New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GEOMETRY	**DOMAIN:** Congruence

CLUSTER: Understand congruence in terms of rigid motions

Students build on their understanding of rigid motions to formalize the definition of congruence that was developed in grade 8. They learn that two-dimensional figures are congruent if one figure is the image of the other figure after a sequence of rotations, reflections, and translations. They specify a series of rigid motions that maps one figure onto another and use the definition to determine whether two objects are congruent. They learn that figures that are congruent can have different orientations, but corresponding side lengths and angle measures are equal. The criteria for triangle congruence (ASA, SAS, SSS, AAS and HL) is determined, based on the definition of congruence in terms of rigid motions.

Grade Level Standard:

GEO-G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- **A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector.**
- **A rotation requires knowing the center/point and the measure/direction of the angle of rotation.**
- **A line reflection requires a line and the knowledge of perpendicular bisectors.**

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Provide a sequence of rigid motions that maps one figure onto another figure to show that the two figures are congruent.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-8. G.2** Know that a two-dimensional figure is congruent to another if the corresponding angles are congruent and the corresponding sides are congruent. Equivalently, two two-dimensional figures are congruent if one is the image of the other after a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that maps the congruence between them on the coordinate plane.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Transformations build on students' intuitive understanding developed in Grade 8. With the help of manipulatives, students observed how rotations, reflections, and translations behave individually and in concert (**NY-8. G.1**, **NY-8. G.2**). Similarly, students' Grade 8 concept of congruence transitions from a hands-on understanding (**NY-8. G.2**) to a precise, formally notated understanding of congruence (**GEO-G.CO.6**).

To show that two figures are congruent, we need to show that there is a sequence of transformations that maps one figure directly onto the other figure.

Students must learn how to describe a sequence of rigid motions, using precise language. If they want to map one figure onto another, they must be able to describe all the necessary steps so that the sequence is reproducible.

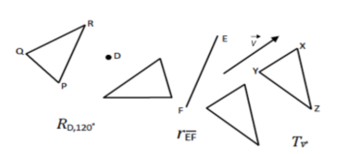
- To rotate a figure, students must state the center of rotation, the angle of rotation, and the direction of rotation. Angle of rotation is counter-clockwise if and only if its degree measure is positive.
- To reflect a figure, students must state the line of refection.
- To translate a figure, students must clearly state the direction and the distance (e.g., a translation such that point B maps to B'). Vectors are useful here.

Example 1:

The first two videos from Khan Academy (content licensed under [CC BY-NC-SA 3.0]) utilize the coordinate plane and demonstrate how a sequence of transformations can be utilized to determine if two given figures are congruent.
[Congruent Shapes and Transformations]
[Non-congruent Shapes and Transformations]
The third video asks students to try mapping one figure onto another by using an interactive widget in order to determine if the figures are congruent.
[Practice Congruence & Transformations]
Students can be presented with similar situations and utilize patty paper to assist with the transformations involved in their determined sequence.

When deciding if or showing that two figures are congruent, based on the definition of congruency, students should be stating that each rigid motion preserves segment length and angle measure. So, if a sequence of rigid motions can be found to map one figure to the other, then the preimage and image figures are congruent. This supports work with standard GEO-G.CO.7 Use the definition of congruence in terms of rigid motions to show two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Example 2: The following example is taken from [EngageNY Geometry Module 1], Lesson 19.



Composite functions are not an expectation of this course. However, they can be utilized to describe a sequence of transformations.

$$T_{\vec{v}}\left(r_{\overline{EF}}\left(R_{D,120^\circ}(\Delta PQR)\right)\right) = \Delta XYZ$$

To map ΔPQR to ΔXYZ, we would first rotate ΔPQR, $120°$ around point D. Then reflect the image over line EF. Finally, we would translate the second image with respect to vector V, to obtain ΔXYZ. Since each transformation is a rigid motion, $\Delta PQR \cong \Delta XYZ$.

$\triangle PQR$ is congruent to $\triangle XYZ$ because rigid motions map point P to point X, point Q to point Y, and point R to point Z. Rigid motions map segments onto segments of equal length and angles onto angles of equal measure.

Similarly, students can do the following:

- Draw and label ΔPQR on your paper.
- Use your construction tools (or patty paper) to apply one of each of the rigid motions we have studied to it, in a sequence of your choice.
- Describe your sequence of transformations using precise mathematical language and/or notation.
- Label your resulting image as ΔZYX.
- On a separate piece of paper, trace the set of figures in your sequence, but do NOT include the center of rotation, the line of reflection, or the vector of the applied translation. Swap papers with a partner and determine the sequence of transformations your partner used to render $\Delta PQR \cong \Delta ZYX$. Describe the sequence of transformations using precise mathematical language and/or notation.

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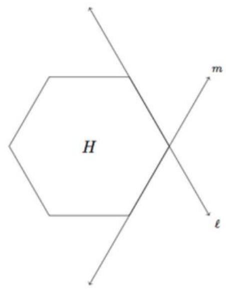
Example 3: Creating patterns by using a sequence of rigid motions, Illustrative Mathematics Tasks: G-CO Building a tile pattern by reflecting hexagons and G-CO Building a tile pattern by reflecting octagons (Content licensed under CC BY-NC-SA 4.0).

Building a tile pattern by reflecting hexagons

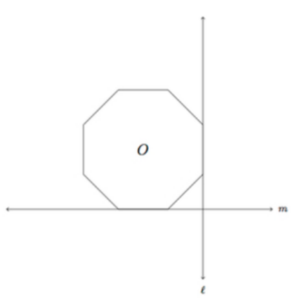
Below is a picture of a regular hexagon, which we denote by H, and two lines denoted ℓ and m, each containing one side of the hexagon:



a. Draw the reflection of the hexagon H over line ℓ.
b. Draw the reflection of the hexagon H over line m.
c. Show that H and its reflections over the six lines containing its sides make the following pattern:

Reflecting Octagons

Below is a picture of a regular octagon, which we denote by O, and two lines denoted by ℓ and m, each containing one side of the octagon:



a. Draw the reflection of hexagon O over line ℓ.
b. Draw the reflection of hexagon O over line m.
c. Draw the reflection of the hexagon obtained in part a over line m.
d. Show that the quadrilateral enclosed by the four octagons is a square.

One possible path to constructing an argument as to why the resulting enclosed quadrilateral is a square could involve discussing the interior/exterior angle measures of a regular octagon, reinforcing work connected to GEO-G.CO.3.