# New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

| **GEOMETRY** | **DOMAIN:** Similarity, Right Triangles, and Trigonometry |
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**CLUSTER: Apply trigonometry to general triangles.**
With the introduction of the formula $A = 1/2 \ ab \sin(C)$, students discover how prior knowledge of trigonometric ratios can help with area calculations in cases where the measurement of the height is not provided. In order to determine the height in these cases, students must draw an altitude to create right triangles within the larger triangle. With the creation of the right triangles, students will set up the necessary trigonometric ratios to express the height of the triangle (**GEO-G.SRT.8**). Students will carefully connect the meanings of formulas to the diagrams they represent.

## Grade Level Standard:

**GEO-G.SRT.9** Justify and apply the formula $A = 1/2 \ ab \sin(C)$ to find the area of any triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

### PERFORMANCE/KNOWLEDGE TARGETS
### (measurable and observable)

- Recall how to transform the equation $A = \frac{1}{2}bh$ to $A=\frac{1}{2} \ ab \sin(C)$.
- Prove that the area of a triangle is one-half times the product of two side lengths times the sine of the included angle.
- Solve problems using this formula.

### ASPECTS OF RIGOR

Procedural Conceptual Application

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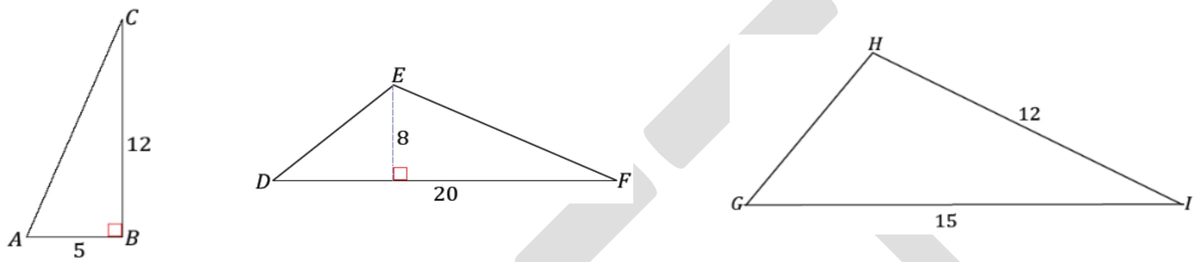
| **MATHEMATICAL PRACTICES** | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |
| **FOUNDATIONAL UNDERSTANDING** | **NY-6. G.1** Find area of triangles, trapezoids, and other polygons by composing into rectangles or decomposing into triangles and quadrilaterals. Apply these techniques in the context of solving real-world and mathematical problems.<br>**NY-7. G.6** Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles and trapezoids.<br>**GEO-G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent ratios for acute angles.<br>**GEO-G.SRT.8** Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems. |

**The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.**

**Example 1: Using Pythagorean Theorem to Determine the Area of a Triangle**

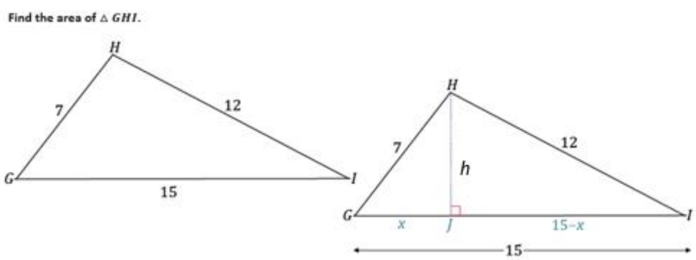
The following is taken from EngageNY Geometry Module 2, lesson 31.

Three triangles are presented below. Determine the areas for each triangle, if possible. If it is not possible to find the area with the provided information, describe what is needed in order to determine the area.




The area of $\Delta ABC$ is 30 square units, and the area of $\Delta DEF$ is 80 square units. There is not enough information to find the height of $\Delta GHI$.

What if the third side length of the triangle were provided? Is it possible to determine the area of the triangle now?




