation Mathematics Learning Standards (2017)*", a resource created by a committee of New York State educators, to support school districts and teachers in their process of aligning new and existing curricula to the Next Generation Mathematics Learning Standards.

Objectives:

- Reflect upon a process for aligning standards-based curricula;
- Facilitate conversations about curricular decisions;
- Introduce important types of revisions;
- Share curricular resources for implementing the Next Generation Mathematics Learning Standards

NYS Next Generation Implementation Timeline

Where Does Curriculum Alignment Fit in?

September 2017:
Adoption of Next Generation Learning Standards

THE ROADMAP AND IMPLEMENTATION TIMELINE

Phase I

Raise Awareness

(Winter 2018 – Winter/Spring 2019)

- Professional development on NYS Next Generation Learning Standards
- Two-day assessments measuring the 2011 P-12 Learning Standards

Phase II

Build Capacity

(Spring 2019 - Summer 2021)

- Professional development continuing on NYS Next Generation Learning Standards
- Two-day assessments measuring the 2011 P-12 Learning Standards

Phase III

Full Implementation

(September 2021 - ongoing)

- Full implementation of the NYS Next Generation Learning Standards

Spring 2022:

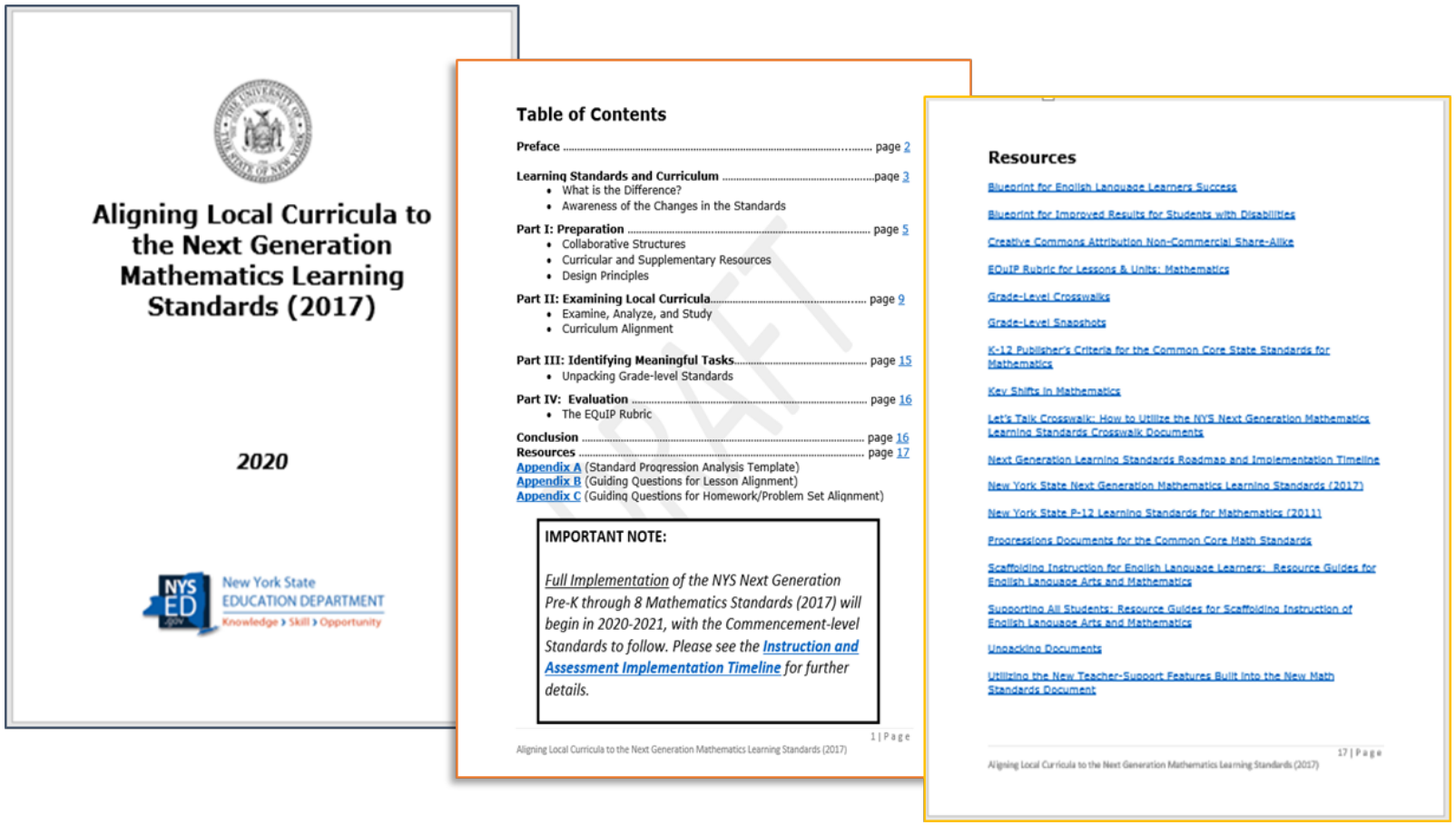
New Grades 3-8 tests measuring the NYS Next Generation Learning Standards.

Algebra I Regents aligned in June 2023

A Guide for Aligning Local Curricula to the Next Generation Mathematics Learning Standards

Curriculum decisions are **locally determined** and this alignment guide is an **optional resource** for school districts to utilize.

This document is designed to assist New York State school districts in the curriculum alignment process so **educators** are **empowered** to do this work.



Learning Standards and Curriculum

What is the difference? Turn and Talk …

Turn and Talk…

With your partner/group, consider the following:

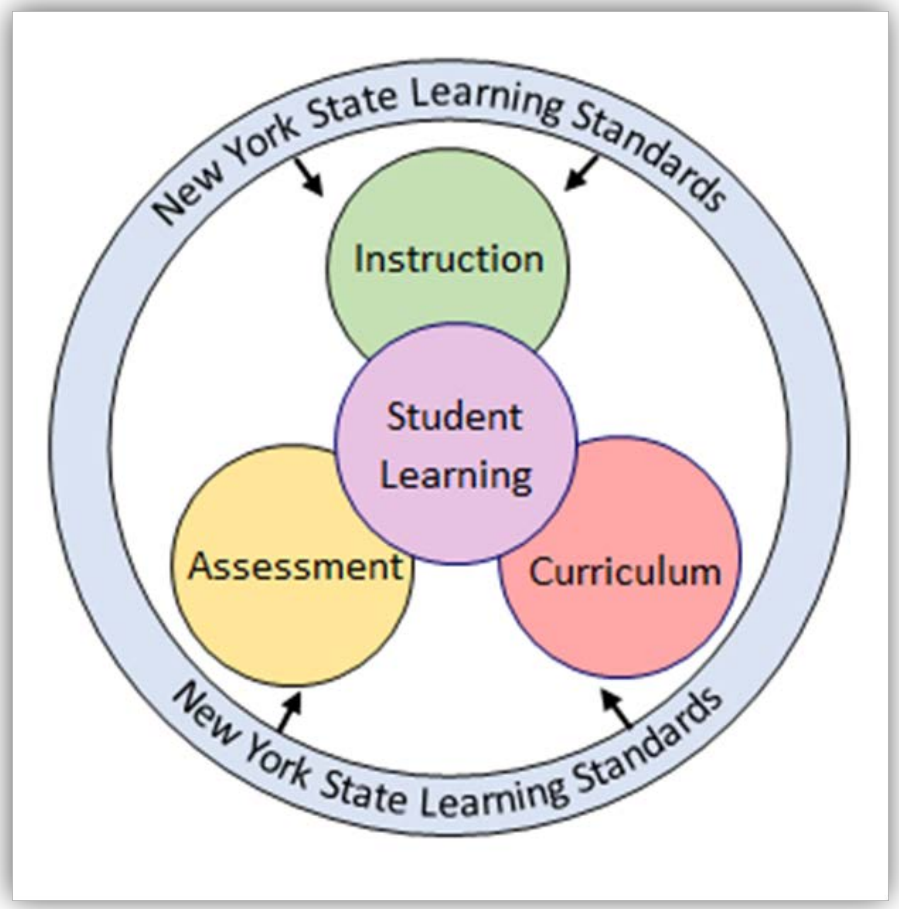
- Come up with a visual that best represents the relationship between the terms standards, curriculum, assessment, and instruction and the impact those terms have on student learning.

Group Share Out…

- What are the common themes displayed in the visuals?
- How do those themes shape curriculum development?

Learning Standards and Curriculum

What is the difference?



- **Standards:** The **knowledge, skills and understanding** that individuals can and do habitually over time because of instruction and learning experiences.
- **Curriculum:** Weaves the learning standards into **a story that builds the capacity of the learner** to access and apply what has been learned.
  - **Curriculum resources:** The materials a district chooses after defining their local curriculum. Well-chosen resources support teachers' delivery of developmentally appropriate, standards-aligned, and culturally-responsive instruction to all students.
- **Instruction:** **Effective teaching** that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.[1]
- **Assessment:** As with any set of standards, they need to be rigorous; they need to demand a balance of conceptual understanding, procedural fluency and application and represent a significant level of **achievement** in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

1. Principles to Actions: Ensuring Mathematical Success for all, NCTM

A Note on the EngageNY Mathematics Curriculum Modules – A Curricular Resource

Key messages:

- The EngageNY Mathematics modules will continue to be a free resource available for educator use. However, NYSED will not be updating the modules to align with the New York State (NYS) Next Generation Mathematics Learning Standards.

- Since the modules are free and an open-source, a school district may adapt the lessons to ensure they align with the Next Generation Mathematics Learning Standards and meet the needs of their local school district.

