Course I Outline

I. Logic 17

A. Truth Value of Simple Sentences 17

1. Closed sentences
2. Open sentences
   a. Replacement set
   b. Solution set
3. Negations

B. Truth Value of Compound Sentences 18

1. Conjunction
2. Disjunction
3. Conditional
4. Related conditionals
   a. Converse
   b. Inverse
   c. Contrapositive
5. Biconditional

C. Tautologies 23

1. Proof using truth tables
2. Applications

II. Algebra 27

A. Operations on Algebraic Expressions 27

1. Review of operations with signed numbers
2. Use of variables
   a. Order of operations
   b. Evaluating algebraic expressions
3. Addition of polynomials
   a. Properties
   b. Combining like terms
   c. Combining fractions with like denominators
4. Multiplication of polynomials
   a. Powers ($x^a \cdot x^b = x^{a+b}$)
   b. Products using monomials
   c. Products of binomials
   d. Equivalent fractions
   e. Combining fractions with unlike denominators
   f. Multiplication of fractions
5. Simplification of algebraic expressions using addition and multiplication

6. Division of polynomials by monomials
   a. Powers ($x^a \div x^b = x^{a-b}$)
      1. Zero and negative exponents
      2. Scientific notation
   b. Simplification of fractions
   c. Division of fractions
7. Factoring
   a. Prime factorization
   b. Common monomial
   c. Binomial factors of a trinomial
   d. Binomial factors of the difference of two squares

B. Solution of Equations and Inequalities 32

1. Open sentences
   a. Solution set
   b. Representation of solution set on number line
2. Techniques for solving linear equations and inequalities
3. Problem solving (linear equations)
4. Techniques for solving quadratic equations
   a. Zero property of multiplication
   b. Solution by factoring
   c. Solution by quadratic formula (optional)
5. Problem solving (quadratic equations)

C. Ratio and Proportion 35

1. Ratio
2. Dimensional Analysis
3. Proportion
4. Solution of proportions
5. Applications
   a. Percent
   b. Similar figures
   c. Direct variation
6. Problem solving

D. Real Numbers 37

1. Rational and irrational numbers
2. Examples of irrational numbers
   a. Non-repeating decimals
   b. Irrational roots
   c. π
3. Rational approximation of irrational numbers
4. Operations with radicals
   a. Simplification
   b. Multiplication and division
   c. Addition and subtraction
5. Applications
   a. Solution of incomplete quadratic equations
   b. Problems involving the Pythagorean theorem
   c. Use of quadratic formula (optional)
6. Problem solving

III. Geometry 41

A. Undefined Terms—Point, Line, and Plane 41

B. Review of Geometric Figures 41

1. Parallel and intersecting lines
2. Angles
   a. Degree measure
   b. Right, acute, obtuse, straight
   c. Supplementary and complementary
   d. Vertical
   e. Alternate interior, corresponding
3. Perpendicular lines
4. Simple closed curves
   a. Polygons
   b. Circle

C. Triangles 43

1. Classification
   a. Scalene, isosceles, equilateral
   b. Acute, right, obtuse
2. Sum of the measures of the angles of a triangle
3. Exterior angle of a triangle
4. Base angles of an isosceles triangle
5. Comparison of triangles
   a. Congruence
   b. Similarity
6. Pythagorean Theorem

D. Quadrilaterals 45

1. Classification and properties
   a. Parallelogram
   b. Rectangle
   c. Rhombus
   d. Square
   e. Trapezoid
2. Sum of the measures of the angles

E. Polygons 46

1. Review of polygons
2. Perimeter
3. Area
4. Similar polygons
   a. Ratio of perimeters
   b. Ratio of areas

F. Circle 48

1. Circumference
2. Area

G. Application 49

1. Problem solving

H. Transformations 50

1. Intuitive notions
   a. Line reflection
   b. Translation
   c. Rotation
   d. Dilation
2. Congruence and similarity

3. Symmetry
   a. Line
   b. Point

I. Solids 53

1. Classification
   a. Prism
   b. Rectangular solid
   c. Pyramid
   d. Right circular cylinder
   e. Cone
   f. Sphere
2. Volume
3. Comparison of volumes of similar solids

