New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GEOMETRY	DOMAIN: Congruence

CLUSTER: Experiment with transformations in the plane.

Students are reintroduced to rigid transformations, specifically rotations, reflections, and translations. In grade 8 (NY-8. G.1–4), students developed an intuitive understanding of the transformations, observing their properties by experimentation. Students now examine each transformation more closely building on their hands-on work and will develop precise definitions that will serve as a logical basis for all theorems that students prove in geometry. Students will differentiate rigid motions from non-rigid motions. Rotations and reflections will be used to verify symmetries within polygons. Using their construction skills, in conjunction with their understanding of transformations, as well as properties of parallel and perpendicular lines, students will perform transformations and will use these transformations to examine correspondence and its place for further upcoming discussion of congruency.

Grade Level Standard:

GEO-G.CO.2 Represent transformations as geometric functions that take points in the plane as inputs and give points as outputs. Compare transformations that preserve distance and angle measure to those that do not.

Note: **Instructional strategies may include drawing tools, graph paper, transparencies and software programs.**

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Represent transformations in the plane using various tools (transparencies, geometry software, etc.).
- Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
- Investigate transformations that preserve distance and angle measure and those that do not (translation vs. horizontal stretch).

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-8. G.1** Verify experimentally the properties of rotations, reflections, and translations. **Notes:** A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. A rotation requires knowing the center/point of rotation and the measure/direction of the angle of rotation. A line reflection requires a line and the knowledge of perpendicular bisectors. **NY-8. G.1a** Verify experimentally lines are mapped to lines, and line segments to line segments of the same length. **NY-8. G.1b** Verify experimentally angles are mapped to angles of the same measure. **NY-8. G.1c** Verify experimentally parallel lines are mapped to parallel lines. **NY-8. G.3** Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates. **Note**: Lines of reflection are limited to both axes and lines of the form $y=k$ and $x=k$, where k is a constant. Rotations are limited to 90 and 180 degrees about the origin. Unless otherwise specified, rotations are assumed to be counterclockwise. **AI-F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y=f(x)$.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

This standard connects the work done with algebraic functions in grade 8 and Algebra I to geometric functions. Some of the representations of and work with algebraic functions included the following (EngageNY Algebra I module 3, lesson 10):

- Study the four representations of a function below. How are these representations alike? How are they different?

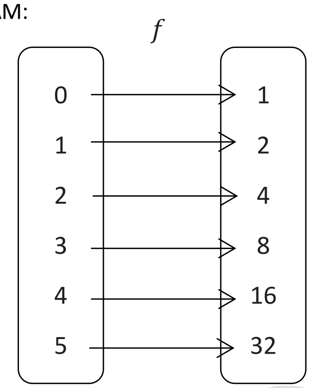
TABLE

Input	0	1	2	3	4	5
Output	1	2	4	8	16	32

MAPPING: Let $f: \{0,1,2,3,4,5\} \to \{1,2,4,8,16,32\}$ be such that $x \mapsto 2^x$.

SEQUENCE: Let $a_{n+1} = 2a_n,\ a_0 = 1$ for $0 \le n \le 4$ where n is an integer.

DIAGRAM:

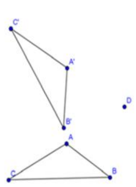


These representations are alike because they all match the same pairs of numbers $(0,1)$, $(1,2)$, $(2,4)$, $(3,8)$, $(4,16)$, and $(5,32)$. They are different because they describe the domain, range, and correspondence differently. The table and the mapping look similar; the input and output are related to domain and range of a function. Evaluating the expression for the given x values returns the output values in the table, and the sequence also generates the output values for the first 6 terms starting at $n = 0$.

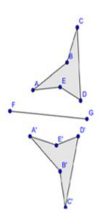
- Let $f(x) = 6x - 3$, and let $g(x) = 0.5(4)^x$. Find the value of each function at a given input, for example, *f* (-10), *g* (4).
- Write three different polynomial functions such that $f(3) = 2$.
- What is the range of each of the following functions?
  $f(x)=9x-1$
  $g(x)=3^{2x}$
  $h(x)=x^2-4$

Defining an ALGEBRAIC FUNCTION as: Given an algebraic expression in one variable, an algebraic function is a function $\boldsymbol{f}: \boldsymbol{D} \to \boldsymbol{Y}$ such that for each real number $\boldsymbol{x}$ in the domain $\boldsymbol{D}$, $\boldsymbol{f}(\boldsymbol{x})$ is the value found by substituting the number $\boldsymbol{x}$ into instances of the variable symbol in the algebraic expression and evaluating. If the domain, ***D,*** is not specified explicitly, the domain is taken to consist of *all* real number values for which the algebraic expression produces a real number. Otherwise, the domain should be specified.

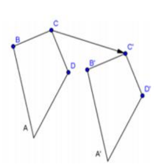
In geometry, students utilize functions to determine how figures are moved and/or altered in the plane. It is critical that students understand transformations as functions that take a set of points as inputs (pre-image), apply a given rule, and output a new location (image). Prime notation is utilized to distinguish the difference between the pre-image and the image. The function notation seen below highlights the connection to previous function work in algebra but is not an expectation of the geometry course. The domain and range for rotations, reflections, translations and dilations are the points of the plane.