Awareness of the Changes in the Standards
The Results of Standards Review: What happened?

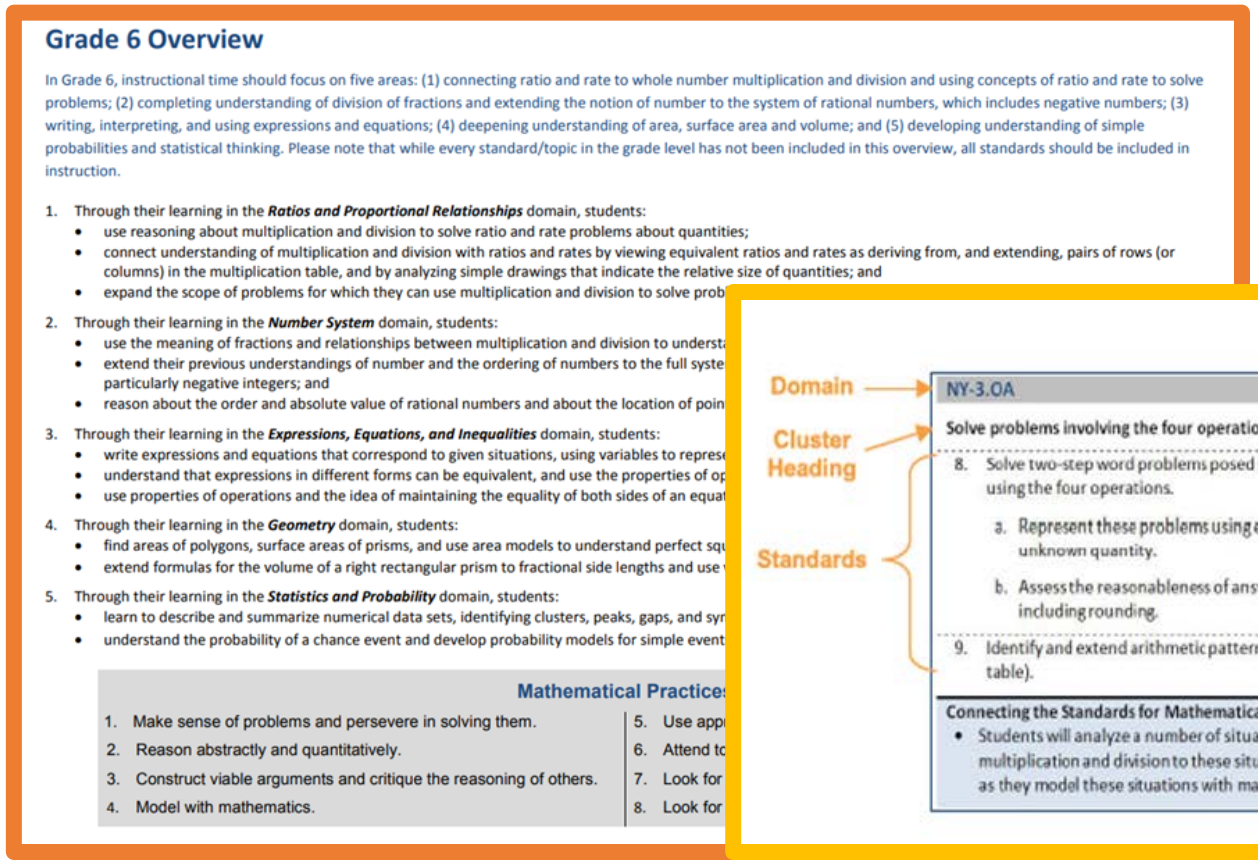
- **Movement of Standards** to different grade levels to improve the focus of major content and skills for each grade-level and course; providing more time for students to develop deep levels of understanding of grade-level appropriate content;
- **Clarification of Standards** to make expectations more clearly defined, without limiting instructional flexibility;
- **Addition and Consolidation** of Standards to improve coherence, focus and reduce redundancy amongst grade levels;
- **Maintain the Rigor** of the Standards by improving the balance of conceptual understanding, procedural skill and application;
- Provide opportunities for students to **Explore** certain standards to ensure that the standards are grade-level appropriate. Exploring a standard allows a student to be introduced to and learn a concept without the expectation of mastering the concept at that grade level.

Awareness of the Changes in the Standards

Where to Go? The Standards Document

[NYS Next Generation Mathematics Learning Standards Document]

[Next Generation Mathematics Teacher Support Features Toolkit]



Grade 6 Overview

In Grade 6, instructional time should focus on five areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; (4) deepening understanding of area, surface area and volume; and (5) developing understanding of simple probabilities and statistical thinking. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Ratios and Proportional Relationships*** domain, students:
   - use reasoning about multiplication and division to solve ratio and rate problems about quantities;
   - connect understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities; and
   - expand the scope of problems for which they can use multiplication and division to solve prob
2. Through their learning in the ***Number System*** domain, students:
   - use the meaning of fractions and relationships between multiplication and division to understa
   - extend their previous understandings of number and the ordering of numbers to the full syste particularly negative integers; and
   - reason about the order and absolute value of rational numbers and about the location of poin
3. Through their learning in the ***Expressions, Equations, and Inequalities*** domain, students:
   - write expressions and equations that correspond to given situations, using variables to repres
   - understand that expressions in different forms can be equivalent, and use the properties of op
   - use properties of operations and the idea of maintaining the equality of both sides of an equat
4. Through their learning in the ***Geometry*** domain, students:
   - find areas of polygons, surface areas of prisms, and use area models to understand perfect squ
   - extend formulas for the volume of a right rectangular prism to fractional side lengths and use
5. Through their learning in the ***Statistics and Probability*** domain, students:
   - learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and syr
   - understand the probability of a chance event and develop probability models for simple event

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use app
6. Attend to
7. Look for
8. Look for

Prekindergarten throu

Domain → NY-3.OA Operations and Algeb

Cluster Heading → Solve problems involving the four operations, and identify and extend patterns in arithmeti

Standards →
8. Solve two-step word problems posed with whole numbers and having whole-number an using the four operations.
   a. Represent these problems using equations or expressions with a letter standing for unknown quantity.
   b. Assess the reasonableness of answers using mental computation and estimation str including rounding.
9. Identify and extend arithmetic patterns (including patterns in the addition table or multiplication table).

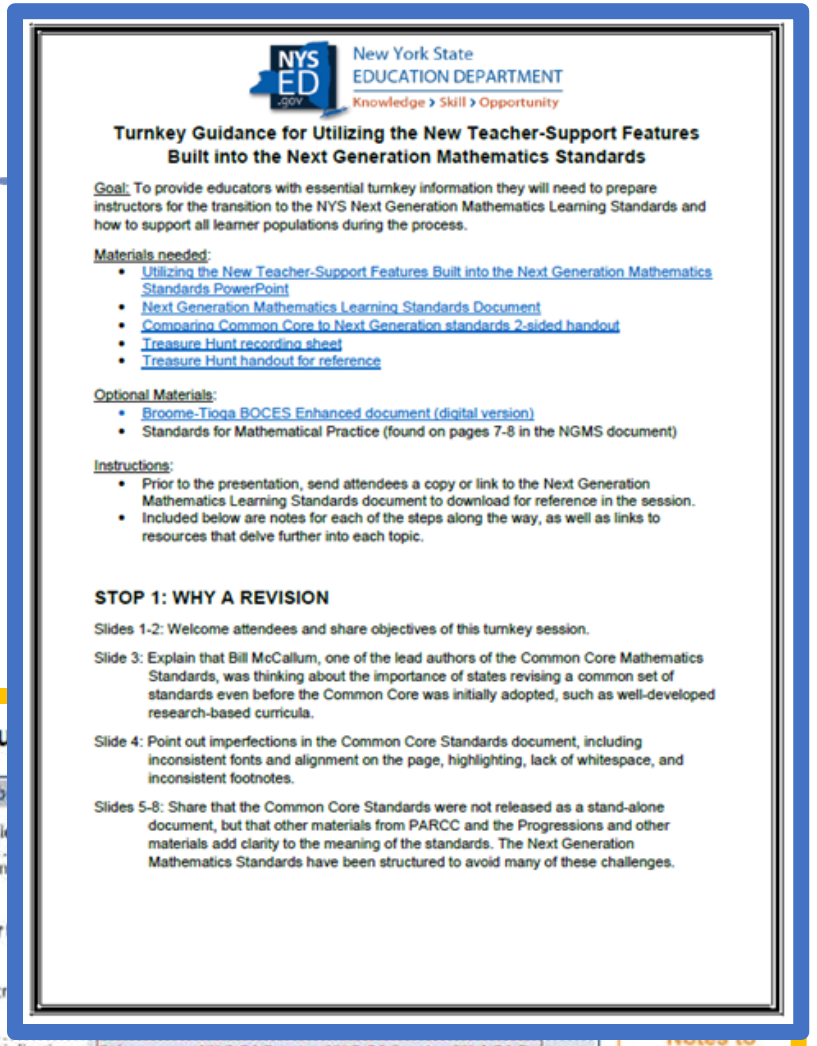
Coherence: NY-2.OA.3 → NY-3.OA.9 → NY-4.OA.5

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (MP.1), look for and make use of structure (MP.7) as they model these situations with mathematical forms (MP.4), and attend to precision (MP.6) as they distinguish different kinds of situations over time (MP.8).[14]

Clarify & Connect Standards

Citation



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Turnkey Guidance for Utilizing the New Teacher-Support Features Built into the Next Generation Mathematics Standards

Goal: To provide educators with essential turnkey information they will need to prepare instructors for the transition to the NYS Next Generation Mathematics Learning Standards and how to support all learner populations during the process.