IV. Analytic Geometry 55

A. Review of Rectangular Coordinate System 55

B. Linear Relations and Slope 55
   1. Graphing linear relations
   2. Slope of a line
   3. Slope intercept form

C. Direct Variation 56

D. Graphing Linear Inequalities 57

E. Graphic Solution of Systems of Linear Equations 58

F. Algebraic Solution of Systems of Linear Equations 59
   1. Substitution method
   2. Addition-subtraction method
   3. Application
   4. Problem solving

G. Graphic Solution of Systems of Linear Inequalities 60

H. Area Using Coordinates 60

I. Optional Graphing 61
   1. $y = |x|$
   2. $y = |x|$
   3. $|x| + |y| = c$
   4. $y = ax^2$
   5. $y = ax^2 + b$

V. Probability 63

A. Review of Terms 63

B. Probability of Single Events 63
   1. Theoretical vs. empirical probability
   2. Simple probability and notation
   3. Problems involving AND
   4. Problems involving OR
   5. Probability of the complement of an event

C. Counting Procedures 67
   1. Sample space
      a. List of ordered pairs or n-tuples
      b. Tree diagrams
      c. Dot graphs

2. Counting Principle
3. Permutations
   a. Factorial notation
   b. ${}_nP_n$
   c. ${}_nP_r$

D. Probability of Events Occurring Jointly 71

1. Extension of Counting Principle

VI. Statistics 73

A. Collecting and Organizing Data 73

1. Sampling or census
2. Tally, chart, frequency table
3. Frequency histogram
4. Cumulative frequency histogram

B. Measures of Central Tendency 75

1. Mean
2. Median
3. Mode

C. Other Significant Measures 75

1. Quartiles
2. Percentiles

A Note on Time and Sequence

It is difficult to supply a meaningful time frame for an integrated course since it is intended that topics interact with each other in the development of the material. The following chart should be used only as a general frame of reference.

Topic	*Days*
I. Logic	10–15
II. Algebra	55–65
III. Geometry	35–45
IV. Analytic Geometry	15–20
V. Probability	5–10
VI. Statistics	5–10

Depending on the students' backgrounds, time allotments for one or more of these topics may vary significantly.

It should be noted that the time suggestion is not intended to imply that these topics should be taught in this order or as separate blocks of material. Algebraic techniques will be used throughout the course in the development of other topics.

Course II Outline

I. Logic 15

A. Review of Logic from Course I 15

1. Logical connectives and truth values
2. Related conditionals
   a. converse
   b. inverse
   c. contrapositive

B. Laws of Reasoning 16

1. Law of Detachment
2. Law of Contrapositive
3. Law of Disjunctive Inference
4. Law of Syllogism
5. DeMorgan's Laws

C. Development of Notions of Proof 20

1. Direct
2. Indirect

II. Mathematical Structure and Related Algebraic Skills 23

A. Review of Properties of the Real Numbers 23

1. Closure
2. Commutative property
3. Associative property
4. Identity element
5. Inverse element
6. Distributive property
7. The importance of the properties of the real numbers

B. Review of Algebraic Skills 26

1. Addition of polynomials
2. Multiplication of polynomials
3. Simplification of algebraic expressions
4. Division of polynomials by monomials
5. Factoring
6. Solution of linear equations

C. Extension of Algebraic Skills 29

1. Factoring
2. Simplifying algebraic fractions with polynomial denominators
3. Multiplication and division of fractions with polynomial denominators
4. Addition and subtraction of fractions

D. Groups and Fields (Optional) 31
1. Groups
a. definition
b. infinite systems
c. finite systems
d. theorems
2. Fields
a. definition
b. applications

III. Euclidean Geometry 35
A. Axiomatic Systems 35
1. Undefined terms and definitions
2. Assumptions
3. Theorems and proof
B. Congruence 36
1. Assumptions
2. Isosceles triangle theorems
3. Other congruence theorems (AAS, HL)
C. Inequalities 37
1. Angle relationships
2. Relationships of lengths of sides
D. Parallelism 37
1. Definitions and assumptions
2. Theorems
3. Angle measure in triangles
4. Classification and properties in quadrilaterals
E. Similarity 39
1. Definition of similar polygons
2. Similar triangles
3. Pythagorean relationship
4. Special right triangles
5. Right triangle trigonometry
F. Constructions and Proofs 41
1. Basic constructions
2. Other constructions

