New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 5	DOMAIN: Numbers and Operations-Fractions

CLUSTER: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students use their knowledge of fractions, of multiplication and division, and of the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. Students make connections between models (e.g., area models) and equations while reasoning about their results. Students interpret multiplication in Grade 3 as equal groups, and in Grade 4 students begin understanding multiplication as a comparison (times as much). Students will now extend their understanding of multiplication to include scaling, where they reason about the size of products when quantities are multiplied by numbers larger than 1 and smaller than 1.

Grade Level Standard:

NY-5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
NY-5.NF.7a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients.
NY-5.NF.7b. Interpret division of a whole number by a unit fraction and compute such quotients.
NY-5.NF.7c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Solve problems, including real-world problems, involving the division of a unit fraction by a whole number and a whole number divided by a unit fraction. Explain and illustrate using a visual fraction model or equation.
- Use related multiplication to explain the quotient that results from the division of a unit fraction by a whole number and a whole number divided by a unit fraction.

ASPECTS OF RIGOR

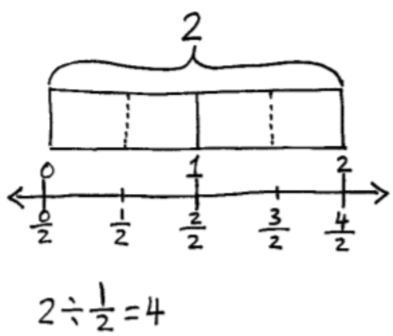
Procedural Conceptual Application

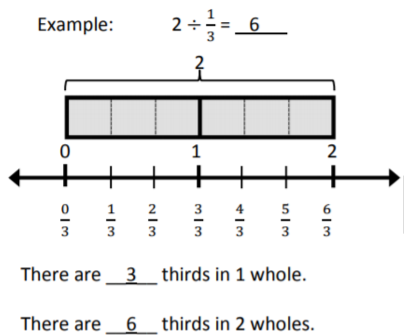
MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. **NY-3.OA.6** Understand division as an unknown-factor problem. **NY-3.NF.1** Understand a unit fraction, $\frac{1}{b}$, is the quantity formed by 1 part when a whole is partitioned into *b* equal parts. Understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. **NY-4.NF.4** Apply and extend previous understandings of multiplication to multiply a whole number by a fraction. **NY-5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **NY-5.NF.4a** Interpret the product $\frac{a}{b} \times q$ as *a* parts of a partition of q into *b* equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. **NY-5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Using the relationship between division and multiplication, students use tape diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number. Utilizing prior understanding of equal sharing and the division of whole numbers (both partitive and measurement), students reason about the size of the quotient when answering a division problem such as how many fourths are in 5, or $5 \div \frac{1}{4}$. How will the size of the quotient relate to 5, will it be larger or smaller? Students start to make connections to "invert and multiply" when answering how many $\frac{1}{4}$ are in 5 (measurement) and 5 is $\frac{1}{4}$ group of what size (partitive). They also reason about the size of the unit when $\frac{1}{4}$ is partitioned into 5 equal parts: $\frac{1}{4} \div 5$. How will the size of the quotient relate to $\frac{1}{4}$? Division of a fraction by a fraction is not an expectation until grade 6 with standard NY-6.NS.1.

Example 1: Divide a whole number by a unit fraction
The following is taken from EngageNY Grade 5 Module 4, Lesson 25.

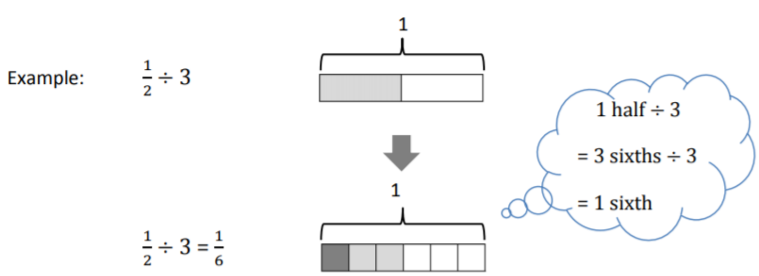




Student should relate the division problem to multiplication (NY-4.NF.4), such as
$4 \times \frac{1}{2} = 2$
4 of $\frac{1}{2} = 2$ (Four halves are the same as two wholes)

Student should relate the division problem to multiplication (NY-4.NF.4), such as
$6 \times \frac{1}{3} = 2$
6 of $\frac{1}{3} = 2$ (Six thirds are the same as two wholes)

Example 2: Divide a unit fraction by a whole number (and whole number by a unit fraction)
The following is taken from EngageNY Grade 5 Module 4, Lesson 26.