Materials needed:
- Utilizing the New Teacher-Support Features Built into the Next Generation Mathematics Standards PowerPoint
- Next Generation Mathematics Learning Standards Document
- Comparing Common Core to Next Generation standards 2-sided handout
- Treasure Hunt recording sheet
- Treasure Hunt handout for reference

Optional Materials:
- Broome-Tioga BOCES Enhanced document (digital version)
- Standards for Mathematical Practice (found on pages 7-8 in the NGMS document)

Instructions:
- Prior to the presentation, send attendees a copy or link to the Next Generation Mathematics Learning Standards document to download for reference in the session.
- Included below are notes for each of the steps along the way, as well as links to resources that delve further into each topic.

STOP 1: WHY A REVISION

Slides 1-2: Welcome attendees and share objectives of this turnkey session.

Slide 3: Explain that Bill McCallum, one of the lead authors of the Common Core Mathematics Standards, was thinking about the importance of states revising a common set of standards even before the Common Core was initially adopted, such as well-developed research-based curricula.

Slide 4: Point out imperfections in the Common Core Standards document, including inconsistent fonts and alignment on the page, highlighting, lack of whitespace, and inconsistent footnotes.

Slides 5-8: Share that the Common Core Standards were not released as a stand-alone document, but that other materials from PARCC and the Progressions and other materials add clarity to the meaning of the standards. The Next Generation Mathematics Standards have been structured to avoid many of these challenges.

Awareness of the Changes in the Standards

Where to Go? Grade-Level Snapshots

New York State Next Generation Mathematics Learning Standards

This document is intended to help educators identify the key changes that have occurred to the content standards for this grade level/course and to assist with designing curriculum and lessons aligned to the NYS Next Generation Mathematics Learning Standards. This document does not contain the comprehensive list of learning standards for the grade level/course. The complete list of standards for the grade level/course can be found at NYS Next Generation Mathematics Learning Standards.

Grade 7 Snapshot

Standards New to Grade 7

NY-7.SP.1 Construct and interpret box-plots, find the interquartile range, and determine if a data point is an outlier (Box-plots are no longer introduced in grade 6). Students are not expected to construct box-plots that include outliers in the data, but they are expected to interpret box-plots that may contain outliers.

Standards Moved from Grade 7

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. Moved to grade 6 (NY-6.SP.1b).
7.SP.2 Moved generate multiple samples to grade 6 (NY-6.SP.1c). Merged "use data from a random sample to draw inferences about a population with an unknown characteristic of interest" into standard NY-7.SP.4.
7.SP.5-7b Probability of simple events; these standards were moved to grade 6 (NY-6.SP.6, 7, 8, 8a, 8b).

Highlights/Instructional Considerations

NY-7.RP.1 Problems may include ratios of lengths, areas, and other quantities measured in like or different units, including across measurement systems.
NY-7.RP.2a Students may utilize a strategy of their choice when deciding whether two quantities are in a proportional relationship.
NY-7.RP.3 Percent problems include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.
NY-7.NS.3 Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
NY-7.EE.4 Solving equations that contain variables on both sides is not an expectation in grade 7.
NY-7.EE.4a *Leading to* may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard.
NY-7.EE.4b Added $px + q \geq r$ and $px + q \leq r$; *Leading to* may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard.
NY-7.G.2 Draw triangles when given measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
NY-7.G.3 Plane sections are parallel or perpendicular to the base of right rectangular prisms and right rectangular pyramids.
NY-7.G.4 Students are applying the formulas for the area and circumference of a circle to solve problems, no informal derivation of relationship between circumference and area of a circle. Students are not expected to calculate the radius of a circle given its area.
NY-7.G.5 Solving equations for an unknown angle in a figure will involve linear expressions on one side of the equation. Solving equations that contain variables on both sides is not an expectation in grade 7.
NY-7.G.6 Quadrilaterals is replaced with trapezoids and the inclusive definition of trapezoid will be utilized, which implies parallelograms are included; surface area problems involve right prisms and right pyramids composed of triangles and trapezoids; volume problems involve right triangular prisms and right rectangular prisms; right prisms include cubes.
NY-7.SP.3 Students do not need to measure the difference between the centers by expressing it as a multiple of a measure of variability, they are informally assessing the degree of visual overlap of two quantitative data distributions.
NY-7.SP.4 Measures of center are mean, median, and mode. The measures of variation include range and the interquartile range.

Grade-Level Snapshots

Standards New to Grade

Standards Moved from Grade

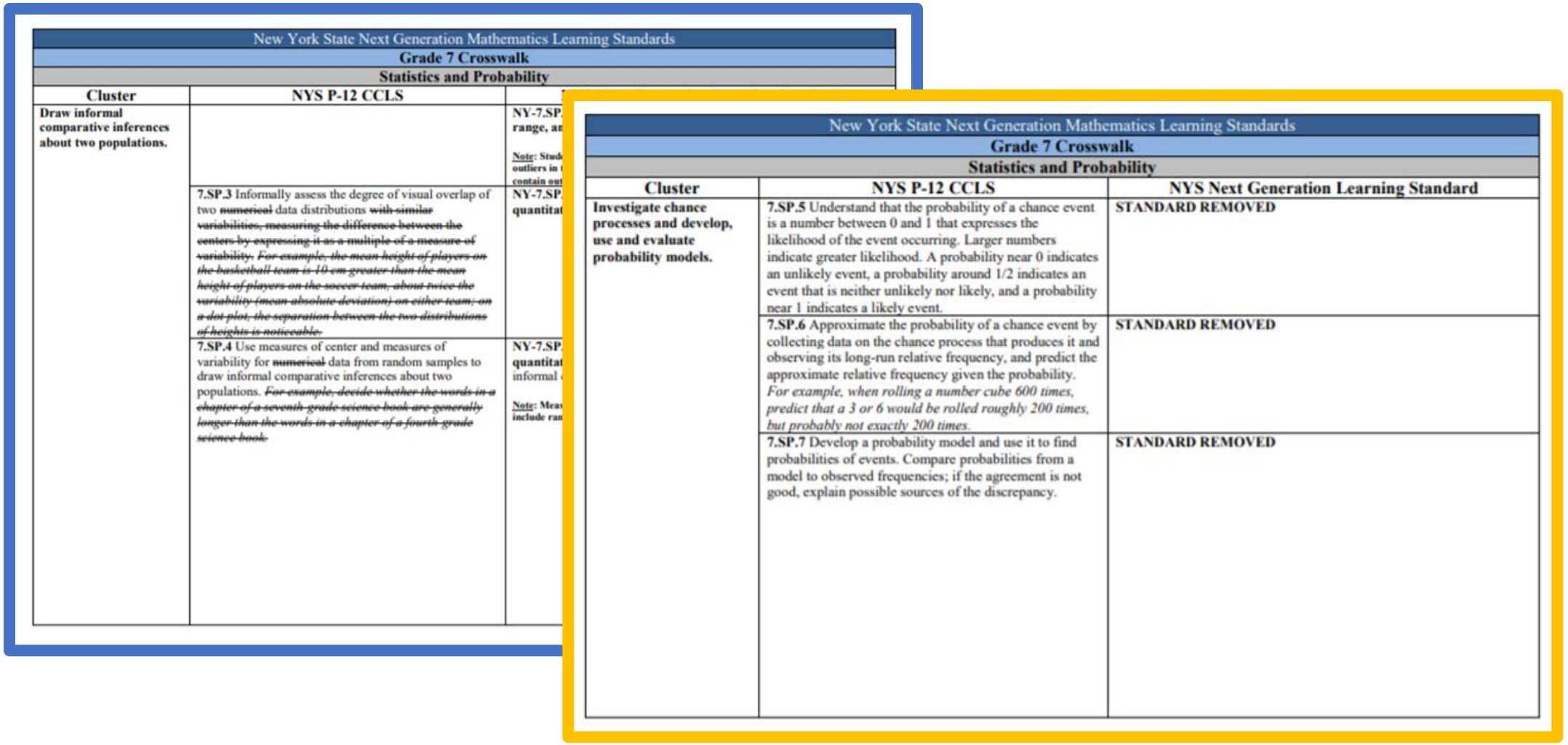
Highlights/ Instructional Considerations

Awareness of the Changes in the Standards

Where to Go? Grade-level Crosswalks

Crosswalk

Two-column Side by Side



New York State Next Generation Mathematics Learning Standards

Grade 7 Crosswalk

Statistics and Probability

Cluster	NYS P-12 CCLS	NY-7.SP
Draw informal comparative inferences about two populations.		**NY-7.SP range, ar** <u>Note</u>: Stud outliers in contain out
	7.SP.3 Informally assess the degree of visual overlap of two ~~numerical~~ data distributions ~~with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*~~	**NY-7.SP quantitat**
	7.SP.4 Use measures of center and measures of variability for ~~numerical~~ data from random samples to draw informal comparative inferences about two populations. ~~*For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*~~	**NY-7.SP quantitat** informal <u>Note</u>: Mea include ra