IV. Analytic Geometry 45
A. Review of the Rectangular Coordinate System 45
B. Parallel and Perpendicular Lines 45
1. Parallel lines
2. Perpendicular lines
C. Formulas 46
1. Length of a segment
2. Midpoint of a segment
D. Equation of a Line 47
1. Point-slope form
2. Slope-intercept form

E. Area 48
1. Review of formulas
2. Application to coordinate geometry
F. Transformations 49
1. Reflection in a line
2. Reflection in a point
3. Translations
4. Dilations
G. Conics 51
1. Circle
2. Parabola
3. Other conics
4. Graphic solution of systems of equations
H. Locus 53
1. Definition
2. Basic loci
a. At a fixed distance from a point
b. At a fixed distance from a line
c. Equidistant from two points
d. Equidistant from two parallel lines
e. Equidistant from two intersecting lines
3. Compound locus
4. A special locus
I. Analytic Proof 54
V. Equations and Inequalities 57
A. Review 57
1. Linear equations and inequalities
2. Proportions
B. Fractional Equations 59
C. Quadratic Equations 59
1. Solution by factoring
2. Solution by completing the square
3. Solution by formula
4. Applications
D. Pairs of Equations 62
1. Review of pairs of linear equations
2. Quadratic-linear pair
VI. Probability and Combinatorics 65
A. Review of Fundamentals 65
1. Probability concepts
2. Permutation concepts
a. n objects, all different, taken n at a time
b. n objects, all different, taken r at a time, $r \leq n$
B. Extension of Work with Permutations 66
1. Number of permutations of n objects, r identical, taken all at a time
2. Number of permutations of n objects, r identical, s identical, t identical, etc., taken all at a time

C. Combinations 67

1. Distinction between combination and permutation
2. Combinations of n objects, taken r at a time, $r \leq n$
3. Applications

A Note on Time and Sequence

It is difficult to supply a meaningful time frame for an integrated course since it is intended that topics interact with each other in the development of the material. The following chart should be used only as a general frame of reference.

Topic	*Days*
I. Logic	10 - 15
II. Mathematical Structure and Related Algebraic Skills	10 - 15
III. Euclidean Geometry	60 - 65
IV. Analytic Geometry	30 - 35
V. Equations and Inequalities	15 - 20
VI. Probability and Combinatorics	10 - 15
TOTAL	135-165

Depending on the students' backgrounds, time allotments for one or more of these topics may vary significantly.

The above chart is not intended to imply that the topics should be taught in that order or as separate blocks of material.

Course III Outline

I. **The Complex Number System** 15

A. Review of the real numbers 15
1. Subsets of the reals
2. Field properties

B. Review and extension of algebraic techniques 16
1. Factoring
2. Laws of exponents
3. Simplifying expressions with integral exponents (positive, negative, zero)
4. Simplifying radical expressions
5. Rationalizing denominators of fractions with radicals
6. Simplifying algebraic fractions
7. Multiplication and division of fractions
8. Addition and subtraction of fractions
9. Simplifying complex fractions

C. Review and extension of solutions of equations 18
1. Absolute value
2. Quadratic (factoring and quadratic formula)
3. Systems of equations (linear and quadratic)
4. Fractional equations
5. Equations with radicals

D. Review and extension of solutions of inequalities 20
1. Linear
2. Absolute value
3. Quadratic

E. Complex Numbers 21
1. The imaginary unit
2. Standard form of complex numbers
3. Operations (addition, subtraction, multiplication, division) with complex numbers
4. Solution of quadratic equations with complex roots
   a. nature of the roots of a quadratic
   b. sum and product of roots
   c. relationship between the quadratic equation and the quadratic function
   d. relationship between the nature of the roots of the quadratic equation and the graph of the quadratic function