How can the height be calculated? By applying the Pythagorean theorem to both created right triangles to find $x$.

$$h^2 = 49 - x^2 \qquad\qquad h^2 = 144 - (15 - x)^2$$

$$49 - x^2 = 144 - (15 - x)^2$$
$$49 - x^2 = 144 - 225 + 30x - x^2$$
$$130 = 30x$$
$$x = \frac{13}{3}$$
$$GJ = \frac{13}{3}, IJ = \frac{32}{3}$$

Solve for $h$:

$$h^2 = 49 - x^2$$
$$h^2 = 49 - (\tfrac{13}{3})^2$$
$$h = \frac{4\sqrt{17}}{3}$$

What is the area of the triangle? Area $= \left(\frac{1}{2}\right)(15)\left(\frac{4\sqrt{17}}{3}\right)$

Area=$10\sqrt{17}$

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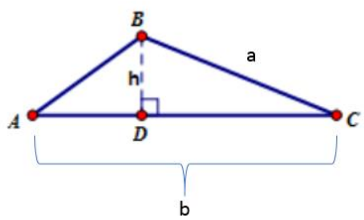
**Example 2: Using Trigonometry to Determine the Area Formula of a Triangle when the Height is Unknown**

To derive the formula $A = 1/2 \ ab \sin(C)$ for the area of a triangle, an auxiliary line is drawn from a vertex perpendicular to the opposite side.

Starting with Δ ABC, drop an altitude to the base of the triangle and label it *h* (for height).




The formula for the area of a triangle is $A = \frac{1}{2}bh$ . Using right triangle trigonometry, sin(C)= $\frac{h}{a}$ .

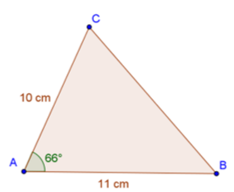
Solving for *h*, $h = a \cdot \sin(\mathrm{C})$.

Substitute $a \cdot \sin(\mathrm{C})$ for *h* in the original formula: $A = \frac{1}{2} b \, (a \cdot \sin(\mathrm{C}))$

$A = \frac{1}{2} ab \sin(\mathrm{C})$

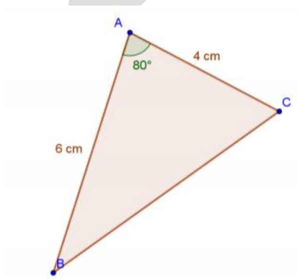
**Example 3:** The following problems are taken from [EngageNY Precalculus and Advanced Topics Module 4](), Lesson 7.

Calculate the area of the triangles below:




Area= $\frac{1}{2}$ (10)(11)·sin(66°)

Area≈ 50.25 $cm^2$




Area= $\frac{1}{2}$ (6)(4)·sin(80°)

Area≈ 11.82 $cm^2$

Students can discuss what is different in the triangles given here in example 3, vs. the triangle that was given in example 1.

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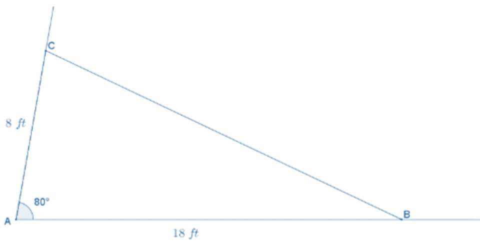
**Example 4:** Taken from in [EngageNY Geometry Module 2](), lesson 31.

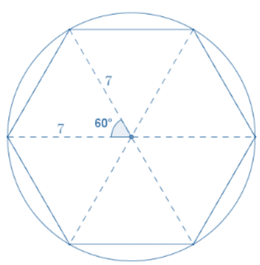
A landscape designer is designing a flower garden for a triangular area that is bounded on two sides by the client's house and driveway. The length of the edges of the garden along the house and driveway are 18 ft. and 8 ft., respectively, and the edges come together at an angle of 80°.




Draw a diagram, and then find the area of the garden to the nearest square foot.

**Example 5:** Taken from in [EngageNY Geometry Module 2](), lesson 31.

A regular hexagon is inscribed in a circle with a radius of 7. Find the perimeter and area of the hexagon.




The regular hexagon can be divided into six equilateral triangular regions with each side of the triangles having a length of **7**. To find the perimeter of the hexagon, solve the following:

6·7=42, so the perimeter of the hexagon is 42 units.

To find the area of one equilateral triangle:

$$\text{Area} = \frac{1}{2}(7)(7)\sin 60$$
$$\text{Area} = \frac{49}{2}\left(\frac{\sqrt{3}}{2}\right)$$
$$\text{Area} = \frac{49\sqrt{3}}{4}$$

The area of the hexagon is six times the area of the equilateral triangle.

$$\text{Total Area} = 6\left(\frac{49\sqrt{3}}{4}\right)$$
$$\text{Total Area} = \frac{147\sqrt{3}}{2} \approx 127.3$$

The total area of the regular hexagon is approximately 127.3 square units.

Task connects to standard GEO-G.CO.3, *Given a regular or irregular polygon, describe the rotations and reflections (symmetries) that carry the polygon onto itself.*

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