New York State Next Generation Mathematics Learning Standards

Grade 7 Crosswalk

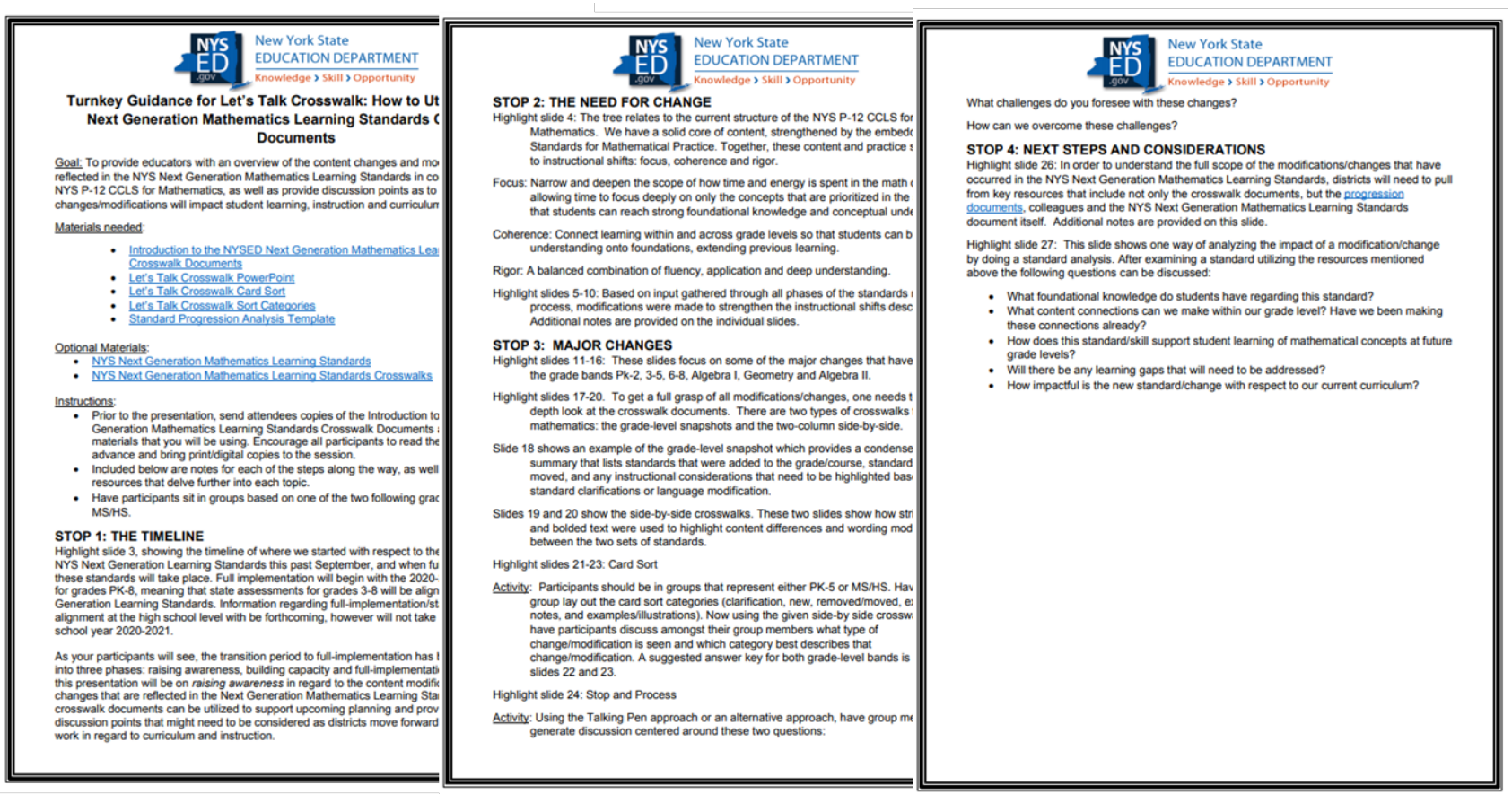
Statistics and Probability

Cluster	NYS P-12 CCLS	NYS Next Generation Learning Standard
Investigate chance processes and develop, use and evaluate probability models.	**7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	**STANDARD REMOVED**
	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	**STANDARD REMOVED**
	7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	**STANDARD REMOVED**

Awareness of the Changes in the Standards

Where to Go? The Toolkit

Crosswalk Toolkit



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Turnkey Guidance for Let's Talk Crosswalk: How to Ut Next Generation Mathematics Learning Standards (Documents

Goal: To provide educators with an overview of the content changes and mo reflected in the NYS Next Generation Mathematics Learning Standards in co NYS P-12 CCLS for Mathematics, as well as provide discussion points as to changes/modifications will impact student learning, instruction and curriculur

Materials needed:

- Introduction to the NYSED Next Generation Mathematics Lea Crosswalk Documents
- Let's Talk Crosswalk PowerPoint
- Let's Talk Crosswalk Card Sort
- Let's Talk Crosswalk Sort Categories
- Standard Progression Analysis Template

Optional Materials:

- NYS Next Generation Mathematics Learning Standards
- NYS Next Generation Mathematics Learning Standards Crosswalks

Instructions:

- Prior to the presentation, send attendees copies of the Introduction to Generation Mathematics Learning Standards Crosswalk Documents materials that you will be using. Encourage all participants to read the advance and bring print/digital copies to the session.
- Included below are notes for each of the steps along the way, as well resources that delve further into each topic.
- Have participants sit in groups based on one of the two following grac MS/HS.

STOP 1: THE TIMELINE

Highlight slide 3, showing the timeline of where we started with respect to the NYS Next Generation Learning Standards this past September, and when fu these standards will take place. Full implementation will begin with the 2020- for grades PK-8, meaning that state assessments for grades 3-8 will be align Generation Learning Standards. Information regarding full-implementation/st alignment at the high school level with be forthcoming, however will not take school year 2020-2021.

As your participants will see, the transition period to full-implementation has into three phases: raising awareness, building capacity and full-implementati this presentation will be on *raising awareness* in regard to the content modific changes that are reflected in the Next Generation Mathematics Learning Sta crosswalk documents can be utilized to support upcoming planning and prov discussion points that might need to be considered as districts move forward work in regard to curriculum and instruction.

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STOP 2: THE NEED FOR CHANGE

Highlight slide 4: The tree relates to the current structure of the NYS P-12 CCLS for Mathematics. We have a solid core of content, strengthened by the embedd Standards for Mathematical Practice. Together, these content and practice to instructional shifts: focus, coherence and rigor.

Focus: Narrow and deepen the scope of how time and energy is spent in the math allowing time to focus deeply on only the concepts that are prioritized in the that students can reach strong foundational knowledge and conceptual und

Coherence: Connect learning within and across grade levels so that students can b understanding onto foundations, extending previous learning.

Rigor: A balanced combination of fluency, application and deep understanding.

Highlight slides 5-10: Based on input gathered through all phases of the standards process, modifications were made to strengthen the instructional shifts desc Additional notes are provided on the individual slides.

STOP 3: MAJOR CHANGES

Highlight slides 11-16: These slides focus on some of the major changes that have the grade bands Pk-2, 3-5, 6-8, Algebra I, Geometry and Algebra II.

Highlight slides 17-20. To get a full grasp of all modifications/changes, one needs t depth look at the crosswalk documents. There are two types of crosswalks mathematics: the grade-level snapshots and the two-column side-by-side.

Slide 18 shows an example of the grade-level snapshot which provides a condense summary that lists standards that were added to the grade/course, standard moved, and any instructional considerations that need to be highlighted bas standard clarifications or language modification.

Slides 19 and 20 show the side-by-side crosswalks. These two slides show how str and bolded text were used to highlight content differences and wording mod between the two sets of standards.

Highlight slides 21-23: Card Sort

Activity: Participants should be in groups that represent either PK-5 or MS/HS. Hav group lay out the card sort categories (clarification, new, removed/moved, e notes, and examples/illustrations). Now using the given side-by side crossw have participants discuss amongst their group members what type of change/modification is seen and which category best describes that change/modification. A suggested answer key for both grade-level bands is slides 22 and 23.

Highlight slide 24: Stop and Process

Activity: Using the Talking Pen approach or an alternative approach, have group me generate discussion centered around these two questions:

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What challenges do you foresee with these changes?

How can we overcome these challenges?

STOP 4: NEXT STEPS AND CONSIDERATIONS

Highlight slide 26: In order to understand the full scope of the modifications/changes that have occurred in the NYS Next Generation Mathematics Learning Standards, districts will need to pull from key resources that include not only the crosswalk documents, but the progression documents, colleagues and the NYS Next Generation Mathematics Learning Standards document itself. Additional notes are provided on this slide.

Highlight slide 27: This slide shows one way of analyzing the impact of a modification/change by doing a standard analysis. After examining a standard utilizing the resources mentioned above the following questions can be discussed:

- What foundational knowledge do students have regarding this standard?
- What content connections can we make within our grade level? Have we been making these connections already?
- How does this standard/skill support student learning of mathematical concepts at future grade levels?
- Will there be any learning gaps that will need to be addressed?
- How impactful is the new standard/change with respect to our current curriculum?