II. **Transformations** 25

A. Review 25
1. Line reflection
2. Rotation
3. Translation
4. Dilation

B. Composition 27
1. General concepts and symbolism
2. Line reflections
a. in parallel lines
b. in intersecting lines
3. Rotations
4. Translations
5. Glide reflections

C. Isometries 29
1. Direct
2. Opposite

D. Application to Graphing 30
1. Inverse functions
2. Symmetries

III. Relations and Functions 33

A. Relations 33
1. Definition
2. Domain and range

B. Functions 34
1. Definition
2. Determining if a relation is a function
3. Inverse variation and the hyperbola
4. One-to-one functions

C. Inverse functions 36
1. Definition and characteristics

D. The Exponential Function 37
1. Rational numbers as exponents
2. The function, $y = a^x$

E. The Logarithmic Function 39
1. $y = \log_a x$ as the inverse of $y = a^x$
2. Working with logarithms
3. Common logarithms
4. Laws used in computation with logarithms
5. Applications

IV. Circles 41

A. Arcs, Angles, and Chords 41
1. Definitions
2. Theorems related to congruence
3. Theorems related to angle measure

B. Chords, Secants, and Tangents 42
1. Definitions
2. Theorems related to angle measure
3. Theorems related to length of segments

V. Trigonometric Functions and Their Inverses 45

A. Definitions 45
1. Angles as rotations
2. Unit circle
3. Radian measure

4. Sine and cosine
5. Tangent
6. Coordinates of a point
7. Reciprocal trigonometric functions

B. Basic Identities 48

C. Function Values 48
1. Special angles
2. Use of tables
3. Reference angles

D. Graphs 49
1. Graphs of basic functions
2. Amplitude and period

E. Related Relations and Functions 49
1. Reflections in the line y = x
2. Inverse functions

VI. Trigonometric Identities and Equations 51

A. Identities 51
1. Review and extension of basic relationships
2. Proving trigonometric identities

B. Equations 52
1. Linear equations
2. Quadratic equations

C. Functions of Sums and Differences of Angles 54
1. Sine, cosine, tangent
2. Applications

D. Double and Half Angle Formulas 56
1. Double angle formulas: sin 2x, cos 2x, tan 2x
2. Half angle formulas: sin x/2, cos x/2, tan x/2

VII. Trigonometry - Triangle Solution 59

A. Area of a Triangle 59

B. General Solution of Triangles 59
1. Law of Sines
   a. finding a side given ASA or AAS
   b. the ambiguous case (SSA)
   c. finding a side given SSA
2. Law of Cosines
   a. finding a side given SAS
   b. finding an angle given SSS
3. Applications

C. Right Triangle Trigonometry 62
1. Definitions of sin A, cos A, and tan A in terms of the legs and hypotenuse of a right triangle
2. Applications

VIII. Probability 65

A. Review of Work with Combinations 65

B. Probability of Exactly r Successes in n Trials of a Bernoulli Experiment 66
1. Definition of Bernoulli experiment
2. Case where r successes are assumed to occur first
3. General case
4. Applications
C. The Binomial Theorem 68

IX. Statistics 69
A. Review of the Measures of Central Tendency 69
B. Σ - Notation 69
C. Measures of Dispersion 70
1. Range
2. Mean absolute deviation
3. Variance
4. Standard deviation
a definition
b. applications
c. grouped data
D. Bell Curves 73

A Note on Time and Sequence

It is difficult to supply a meaningful time frame for an integrated course since it is intended that topics interact with each other in the development of the material. The following chart should be used only as a general frame of reference.

	Topic	*Days*
I.	The Complex Number System	35 – 37
II.	Transformations	10 – 12
III	Relations and Functions	21 – 23
IV.	Circles	10 – 12
V.	Trigonometric Functions and Inverses	25 – 27
VI.	Trigonometric Identities and Equations	10 – 12
VII.	Triangle Trigonometry	10 – 12
VIII.	Probability	8 – 10
IX.	Statistics	8 – 10
	TOTAL	137 –155

Depending on the students' backgrounds, time allotments for one or more of these topics may vary significantly.

The above time suggestion is not intended to imply that these topics should be taught in this order or as separate blocks of material. For example, algebraic techniques will be used throughout the course in the development of other topics.