New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 2	DOMAIN: Operations & Algebraic Thinking

CLUSTER: Add and subtract within 20.

Students' fluency with adding and subtracting numbers within 20 support all the grade 2 level standards within the Operations and Algebraic Thinking domain. Fluently means efficient, accurate, flexible, and appropriate strategy use. It involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (NYS Next Generation Mathematics Learning Standards). Mental strategies help students make sense of number relationships. Mental strategies include counting on, making ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums, and near doubles. Students make sense of the part-whole relationships, work flexibly with numbers and use strategies and properties of operations to solve addition and subtraction problems. By the end of Grade 2, students will know from memory all sums within 20 of two one-digit numbers. To retrieve facts from memory, students must use efficient strategies that they have learned. Building students' strengths with fluency strategies leads to automaticity and recall of facts.

Grade Level Standards:

NY-2.OA.2a. Fluently add and subtract within 20 using mental strategies.
Strategies could include: counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.
NY-2.OA.2b Know from memory all sums within 20 of two one-digit numbers.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Use mental strategies for addition and subtraction.
- Apply the relationship between addition and subtraction when solving addition and subtraction problems and be able to create fact families.
- Apply properties of operations (commutative and associative) to aid in solving addition and subtraction problems.
- Apply knowledge of familiar facts to learn unfamiliar facts.
- Recall from memory all sums of two one-digit numbers.

ASPECTS OF RIGOR

Procedural   Conceptual   Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **NY-K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **NY-K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **NY-1.OA.5** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **NY-1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **NY-1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

The deep extended experiences students have with addition and subtraction in Kindergarten and Grade 1 culminate in Grade 2 with students becoming fluent in adding and subtracting within 20 using the mental Level 2 and 3 strategies as needed. (Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.)

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

"Know from memory" does not indicate that students will memorize all facts through the use of timed fact practice tests. Instead, teachers can build from previous work in Grade 1. In Grade 1, students worked extensively with numbers to gain fluency with sums and difference within 10 (NY-1.OA.6b) and became proficient in counting on which is a Level 2 strategy. They also began to make easier problems to add and subtract within 20 and 100 by making ten and taking from a ten which is Level 3 strategy. (NY-1.OA.6, NY-1.NBT.4-6).

Instruction to support the standards NY-2.OA.2a and NY-2.OA.2b should focus on making sense and using strategies for remembering facts and seeing the relationship between addition and subtraction. Students begin to choose the strategies that they are efficient with; individualized strategies that can be done mentally and quickly. As students begin to master NY-2.OA.2a by engaging in meaningful, strategic practice through problem-based activities, automaticity for memory of all the sums within 20 will progress.

Four Tenants (Aspects/Categories) of Fluency taken from "Assessing Basic Fact Fluency" (Kling, Gina and Jennifer M. Bay Williams. "Assessing Basic Fact Fluency." *NCTM Teaching Children Mathematics* 20 (April 2014):492) are the following:

Accuracy: Assessed as soon as the student responds.
Efficiency: Observed on the basis of how long it takes a student to solve the fact.
Flexibility and Appropriate Strategy Selection: Addressed by such follow-up prompts as
- "How did you figure it out?"
- "Is there another way?"
- "How could you use this strategy to solve this fact?"

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When students are given the opportunity to use concrete objects and pictorial representations first, their abstract understanding and knowing from memory all the sums within 20 of two one-digit numbers will follow. As conceptual understanding is being developed, students can solve problems using the method that works for them.

Example 1: Knowing the Decomposition of Any Number Within 10
Decomposition of single-digit numbers is a foundational skill for fluency with sums and differences to 20. The following is taken from EngageNY Grade 2, Module 1, Lesson 1.

Target Number:

Target Practice: Within 10

Materials: (S) Per set of partners: personal white board, target practice (Lesson 1 Fluency Template 3), 1 numeral die

Assign Partner A and Partner B.
Students write their choice of target number in the circle in the top right of the Target Practice template.
Partner A rolls the die.
Partner A writes the number rolled in the circle at the end of one of the arrows.
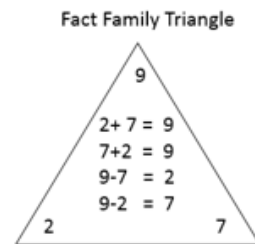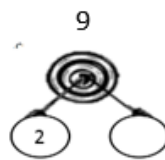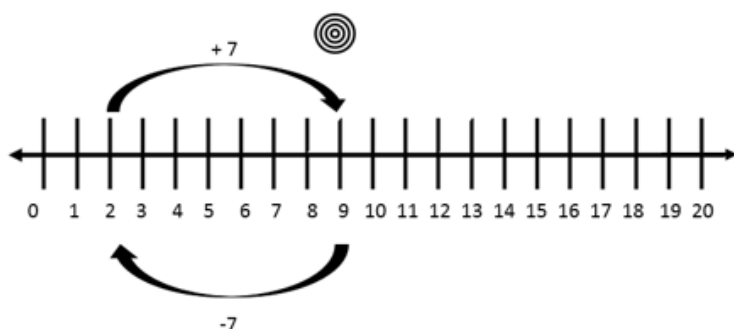Partner B makes a bull's eye by writing the number in the other circle that is needed to make the target.
Adjust the target number as appropriate for each pair of students, focusing on totals of 6, 7, 8, 9, and 10. If the pair demonstrates fluency, challenge them to move into teen numbers!

Multiple representations are encouraged, such as the use of concrete manipulatives (interlocking counting cubes or counters). Students are presented with a group/block of counters or cubes (target). Based on the roll of the die, students can break apart the target accordingly, or without the die, each student (A and B) can randomly break apart their block/group into two groups. Students now can create a number bond that resembles how they broke apart their target and share their results. Activity can be extended for subtraction. Each student has a block/group that represents the target. One student can take target number and then break and hide a part of the target. Other partner needs to find the quantity of the hidden piece/group.