[Let's Talk Crosswalk] and [Next Gen Crosswalk Toolkit]

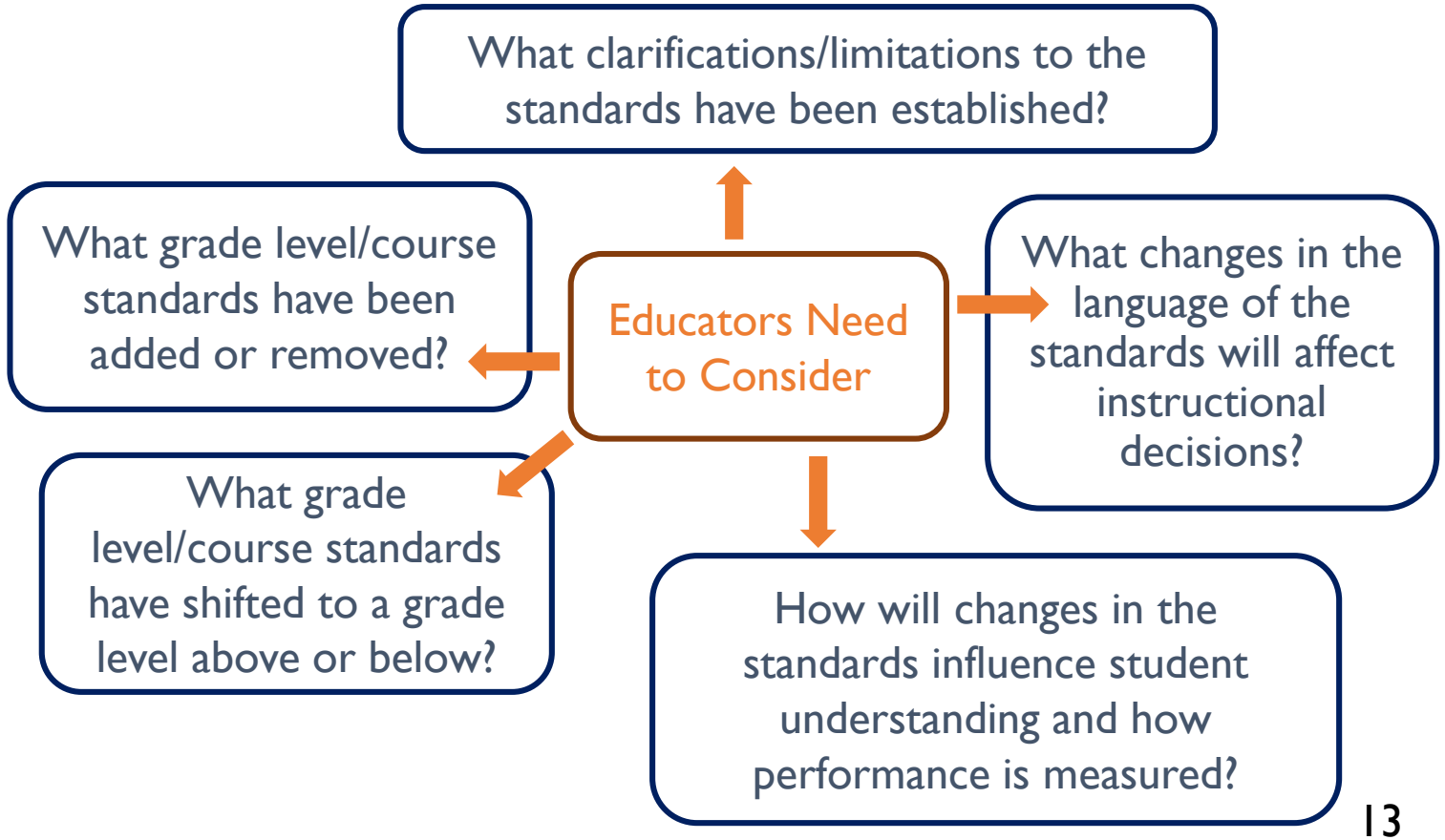
Part I: Preparation
Collaborative Structures

Turn and Talk…

With your partner, consider the following:

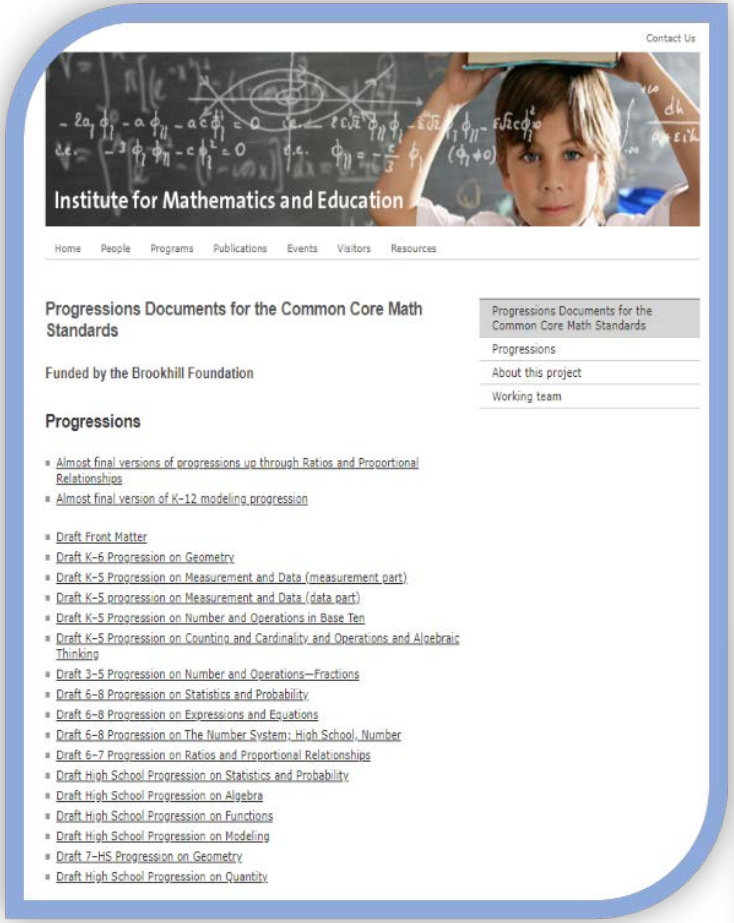
- What are the "Big" changes that have occurred within your grade-level/grade-level band or course that will have the greatest impact on curriculum development?
- What collaborative structures does your district already have in place to support curriculum development?
- What challenges exist?
- What components need to be a part of this process in order to make sure that the curriculum supports all students?

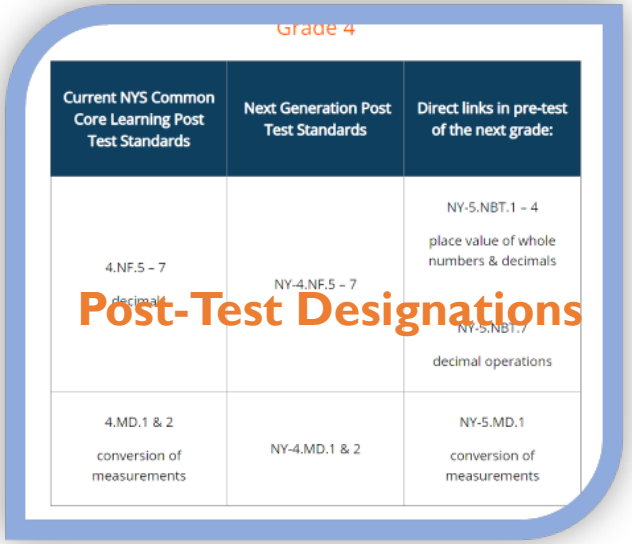
Curriculum Review and **Development** is a process that involves continuous ongoing collaboration and conversation.