Students relate that when dividing a quantity by 3, it is the same as taking $\frac{1}{3}$ of that quantity.
$\frac{1}{2} \div 3 =$
$\frac{1}{3}$ of $\frac{1}{2} =$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

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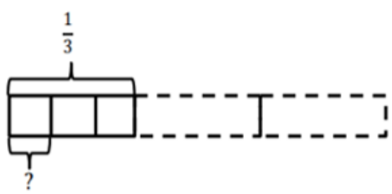
Using story and real-world contexts, students make sense of division problems. When comparing the two division problems $\frac{1}{2} \div 3$ and $3 \div \frac{1}{2}$, and determining if they are the same or different and why, students can consider the following contexts:

- 3 people share $\frac{1}{2}$ lb. of chocolate equally, how much does each person get?
- How many $\frac{1}{2}$ lb. servings are there in 3 lbs. of chocolate?

How are the visual diagrams different for each scenario?

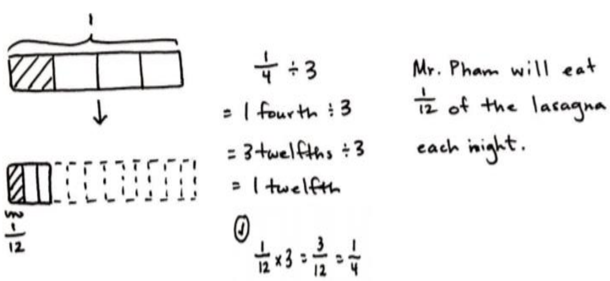
Students can be asked to draw a tape diagram and create their own word problem for expressions such as $\frac{1}{3} \div 5$ or $7 \div \frac{1}{6}$.
A scaffolded task might look like the following:

- Create and solve a story problem about $\frac{1}{3}$ pound of flour that is modeled by the tape diagram below.



Example 3: Solve real-world problems
The following is taken from EngageNY Grade 5 Module 4, Lesson 27.

- Mr. Pham has $\frac{1}{4}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices, so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.



The following problem also involves a conversion of units (NY-5.MD.1). Students should be provided with the conversion factor of 16 ounces = 1 pound.

- A container is filled with blueberries. $\frac{1}{6}$ of the blueberries is poured equally into two bowls.
  What fraction of the blueberries is in each bowl?
  If each bowl has 6 *ounces* of blueberries in it, how many ounces of blueberries were in the full container?
  If $\frac{1}{5}$ of the remaining blueberries is used to make muffins, how many *pounds* of blueberries are left in the container?

Additional word problems can be found throughout lessons 25-28 of Module 4.

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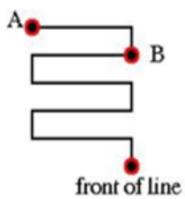
Example 4: Modeling example
Standing in Line, taken from Illustrative Mathematics, content licensed under CC BY-NC-SA 4.0.

Alysha really wants to ride her favorite ride at the amusement park one more time before her parents pick her up at 2:30 pm. There is a very long line at this ride, which Alysha joins at 1:50 pm (point A in the diagram below). Alysha is nervously checking the time as she is moving forward in the line. By 2:03 pm she has made it to point B in line.



What is your best estimate for how long it will take Alysha to reach the front of the line? If the ride lasts 3 minutes, can she ride one more time before her parents arrive?

$13 = \frac{1}{5} \times d$ where d is the total time from the start to the end of the line.
$13 \div \frac{1}{5} = d$
$d = 65$

With the numbers given in the problem, the conclusion is clear that there is not enough time for another ride. It would be easy to change the numbers to make the answer less clear. This would provide an opportunity to also include an estimate of the time it takes to go on the ride and to account for the time it takes to meet up with the parents if they aren't meeting at the ride. This set-up would provide a forum for a lively classroom discussion where students must justify their claims with appropriate assumptions and computations.

This task is a good illustration of MP4, Model with mathematics. This means that it is not just a real-world problem, but we also have to make simplifying assumptions to solve the problem. In this case we are assuming that the line moves in a uniform way, i.e., it takes equal amounts of time to move equal distances.

An extension of the task would be to give non-equally spaced times at different points along the line. Then the students also have to decide if they want to average the numbers, if they want to find a worst-case scenario estimate or an optimistic estimate and come up with a likely range of times it will take to go through the line.