New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 7	DOMAIN: Ratio and Proportional Reasoning

CLUSTER: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Students build upon their reasoning about ratios, rates, and unit rates to formally define proportional relationships and the constant of proportionality. Reasoning is extended about ratios and proportional relationships by computing unit rates for ratios and rates specified by rational numbers. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions. Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation.

Grade Level Standard:

NY-7.RP.2 Recognize and represent proportional relationships between quantities.
NY-7.RP.2a Decide whether two quantities are in a proportional relationship.
Note: Strategies include but are not limited to the following: testing for equivalent ratios in a table and/or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
NY-7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
NY-7.RP.2c Represent a proportional relationship using an equation.
NY-7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Analyze ratios in a table or diagram to determine if the ratios are equivalent and if possible, identify the constant of proportionality/unit rate.
- Calculate the constant of proportionality/unit rate given a verbal description of a proportional relationship.
- Graph ratios on a coordinate plane to determine if the ratios are proportional by observing if the graph is a straight line through the origin.
- Identify the constant of proportionality/unit rate given a graph of a proportional relationship.
- Using a graphical representation of a proportional relationship in context, explain the meaning of any point (x, y), including (0,0).
- Explain that the y-coordinate of the ordered pair (1, r) corresponds to the unit rate and explain its meaning in context.
- Write and explain an equation that models a proportional relationship between two quantities.
- Explain what the constant of proportionality means in the context of a given situation.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b $\neq$ 0 and use rate language in the context of a ratio relationship. **NY-6.RP.3a** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **NY-6.RP.3b** Solve unit rate problems. **NY-6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$; $x - p = q$; $px = q$; and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers. **NY-6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another. Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students work with tables, graphs and equations as frames of reference to understand the constant of proportionality, connecting their work and understanding of unit rates to constants of proportionality. Work with this standard also reinforces work with from standards in the Expressions and Equations (Inequalities) domain, specifically 7.EE.4 *Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.*

What does it mean for there to be a proportional relationship between quantities?
Proportional To (description): Measures of one type of quantity are proportional to measures of a second type of quantity if there is a number k so that for every measure x of a quantity of the first type, the corresponding measure y of a quantity of the second type is given by k; that is, $y = kx$. The number k is called the constant of proportionality.

Steps to determine if the quantities in a table are proportional to each other:

1. For each row (or column), calculate $\frac{B}{A}$, where *A* is the measure of the first quantity and *B* is the measure of the second quantity.
2. If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities in the table are proportional to each other.

Example 1: Identifying a Proportional Relationship
The following example, Pay by the Ounce Frozen Yogurt, is taken from EngageNY Grade 7 Module 1, lesson 2.

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed his dish, and this is what they found. Determine if the cost is proportional to the weight.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

Students can determine the unit rate from each of the four given scenarios and use that reasoning to justify whether there is a proportional relationship between cost and weight.

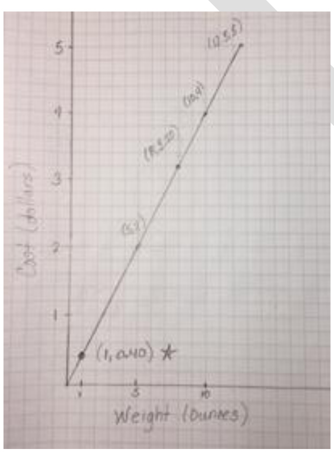
5 dollars for 12.5 ounces
$\frac{5}{12.5} = 0.40$

4 dollars for 10 ounces
$\frac{4}{10} = 0.40$

2 dollars for 5 ounces
$\frac{2}{5} = 0.40$

3.20 dollars for 8 ounces
$\frac{3.20}{8} = 0.40$

Each unit rate is 0.40, meaning that for every 1 ounce, the cost of a dish of yogurt is $0.40.
An equation that models the relationship between cost and weight would be *c*=0.40*w*, where *c* represents the cost of a dish of yogurt that weighs *w* ounces. Students can graph the ratio pairs to see that the proportional relationship forms a straight line that goes through the origin. Should the point (0,0) be on the graph? Why?



Students should be able to take a given point on the graph and explain the meaning behind its coordinates in terms of the situation. Students should determine how to utilize the graph to identify the constant of proportionality. Special attention should be given to locating which point on the graph helps to easily identify the unit rate, in this case the point (1,0.40).
If a dish of yogurt weighs 6 ounces, how could we determine the cost?

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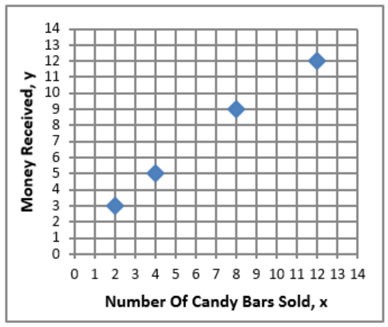
Example 2: Identifying a Non-Proportional Relationship
The following provides an example of where the relationship presented is not proportional, taken from EngageNY Grade 7 Module 1, lesson 5.

- Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold compared to the money he received.

- Is the amount of candy bars sold proportional to the money Isaiah received?
- How do you know?

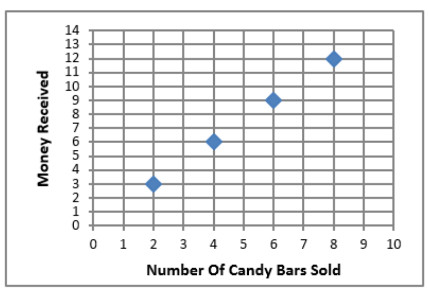
x Candy Bars Sold	y Money Received ($)
2	3
4	5
8	9
12	12



Students should see that there is no constant value that can be multiplied by the number of candy bars sold to get the corresponding money received. Students can be asked to create ratio pairs that would show a proportional relationship between the amount of candy sold and the money received as follows:

- Using the ratio provided, create a table that shows that money received is proportional to the number of candy bars sold. Plot the points in your table on the grid.

x Candy Bars Sold	y Money Received ($)
2	3
4	6
6	9
8	12



Students can write an equation that relates the money received to the number of candy bars sold, identifying the constant of proportionality in this case with 1.5.
y=1.5x
Students can also add a point to the graph that identifies the constant of proportionality (unit rate), in this case (1, 1.5) and explain that point in context. How do the other points help identify the constant of proportionality? Students will need to determine whether the graph involves just the plotting of points or the drawing of a line depending on the characteristics of the quantities involved.

Characteristics of graphs of proportional relationships:
1. Points appear on a line.
2. The line goes through the origin.

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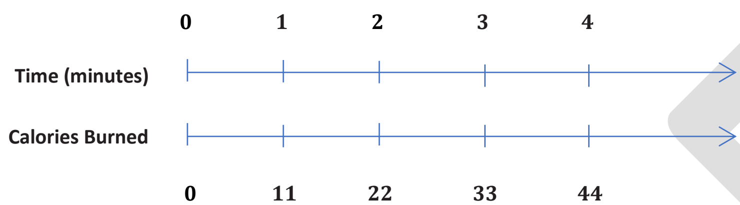
Example 3: To Be or Not to Be Proportional

Students can be presented with various scenarios, deciding if the scenario presented represents a relationship that is proportional, like the following:

- During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many calories (on average) would be burned by completing the activity.

  Calories Burned While Jumping Rope



  Is the number of calories burned proportional to time? How do you know?

- Would you expect the relationship between the number of books a person buys at a bookstore and the total cost of the books to be proportional? Explain your answers and include any assumptions you made? (Proportional Relationships 7.RP.A Conceptual Understanding and Application Mini-Assessment by Student Achieve Partners).

- Joseph earns $15 for every lawn he mows. Is the amount of money he earns proportional to the number of lawns he mows? Make a table to help you identify the type of relationship.

Number of Lawns Mowed	1	2	3	4
Earnings ($)	15	30	45	60

- At the end of the summer, Caitlin had saved $120 from her summer job. This was her initial deposit into a new savings account at the bank. As the school year starts, Caitlin is going to deposit another $5 each week from her allowance. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

Time (in weeks)	0	1	2	3
Account Balance ($)	120	125	130	135

Have students come up with their own scenarios that represent proportional and non-proportional relationships. They should describe the situation and create a table and/or graph to justify why or why not the proportional relationship exists. For proportional relationships, students should identify the constant of proportionality and explain its meaning in the context of the situation. Examples can be found in lessons 4, 5 and 6 of EngageNY Grade 7 Module 1. Once again, depending on the scenario, students will need to determine whether the graph involves just the plotting of points or the drawing of a line depending on the characteristics of the quantities involved.

Example 4: Illustrative Mathematics, Art Class (content licensed under CC BY-NC-SA 4.0)

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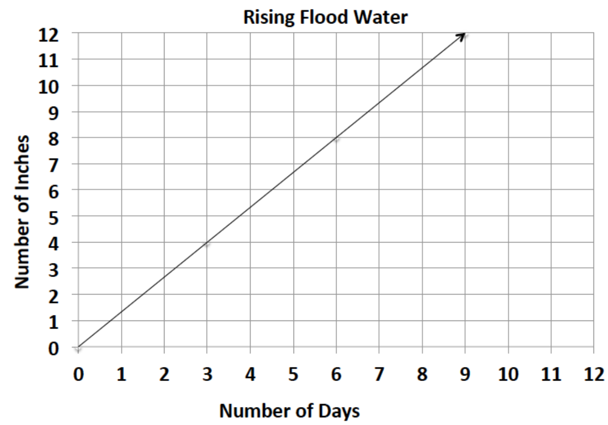
Example 5: Proportional Relationships Involving Fractions

The following example from EngageNY Grade 7 Module 1, lesson 15, shows a proportional relationship that involve fractions.

Using the graph and its title:

- Describe the relationship that the graph depicts.



- Identify two points on the line and explain what they mean in the context of the problem.
- What is the unit rate?
- What point represents the unit rate?