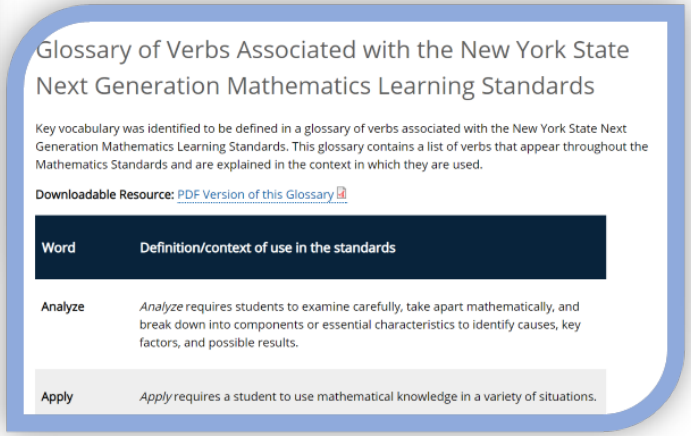


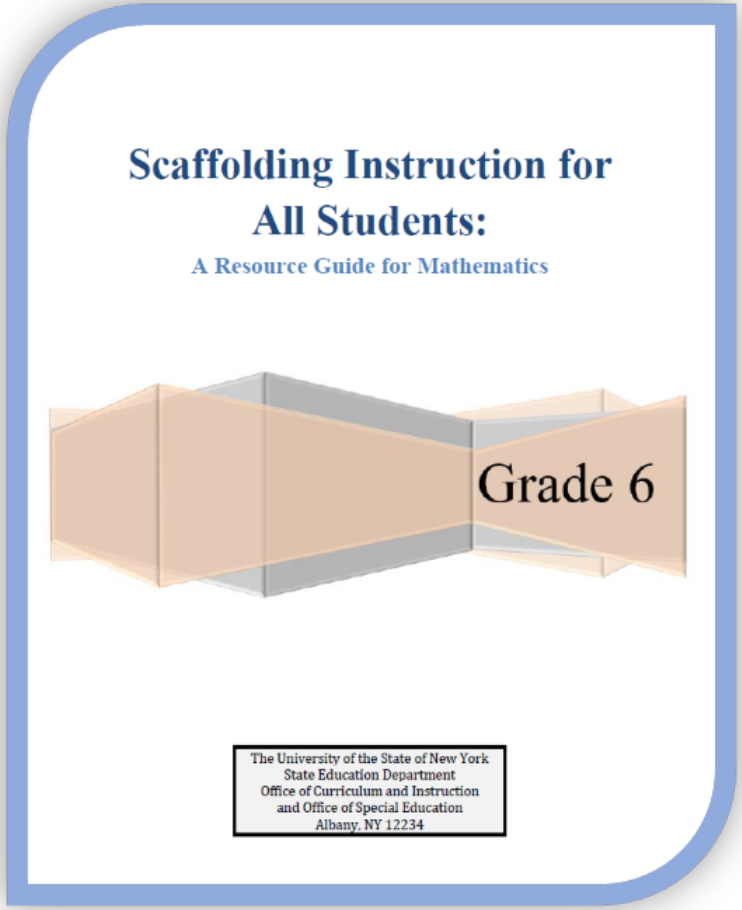
Part I: Preparation

Additional Curricular and Supplementary Resources You Should Know About





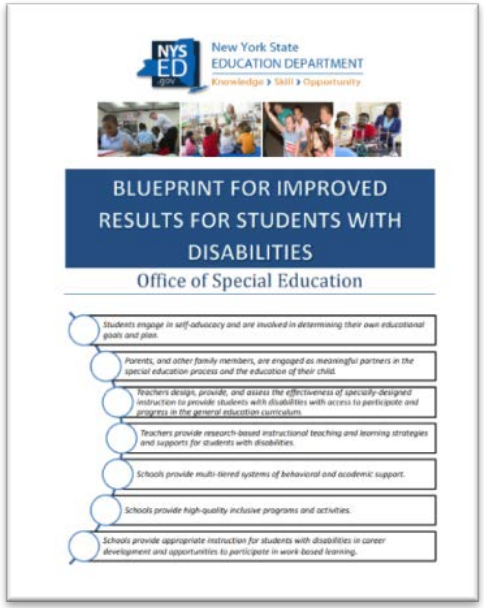




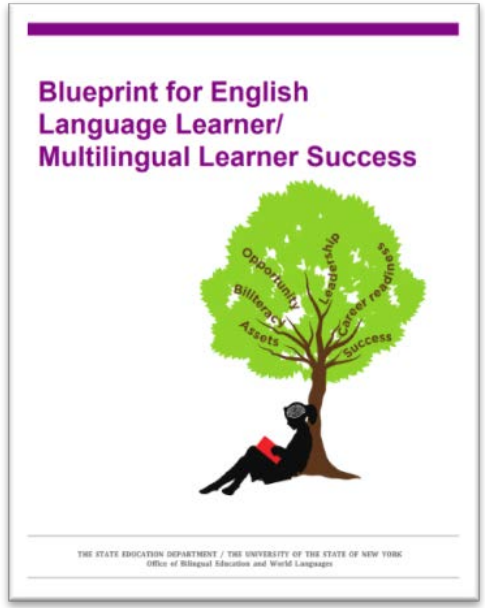
Part I: Preparation
More Additional Curricular and Supplementary Resources You Should Know About

- "No set of grade-specific standards can fully reflect the variation in learning profiles, rates, needs, linguistic backgrounds, and achievement levels of students in any given classroom. When designing and delivering mathematics instruction, educators must consider the cultural context and preceding academic experiences of all students while connecting prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible."

Culturally Responsive Sustaining Education Framework



Blueprint for Improved Results for Students with Disabilities



Blueprint for English Language Learner Success

Part I: Preparation
Additional Curricular and Supplementary Resources

A list of curricular resources is never exhaustive!

Turn and Talk…

With your partner/group, discuss the following:

- What additional resources does your district utilize for mathematics curriculum development?

Part I: Preparation
Design Principles for Standards Alignment

The Foundations for Effective Mathematics Curricula and Instruction

Focus is the emphasis on the major concepts within a domain.

Coherence refers to the progression of mathematics within and across the grade levels.

Rigor refers to ensuring the proper balance of conceptual understanding (C-R-A), procedural fluency and application.

Part II: Examining current Local Curricula

What to Do When A Standard…

- is completely **NEW** to the grade level?
- has **CLARIFICATIONS ADDED** or has been split up into sub-standards?
- has been **MOVED/REMOVED?**
- has **NEW NOTES** added?
- has **NEW SUPPORTING EXAMPLES** and/or **ILLUSTRATIONS?**
- is an **EXPLORE** standard?
- is a new and/or additional **PRE/POST-TEST** standard?

Part II: Examining Current Local Curricula

Standard Progression Analysis Tool

Looking at:

- Standard/Skills
- Within Grade Connections
- Foundational Knowledge
- Subsequent Knowledge
- Potential Gaps

STANDARD PROGRESSION ANALYSIS

Domain: Solve real-world and mathematical problems involving area, surface area and volume.

Standard: NY-6. G.5 Use area and volume models to explain perfect squares and perfect cubes.

Foundational Knowledge	Within Grade Connections	Subsequent Knowledge
Intro of the Square Unit in Grade 3 NY-3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. NY-3.MD.5a Recognize a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. NY-3.MD.5b Recognize a plane figure which can be covered without gaps or overlaps by *n* unit square units. NY-3.MD.7 Relate area to the operations of multiplication and addition. **Intro to the Cube unit in Grade 5** NY-5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. NY-5.MD.3a Recognize that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. NY-5.MD.3b Recognize that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of n cubic units. NY-5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units. NY-5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. NY-5.MD.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. **Intro to Exponents and Powers of 10** NY-5.NBT.2 Use whole-number exponents to denote powers of 10. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	NY-6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. NY-6. G.2 Find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**Intro to Irrational Numbers in Grade 7/8** NY-7.NS.2d Convert a fraction to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. NY-8.NS.1 Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion eventually repeats. Know that other numbers that are not rational are called irrational. NY-8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know square roots of perfect squares up to 225 and know that the square root of a non-perfect square is irrational. cube roots of perfect cubes up to 125. **Operations with Radicals and Completing the Square in Algebra I** AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers. AI-N.RN.3a Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots. AI-A.REI.4 Solve quadratic equations in one variable. Note: Solutions may include simplifying radicals. AI-A.REI.4b Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
Skills (Verbs)		**Areas of Concern/Potential Gaps:**
Recognize the shapes of squares and cubes. **Build/compose** models of squares and cubes from unit squares (cubes). **Draw** squares/cubes. **Write** the area (volume) of a perfect square (cube) using exponent notation. **Explain** why certain whole numbers are not perfect squares (cubes).		

Part II: Examining Current Local Curricula

Examine, Analyze, and Study: Your Turn

Now, it is your turn…

Appendix A STANDARD PROGRESSION ANALYSIS

Domain:

Standard:

Foundational Knowledge	Within Grade Connections	Subsequent Knowledge
Next Generation Mathematics Learning Standards document Achieve the Core Coherence Map		Next Generation Mathematics Learning Standards document Achieve the Core Coherence Map
	Skills (Verbs)	**Areas of Concern/Potential Gaps:**
	Glossary of Verbs Associated with the NGMLS	

Part II: Examining Current Local Curricula

Examine, Analyze, and Study Activity

Table Work:

- Pick a grade-level standard that reflects one of the *seven cases* listed.
- Discuss and answer the guiding questions provided specific to your *case*, in the guide found on pages 10-14.
- Complete the progression analysis template for your grade-level standard.

When a standard:

- is completely **NEW** to the grade level?
- has **CLARIFICATIONS ADDED** or has been split up into sub-standards?
- has been **MOVED/REMOVED?**
- has **NEW NOTES** added?
- has **NEW SUPPORTING EXAMPLES** and/or **ILLUSTRATIONS?**
- is an **EXPLORE** standard?
- is a new and/or additional **PRE/POST-TEST** standard?

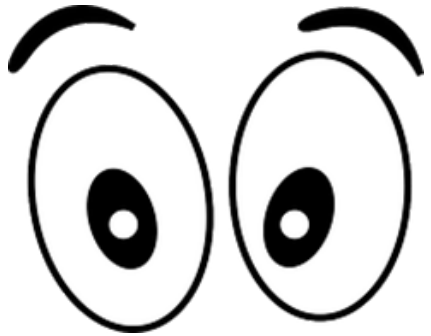
How Do You Identify Meaningful Tasks?
Notice and Wonder

Patterns—Original

The table of value below describes the perimeter of each figure in the pattern of blue tiles. The perimeter P is a function of the number of tiles t.

t	1	2	3	4
P	4	6	8	10

1. Choose a rule to describe the function in the table.
   - A. $P = t + 3$
   - B. $P = 4t$
   - C. $P = 2t + 2$
   - D. $P = 6t - 2$
2. How many tiles are in the figure if the perimeter is 20?
3. Graph the function.



What Do You Notice?