Student discourse is essential. Students should be encouraged to write and translate the number sentences that model the addition/subtraction problem (the fact family). The writing of the number sentences (generating of the fact families) and the speaking/translating help to reinforce the connection between addition and subtraction, and with developing fluency.
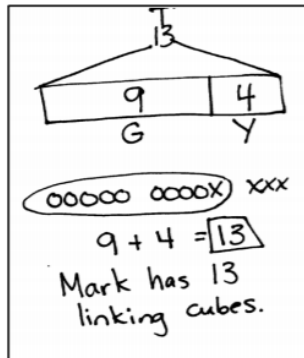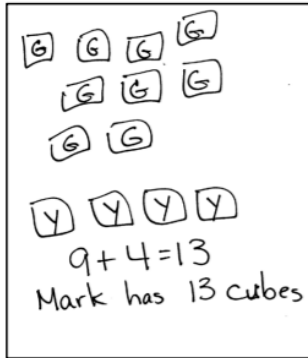
Students can also utilize a desk number line.

Fact Family Triangle

9

2+ 7 = 9
7+2 = 9
9-7 = 2
9-2 = 7

2   7

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 2: Developing Strategies through Problem Solving/Creating Story Problems

Word problems allow for students to develop meaningful language, write number sentences that relate to strategies used and reinforce work done with grade level standard NY-2.OA.1a.

- Making a Ten (Addition) taken from EngageNY Grade 2 Module 1, lesson 4.
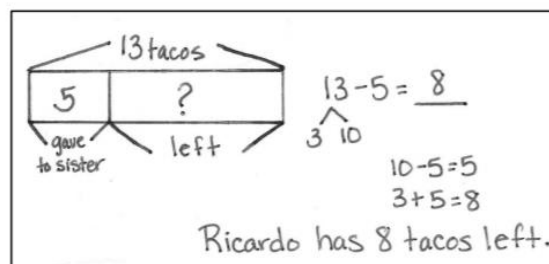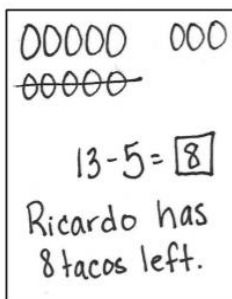
    Mark had a stick of 9 green linking cubes. His friend gave him 4 yellow linking cubes. How many linking cubes does Mark have now?

    Use a story to model/match the number sentence. Students can create their own stories/problems that model $9 + 4 = 13$. For example, my teacher had 9 linking cubes in the bucket and then she picked up 4 more off the floor and put them in the bucket. Now she has 13 cubes all together. To help with automaticity of facts, students could also create their own stories/problems that model other facts in the fact family ($4 + 9 = 13, 13 - 4 = 9, 13 - 9 = 4$).

- Take from Change Unknown problem (Subtraction) taken from EngageNY Grade 2 Module 1, lesson 7.

    Ricardo gave 5 tacos to his sister. He started with 13. How many tacos does Ricardo have left?
    The missing part can be found by subtracting or adding on.

    Students can create their own stories/problems that model 13 – 5 = 8. For example, Mom put 13 tacos on a plate and I ate 5 of them. There were 8 left over for my sisters.
    To help with automaticity of facts, students could also create their own stories/problems that model other facts in the fact family. For example, Ricardo ate 8 tacos, and his sister ate 5 tacos. There were 13 tacos all together.

    A flexible understanding of the relationship of addition to subtraction and parts to total is also necessary for the use of Level 3 strategies.

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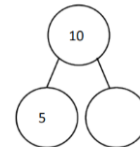
Example 3: Knowing Partners to 10 (Pairs to 10 are a foundational skill for mastery of sums and differences to 20)
The following are taken from EngageNY Grade 2 Module 1, lesson 1.

- Pairs to Ten with Number Bonds

  Use the number bond for addition and subtraction, showing part, part, whole relationships.
  Materials: (S) Personal white board
  T: I'll show a number bond, and you tell me the missing part to make 10.

  T: (Draw the bond shown to the right.)
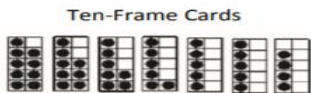 S: 5.
  T: (Erase the 5 and write 8.)
  S: 2.
  Continue with the following suggested sequence: 9, 7, 3, 6, 4, 1, 10, and 0.
  T: With your partner, take turns saying pairs to make 10. Partner A, you will go first for now.

- Ten Frame Flash

  Materials: T) Ten-frame cards (Fluency Template 1), 5-group column cards (Fluency Template 2)

  By alternating between ten-frame and 5-groups column cards, students develop flexible perception of numbers 6–10 in two parts, with one part as 5. This activity practices the core fluency objective from Grade 1, adding and subtracting within 10. The teacher flashes a ten-frame card for 2–3 seconds and guides students to respond on a signal. Students then generate a number sentence to get to 10.

Example 4: Knowing Teen Numbers as 10 + n
The following is taken from EngageNY Grade 2 Module 1, lesson 1.

- Say Ten as Ten Plus Facts

  Students say addition sentences for teen numbers when one addend is 10. Alternate between the regular way and the Say Ten way.
  T: If I say ten 2, you say 10 + 2 = 12.
  T: What do you say if I say thirteen?
  S: 10 + 3 = 13.
  T: Yes! You guessed the pattern. Here's another. Ten 5.
  S: 10 + 5 = 15.
  T: Fourteen.
  S: 10 + 4 = 14.
  Use the following suggested sequence: ten 6, seventeen, eighteen, ten 5, eleven, ten 8, ten 1, etc.