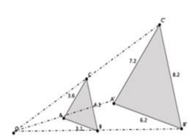
Rotation: $R_{D,\theta}(\Delta ABC) = \Delta A'B'C'$
Where D is the center of the rotation and θ is the angle of rotation, positive for counter-clockwise, negative for clockwise.



Reflection: $r_{\overline{FG}}$ (ABCDE) = A'B'C'D'E'



Translation: $T_{\overrightarrow{CC'}}$(ABCD)=A'B'C'D'



Dilation: $D_{O,2}(\Delta ABC) = \Delta A'B'C'$

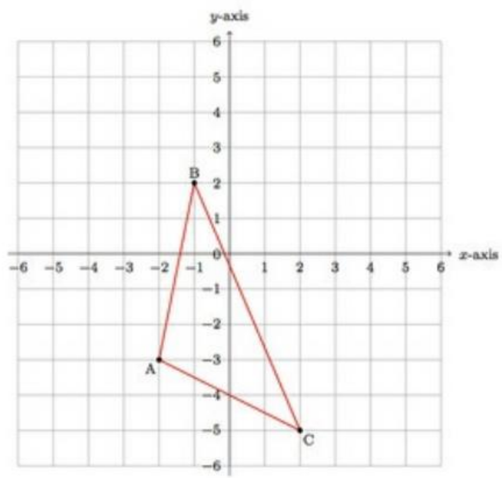
The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

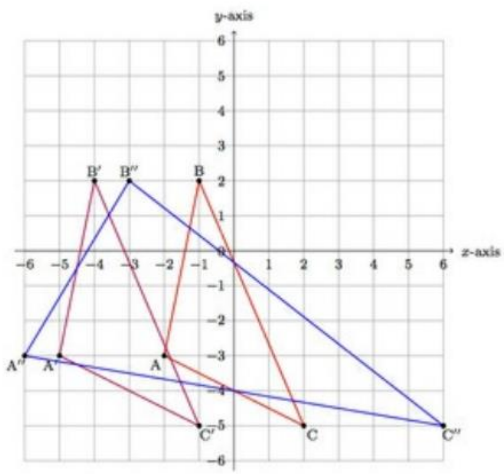
Students should see that rigid transformations (rotations, reflections and translations) preserve distance, angle measure, parallelism, and collinearity between the pre-image and image. Orientation can also be examined.

This Horizontal Stretch of the Plane (Content licensed under CC BY-NC-SA 4.0) activity below is taken from Illustrative Mathematics. Although horizontal stretch *f*(*kx*) is not an expectation in Algebra I, it is still an expectation in Geometry because of its connection to dilations. Stretches (both horizontal and vertical) should be discussed regarding how they relate to other transformations, meaning that students understand which transformations preserve both distance and angle measure, just one, or neither.

Suppose *f* is the map of the plane which takes each point (*x, y*) to the point (*x-3, y*) and *g* is the map of the plane that takes each (*x, y*) to (*3x, y*).
ABC maps to *A'B'C'* after (*x, y*) → (*x*-3, *y*)
ABC maps to *A''B''C''* after (*x, y*) → (*3x, y*)

a. Show the image of $\Delta\ ABC$ after applying each of the maps *f* and *g*.





b. Does *f* preserve distances and angles? Explain.
c. Does *g* preserve distances and angles? Explain.

The goal of this task is to compare a transformation of the plane (translation) which preserves distances and angles to a transformation of the plane (horizontal stretch) which does not preserve either distances or angles. The fact that horizontal stretch does not preserve distances can be seen from the pictures or using the Pythagorean theorem. The only line segments whose distance is preserved by this horizontal stretch are vertical lines and the teacher may wish to have students investigate this as a follow up question. The fact that horizontal stretch does not preserve angles is visibly clear but requires more careful thinking. In the solution a similarity argument is provided but students familiar with trigonometry could also calculate the angles in the different triangles and verify in this way that horizontal stretch changes the angles of this triangle.

The task does not specify that *f* is a translation to the left by 3 units and that is *g* a horizontal stretch by a factor of 3. The teacher should prompt students for a geometric description of what *f* and *g* do so that students understand what transformations of the plane these functions represent.

Points that are their own image under transformations are fixed points and help further define and distinguish rigid transformations, as seen in the Illustrative Mathematics Task, Fixed Points of Rigid Motions. (Content licensed under CC BY-NC-SA 4.0)

Other tasks could include the following:

- A line segment is dilated by a scale factor of 2 centered at a point not on the line segment. Describe the relationship between the given line segment and its image.
- Δ*ABC* has coordinates *A* (1,1), *B* (4,1), and *C* (4,5). Graph and label Δ*A" B"C"*, the image of Δ*ABC* after the translation five units to the right and two units up followed by the reflection over the line *y=0*.