Patterns—Modified

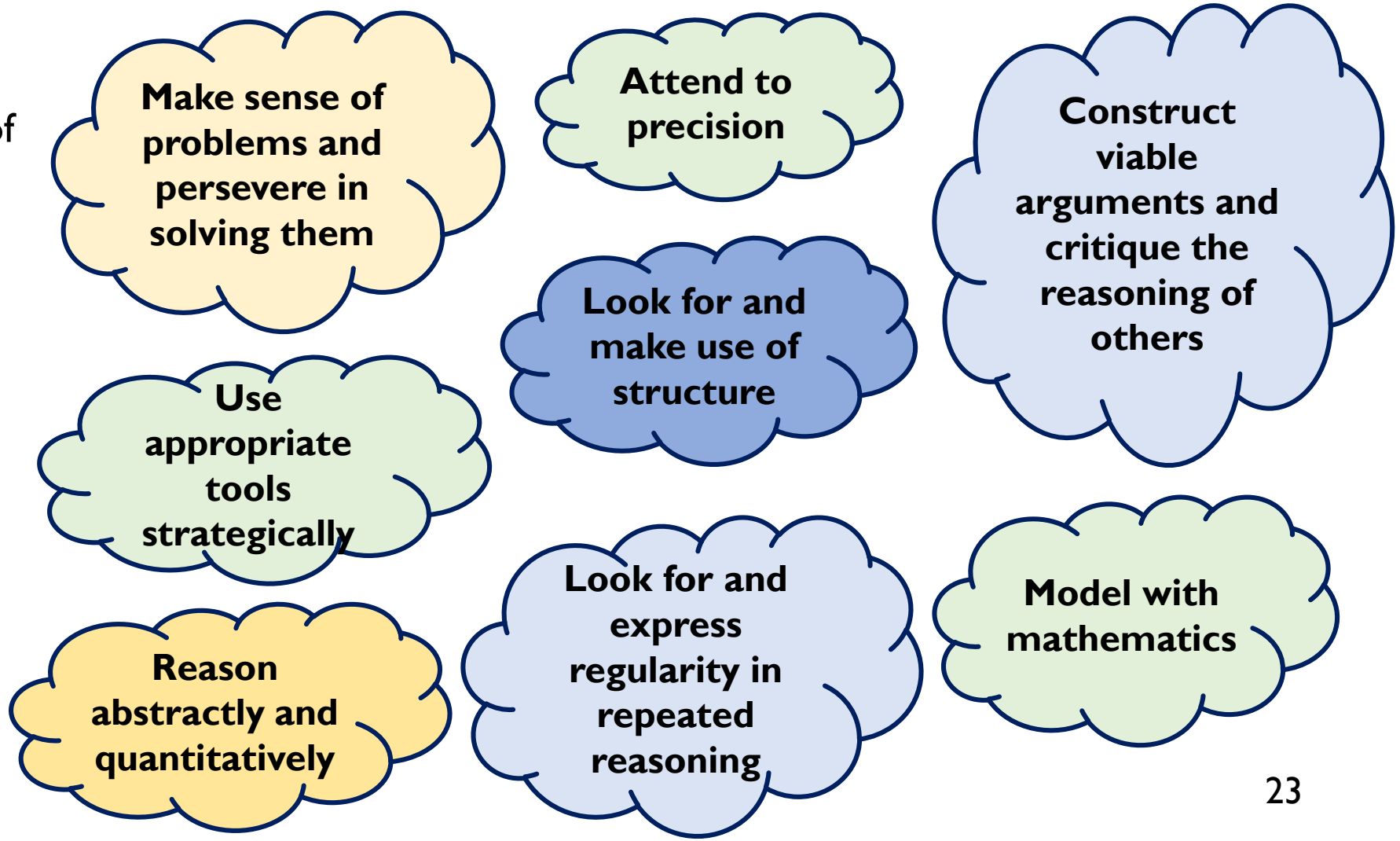
The diagram below shows the first four figures in the square pattern. The first figure has one square. For each additional figure, one new square is added.

1. Compute the perimeter of the first four figures.
2. Draw the fifth and sixth figures and compute their perimeters.
3. Describe in words how to draw the 10th figure and what the perimeter would be.
4. Describe how to find the perimeter of the 100th figure and explain how the description relates to the pattern of tiles.
5. Write a rule to relate the perimeter P to the number of tiles t. Explain how each part of your formula relates to the tile pattern.
6. Graph the function. Why does it make sense that the graph has the shape it does in relation to the pattern?

Part III: Identifying Meaningful Tasks
Standards for Mathematical Practice (SMP)

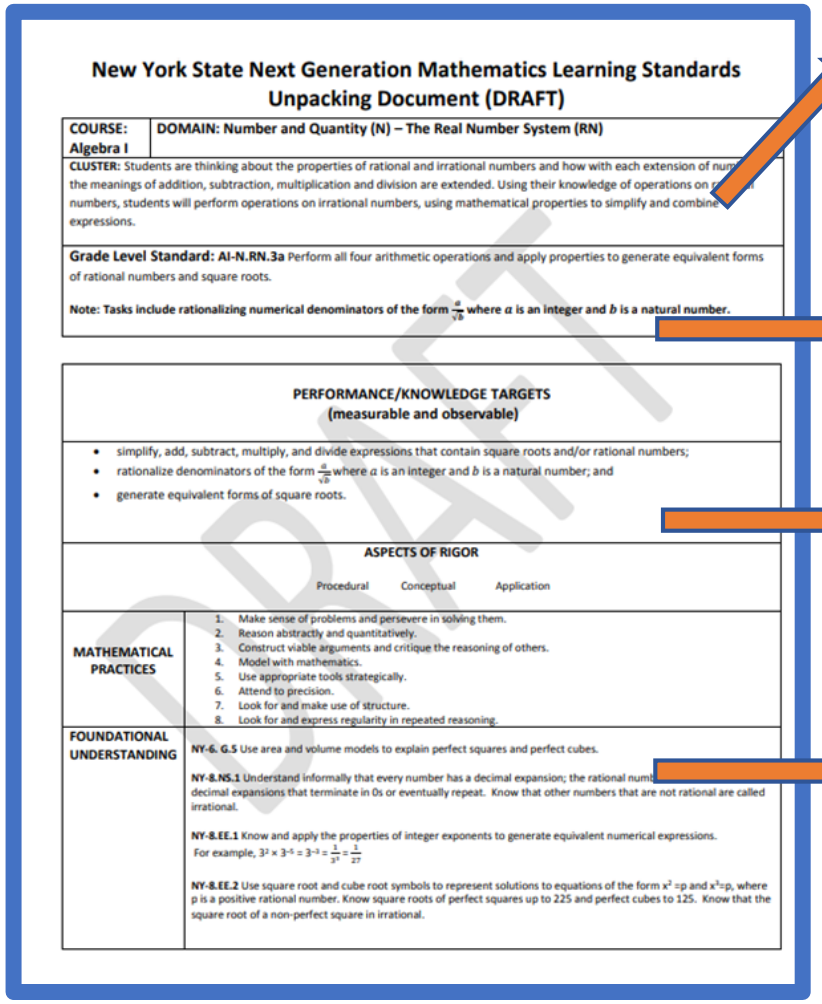
Turn and Talk...

- What is the importance of students engaging in activities that merge content with practice?
- How do you already promote the practices within your curriculum?
- How could you further promote the practices within your curriculum?
- Specific to the grade-level standard chosen, how would you engage the students in the SMP?



Part III: Identifying Meaningful Tasks

Unpacking Grade-level Standards: Page One



New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

COURSE: Algebra I	DOMAIN: Number and Quantity (N) – The Real Number System (RN)

CLUSTER: Students are thinking about the properties of rational and irrational numbers and how with each extension of num… the meanings of addition, subtraction, multiplication and division are extended. Using their knowledge of operations on r… numbers, students will perform operations on irrational numbers, using mathematical properties to simplify and combine… expressions.

Grade Level Standard: AI-N.RN.3a Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots.

Note: Tasks include rationalizing numerical denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number.

PERFORMANCE/KNOWLEDGE TARGETS (measurable and observable)

- simplify, add, subtract, multiply, and divide expressions that contain square roots and/or rational numbers;
- rationalize denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number; and
- generate equivalent forms of square roots.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-6. G.5** Use area and volume models to explain perfect squares and perfect cubes. **NY-8.NS.1** Understand informally that every number has a decimal expansion; the rational numb… decimal expansions that terminate in 0s or eventually repeat. Know that other numbers that are not rational are called irrational. **NY-8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ **NY-8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know square roots of perfect squares up to 225 and perfect cubes to 125. Know that the square root of a non-perfect square is irrational.

What is the unifying idea of the cluster? Does this unifying idea need to be modified based on standard changes that have occurred in the cluster/domain?

Grade-level Overviews

Has the standard been re-written to give a different emphasis or to be more inclusive?

Notes and Clarifications

Measurable and Observable statements that describe what success looks like when the learning goal is reached.

Coherence Links

What foundational and supporting standards have been revised, moved or eliminated? What will the effect be?

Achieve the Core Coherence Map

NYS Next Generation Mathematics Learning Standards Unpacking Documents

A Guide to Unpacking the Next Generation Mathematics Learning Standards and Template

Part III: Identifying Meaningful Tasks

Unpacking Grade-level Standards

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

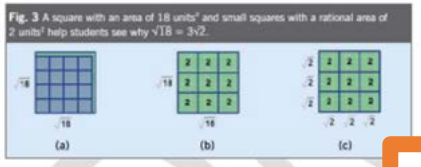
1. Equivalent forms of Rational Numbers: Lessons on infinite and finite decimal expansions of rational numbers can be found in EngageNY Grade 8 Module 7, topic B (lessons 6-14). Keep in mind that students may not have been exposed to representing a rational number expressed as a repeating decimal in fraction form. This is no longer a grade-level expectation for grade 8.

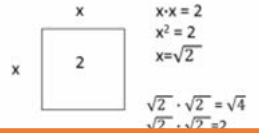
2. Simplifying Square Roots: Lessons on simplifying radicals, as well as performing operations (rationalizing denominators) can be found in EngageNY Grade 8 Module 7, lesson 4, as well as in the Geometry Module 2, lessons 22 and 23.

3. Visual/Geometric Representation for Simplifying Radicals (Taken from Radical Thoughts on Simplifying Square Roots, Kyle T. Schultz and Stephen F. Bismarck, Vol. 19, No. 4, November 2013 MATHEMATICS TEACHING IN THE MIDDLE SCHOOL). This representation connects with work done in grade 6 (NY-6. G.5) with respect to using area models to represent perfect squares.

