New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

Course: Algebra II	Functions DOMAIN: Trigonometric Functions

CLUSTER: Extend the domain of trigonometric functions using the unit circle.
Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students extend trigonometric functions to all (or most) real numbers. A new unit of rotational measure is introduced, a radian. To reinforce their understanding of trigonometric functions, students begin building fluency with the values of sine, cosine, and tangent at π/6, π/4, π/3, π/2, etc. Students make sense of periodic phenomena and use the structure of the unit circle to develop and justify conjectures about the behavior of trigonometric functions, allowing them to evaluate the trigonometric functions for values of θ in all four quadrants.

Grade Level Standard:
AII-F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Note: Focus of this standard is on cos(x), sin(x) and tan(x).

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Students will be able to look at the graph of a trigonometric function and recognize its periodic nature and determine its period;
- students will be able to relate the unit circle values with the corresponding points on the graph of a sine, cosine and tangent function: and
- students will be able to determine whether sine, cosine or tangent is even, odd or neither based upon the graph of the function and the unit circle.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**GEO-G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **AII-F.BF.3b** Using f(x) + k, k f(x), f(kx), and f (x + k): i) identify the effect on the graph when replacing f(x) by f(x) + k, k f(x), f(kx), and f (x + k) for specific values of k (both positive and negative); ii) find the value of k given the graphs; iii) write a new function using the value of k; and iv) use technology to experiment with cases and explore the effects on the graph. Include recognizing even and odd functions from their graphs. Include recognizing even and odd functions from their graphs.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Please see Engage NY Precalculus Module 4, lesson 2 for additional information and connections.

Example 1:

Students are presented with graphs of the sine, cosine and tangent functions on the interval -2π to 6π, with the x-axis scaled every $\frac{\pi}{2}$. Students are asked to determine how they would explain to another student the repetitive nature of the graph and what real life applications such a periodic function might have. Possible answers include tides, breathing and temperature.

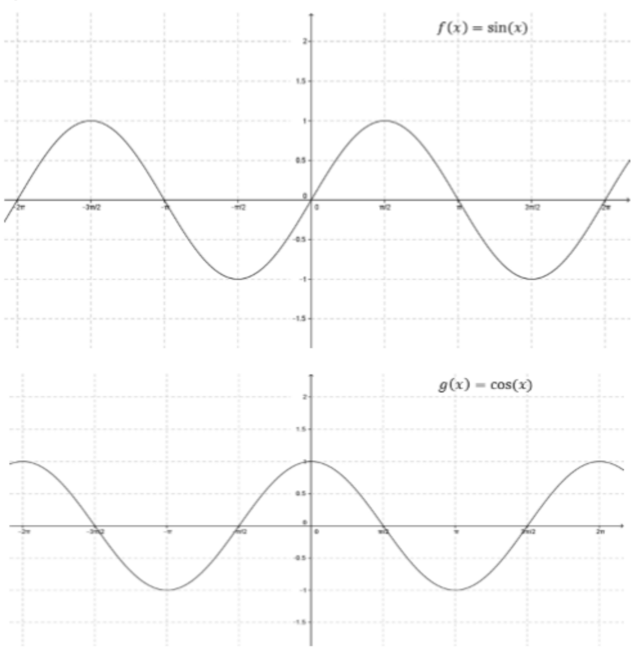
Students should understand that periodic motion is a motion that repeats itself. A periodic function can be defined as one that repeats its values in regular intervals or periods. Students can be given various graphs and asked to identify which are examples of periodic functions and which are not. Students can draw their own and justify whether or not their graphs represent periodic functions (brainstorming real-world models that represent periodic behavior).

A function f whose domain is a subset of the real numbers is said to be periodic with period P>0 if the domain of f contains x+P whenever it contains x, and if $f(x+P) = f(x)$ for all real numbers x in its domain.
Students should see that for sin(x) and cos(x), if you were to shift either graph horizontally by 2π , the resulting shape would be identical to the original function (the graph repeats every 2π). Similarly, for tan(x), a shift horizontally by π produces an identical shape.
sin (θ+2π)=sin (θ)
cos (θ+2π)=cos (θ)
tan (θ+π)=tan (θ)

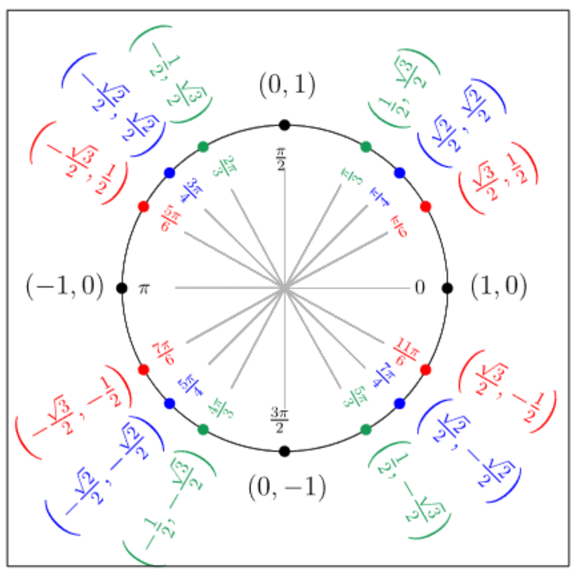
Example 2:

Students are given a sine graph on the interval -2π to 2π, with coordinates of the special angles labeled, including +/- multiples of $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$. Students will plot those coordinates on the unit circle and describe the relationship of the unit circle and the sine graph function. Students will repeat this process for cosine and tangent.

Let's look again at the graphs of the functions $f(x) = \sin(x)$ and $g(x) = \cos(x)$. Describe the symmetry of the graphs.



- *The graph of $f(x) = \sin(x)$ seems to be symmetric about the origin (or seems to have 180° rotational symmetry), and the graph of $g(x) = \cos(x)$ seems to be symmetric about the y-axis.*



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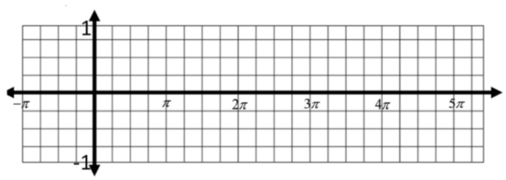
Example 3: The following is from lesson 10 of Engage NY Algebra II, Module 2.

Consider the function $f(x) = \sin(x)$ where x is measured in radians.
Graph $f(x) = \sin(x)$ on the interval $[-\pi, 5\pi]$ by constructing a table of values. Include all intercepts, relative maximum points, and relative minimum points of the graph. Then, use the graph to answer the questions that follow.

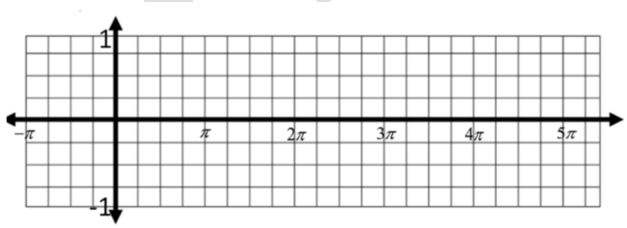
x	
$f(x)$	



- Using one of your colored pencils, mark the point on the graph at each of the following pairs of $\boldsymbol{x}$-values.
  $x = -\frac{\pi}{2}$ and $x = -\frac{\pi}{2} + 2\pi$
  $x = \pi$ and $x = \pi + 2\pi$
  $x = \frac{7\pi}{4}$ and $x = \frac{7\pi}{4} + 2\pi$
- What can be said about the y-values for each pair of x-values marked on the graph?
- Will this relationship hold for any two x-values that differ by 2π? Explain how you know.
- Based on these results, make a conjecture by filling in the blank below.
  For any real number x, $\sin(x + 2\pi) =$ ___________.

Consider the function $g(x) = \cos(x)$ where x is measured in radians.
Graph $g(x) = \cos(x)$ on the interval $[-\pi, 5\pi]$ by constructing a table of values. Include all intercepts, relative maximum points, and relative minimum points. Then, use the graph to answer the questions that follow.

x	
$g(x)$	



Have students repeat questions a-d from above, leading to the conjecture that for any real number x, $\cos(x + 2\pi) =$ ___________.

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Example 4:

Students will be asked to describe the symmetry of the three trigonometric functions to determine whether they are even or odd. Students will be asked if f(x)= sin(x), what are the values of f $(\frac{\pi}{6})$, f $(-\frac{\pi}{6})$, f $(\frac{5\pi}{6})$, f $(-\frac{5\pi}{6})$. Next, they repeat the process if f(x)=cos(x) and f(x)=tan(x).

And how does this relate to our understanding of the symmetry of the sine and cosine functions?

- Since $\sin(-\theta) = -\sin(\theta)$, $f(x) = \sin(x)$ is an odd function with symmetry about the origin. (If students do not suggest that the sine function is an odd function, remind them that an odd function is a function f for which $f(-x) = -f(x)$ for any x in the domain of f.)
- Since $\cos(-\theta) = \cos(\theta)$, $g(x) = \cos(x)$ is an even function with symmetry about the y-axis. (If students do not suggest that the cosine function is an even function, remind them that an even function is a function f for which $f(-x) = f(x)$ for any x in the domain of f.)

Use your understanding of the symmetry of the sine and cosine functions to determine the value of $\tan(-\theta)$ for all real-numbered values of θ. Determine whether the tangent function is even, odd, or neither.

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\frac{\sin(\theta)}{\cos(\theta)} = -\tan(\theta)$$

The tangent function is odd.