New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

COURSE: Algebra I	**DOMAIN: Number and Quantity (N) – The Real Number System (RN)**
CLUSTER: Students are thinking about the properties of rational and irrational numbers and how with each extension of number, the meanings of addition, subtraction, multiplication and division are extended. Using their knowledge of operations on rational numbers, students will perform operations on irrational numbers, using mathematical properties to simplify and combine expressions.	
Grade Level Standard: AI-N.RN.3a Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots. **Note: Tasks include rationalizing numerical denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number.**	

PERFORMANCE/KNOWLEDGE TARGETS (measurable and observable)	
• simplify, add, subtract, multiply, and divide expressions that contain square roots and/or rational numbers; • rationalize denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number; and • generate equivalent forms of square roots.	
ASPECTS OF RIGOR Procedural Conceptual Application	
MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-6. G.5** Use area and volume models to explain perfect squares and perfect cubes. **NY-8.NS.1** Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers that are not rational are called irrational. **NY-8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ **NY-8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form x^2 =p and x^3=p, where p is a positive rational number. Know square roots of perfect squares up to 225 and perfect cubes to 125. Know that the square root of a non-perfect square in irrational.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

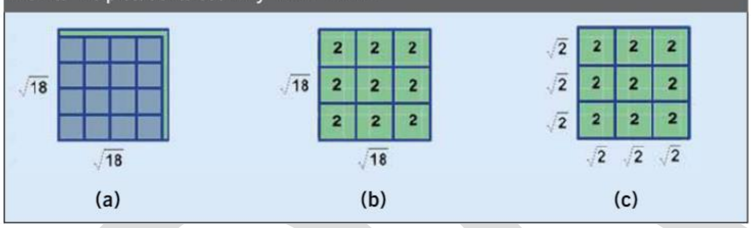
1. Equivalent forms of Rational Numbers: Lessons on infinite and finite decimal expansions of rational numbers can be found in EngageNY Grade 8 Module 7, topic B (lessons 6-14). Keep in mind that students may not have been exposed to representing a rational number expressed as a repeating decimal in fraction form. This is no longer a grade-level expectation for grade 8.

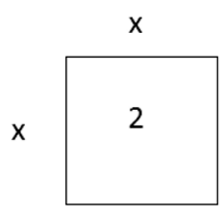
2. Simplifying Square Roots: Lessons on simplifying radicals, as well as performing operations (rationalizing denominators) can be found in EngageNY Grade 8 Module 7, lesson 4, as well as in the Geometry Module 2, lessons 22 and 23.

3. Visual/Geometric Representation for Simplifying Radicals (Taken from Radical Thoughts on Simplifying Square Roots, Kyle T. Schultz and Stephen F. Bismarck, Vol. 19, No. 4, November 2013 MATHEMATICS TEACHING IN THE MIDDLE SCHOOL). This representation connects with work done in grade 6 (NY-6. G.5) with respect to using area models to represent perfect squares.

To show the geometric simplification of $\sqrt{18}$, students use prior knowledge of perfect squares and realize that a square with an area of 18 square units cannot be partitioned into a square-shaped array. This square, nevertheless, can still be partitioned into a square-shaped array of smaller squares, provided that the area of these smaller squares is rational even if the side lengths are irrational. In this case, the square with an area of 18 square units can be partitioned into 9 squares, each with an area of 2 square units (see fig. 3 c). This new configuration provides an alternative way to calculate $\sqrt{18}$ by examining the length of the side of a square whose area is 18 square units. Notice that the side length of the large square is equivalent to three side lengths of one smaller square. The area of each small square is 2 square units, so the side length of each small square is $\sqrt{2}$ units. Thus, $\sqrt{18} = 3\sqrt{2}$. To connect this model to previous work with area models and perfect squares, the search for a perfect-square factor of 18 coincides with identifying how to partition the square into smaller squares with whole-number areas. The expression $\sqrt{9 \cdot 2}$ is equivalent to the length of the side of a square array of 9 squares each with an area of 2 square units. In this case, $\sqrt{2}$ is the length of a partitioned side of the large square into the nine small squares.

Fig. 3 A square with an area of 18 units^2 and small squares with a rational area of 2 units^2 help students see why $\sqrt{18} = 3\sqrt{2}$.





$x \cdot x = 2$
$x^2 = 2$
$x = \sqrt{2}$

$\sqrt{2} \cdot \sqrt{2} = \sqrt{4}$
$\sqrt{2} \cdot \sqrt{2} = 2$

Students can create visual/geometric representations for other radicals, i.e. $\sqrt{52}$, $\sqrt{24}$, $\sqrt{80}$**.**

4. Examples of Arithmetic Operations involving Radicals:

1. $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$
2. $\sqrt{2} + 3\sqrt{2} - 7\sqrt{2} = -3\sqrt{2}$
3. $\sqrt{2} + 2\sqrt{8} = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$
4. $\sqrt{3} + 3\sqrt{5} + 5\sqrt{3} - 4\sqrt{5} = \sqrt{3} + 5\sqrt{3} + 3\sqrt{5} - 4\sqrt{5} = 6\sqrt{3} - \sqrt{5}$
5. $3\sqrt{5} \cdot 4\sqrt{15} = 3 \cdot 4 \cdot \sqrt{5}\sqrt{15} = 12\sqrt{75} = 12\sqrt{25}\sqrt{3} = 12 \cdot 5\sqrt{3} = 60\sqrt{3}$
6. $\frac{3\sqrt{14}}{2\sqrt{7}} = \frac{3}{2}\sqrt{\frac{14}{7}} = \frac{3}{2}\sqrt{2}$
7. $\frac{5}{\sqrt{6}} = \frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$
8. $\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$
9. $\frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$