To show the geometric simplification of $\sqrt{18}$, students use prior knowledge of perfect squares and realize that a square with an area of 18 square units cannot be partitioned into a square-shaped array. This square, nevertheless, can still be partitioned into a square-shaped array of smaller squares, provided that the area of these smaller squares is rational even if the side lengths are irrational. In this case, the square with an area of 18 square units can be partitioned into 9 squares, each with an area of 2 square units (see fig. 3 c). This new configuration provides an alternative way to calculate $\sqrt{18}$ by examining the length of the side of a square whose area is 18 square units. Notice that the side length of the large square is equivalent to three side lengths of one smaller square. The area of each small square is 2 square units, so the side length of each small square is $\sqrt{2}$ units. Thus, $\sqrt{18} = 3\sqrt{2}$. To connect this model to previous work with area models and perfect squares, the search for a perfect-square factor of 18 coincides with identifying how to partition the square into smaller squares with whole-number areas. The expression $\sqrt{9 \cdot 2}$ is equivalent to the length of the side of a square array of 9 squares each with an area of 2 square units. In this case, $\sqrt{2}$ is the length of a partitioned side of the large square into the nine small squares.

Fig. 3 A square with an area of 18 units² and small squares with a rational area of 2 units² help students see why $\sqrt{18} = 3\sqrt{2}$.





$x \cdot x = 2$

$x^2 = 2$

$x = \sqrt{2}$

$\sqrt{2} \cdot \sqrt{2} = \sqrt{4}$

Students can create visual/geometric representations for other radicals.

4. Examples of Arithmetic Operations involving Radicals:

1. $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$
2. $\sqrt{2} + 3\sqrt{2} - 7\sqrt{2} = -3\sqrt{2}$
3. $\sqrt{2} + 2\sqrt{8} = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$
4. $\sqrt{3} + 3\sqrt{3} + 5\sqrt{3} - 4\sqrt{3} = \sqrt{3} + 5\sqrt{3} + 3\sqrt{3} - 4\sqrt{3} = 6\sqrt{3}$
5. $3\sqrt{5} \cdot 4\sqrt{15} = 3 \cdot 4 \cdot \sqrt{5}\sqrt{15} = 12\sqrt{75} = 12\sqrt{25}\sqrt{3} = 12 \cdot 5\sqrt{3}$
6. $\frac{3\sqrt{14}}{2\sqrt{7}} = \frac{3}{2}\sqrt{\frac{14}{7}} = \frac{3}{2}\sqrt{2}$
7. $\frac{5}{\sqrt{6}} = \frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$
8. $\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$
9. $\frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$

What should be the integral parts of lessons surrounding a content standard ?

- Tap In
- Develop Conceptual Understanding
- Make Connections

Concrete
Representational
Abstract

RADICAL EQUATIONS Openmiddle.com

Directions: Using the digits 0-9 at most one time each, make both of these equations true.

$\sqrt{\square\square} = \square\sqrt{\square}$

$\sqrt{\square\square} = \square$

$x^2 - 4x + 4 = 13$

Compare solution strategies

Completing the Square vs. Quadratic Formula

$2 \pm \sqrt{13}$ $\qquad$ $\frac{4 \pm \sqrt{52}}{2}$

Do the methods of solution yield the same results? Examine the graph.

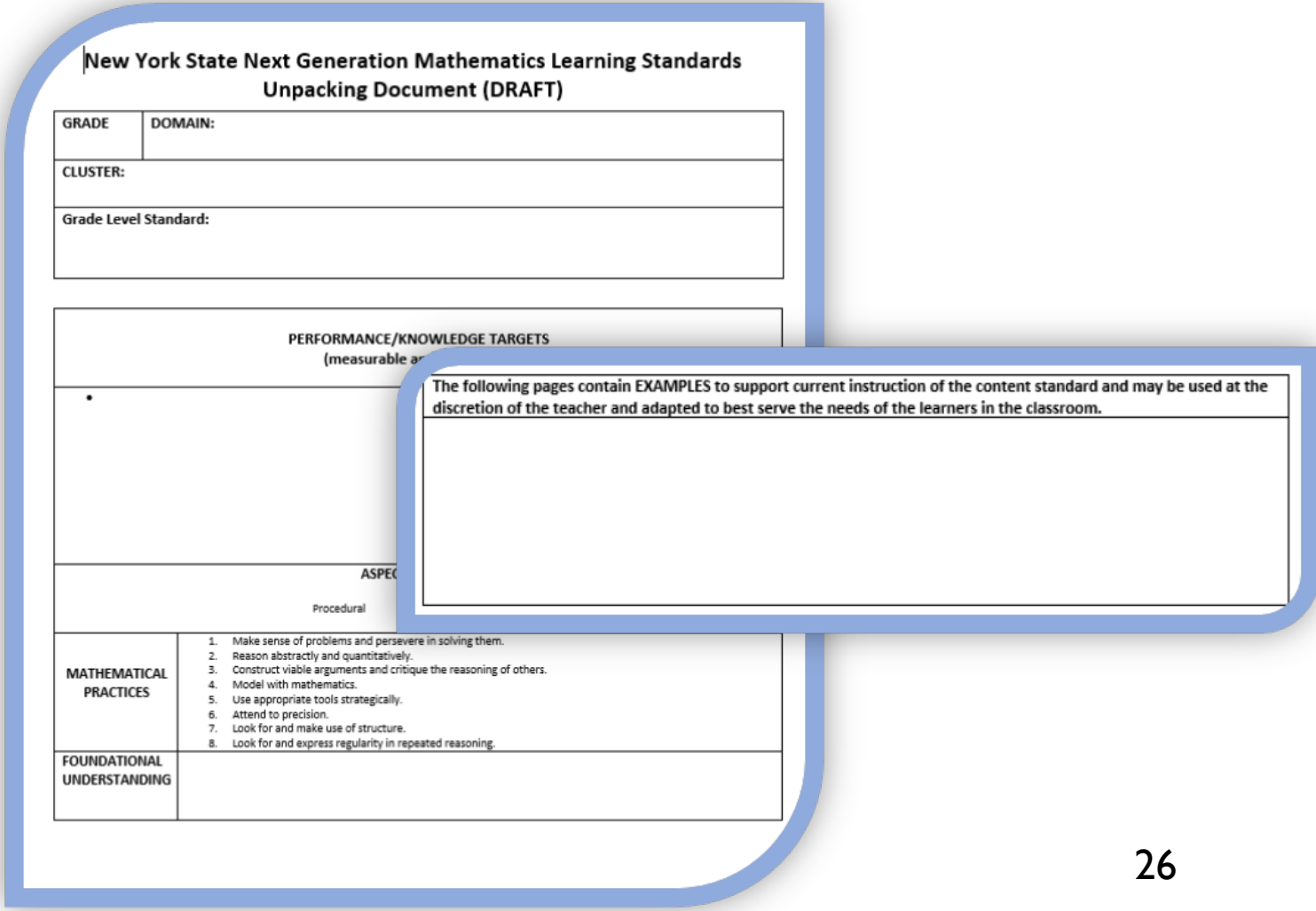
Part III: Identifying Meaningful asks

Unpacking Grade-level Standards Activity

Table Work:

- Using your standard progression analysis template as a guide, identify meaningful tasks specific to your standard.

- Complete an unpacking document template for your standard.


New York State Next Generation Mathematics Learning Standards
Unpacking Document (DRAFT)

GRADE	DOMAIN:
CLUSTER:	
Grade Level Standard:	
PERFORMANCE/KNOWLEDGE TARGETS (measurable a	
•	
ASPE	
Procedural	
MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Part IV: Evaluation – Putting it All Together
Final Thoughts – The Emphases

The clusters of the NYS CCLSM are divided into three categories recommended for instructional and assessment emphasis. These categories are **major** clusters, **supporting** clusters and **additional** clusters. Through the school year 2020-2021, NYS's grades 3-8 assessments aligned to the NYS P-12 CCLSM will reflect these content emphases.

When the NYS Next Generation Mathematics Learning Standards are implemented, which is school year 2021-2022 for grades 3-8, these content emphases will no longer be applicable. They will however, still be in effect for the High School courses until they transition over to the NYS Next Generation Mathematics Learning Standards. At that point, there will be no labels of Major, Supporting, and Additional.

Local work surrounding the prioritizing of the NYS Next Generation Mathematics Learning Standards can help educators identify and understand how collectively content standards of a grade level work together.
What are the areas of "focus" for the grade levels? Grade-level overviews found in the NYS Next Generation Mathematics Standards document would be a good discussion starting point for that work. Work centered around identifying **within-grade level connections that support the areas of focus** might be something for your educators to engage in.

Part IV: Evaluation – Putting it All Together
Final Thoughts

What are your next steps for curriculum development and alignment?

In what ways is the guide helpful for curriculum alignment?

What are some final thoughts or questions for the group?

Where can you find more information to benefit curriculum alignment?

Moving Forward

Part IV: Evaluation – Putting it All Together
Final Thoughts – A Rubric

EQuIP Rubric for Lessons & Units: Mathematics

Grade: **Mathematics Lesson/Unit Title:** **Overall Rating:**

I. Alignment to the Depth of the CCSS	II. Key Shifts in the CCSS	III. Instructional Supports	IV. Assessment
The lesson/unit aligns with the letter and spirit of the CCSS: ○ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ○ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ○ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	*The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:* ○ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades. ○ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions. ○ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following: – **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations. – **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding. – **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	*The lesson/unit is responsive to varied student learning needs:* ○ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media. ○ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline. ○ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking. ○ Addresses instructional expectations and is easy to understand and use. ○ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners. – Supports diverse cultural and linguistic backgrounds, interests and styles. – Provides extra supports for students working below grade level. – Provides extensions for students with high interest or working above grade level. *A unit or longer lesson should:* ○ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share). ○ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently. ○ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ○ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	*The lesson/unit regularly assesses whether students are mastering standards-based content and skills:* ○ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ○ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ○ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. *A unit or longer lesson should:* ○ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.
Rating: 3 2 1 0	**Rating: 3 2 1 0**	**Rating: 3 2 1 0**	**Rating: 3 2 1 0**

QUESTIONS?

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