Utilizing the New Teacher-Support Features

New York State Next Generation Mathematics Learning Standards 2017

Andrew Mitchell
Andrew.Mitchell@wflboces.org
@gadfly1974

Paul Volkert
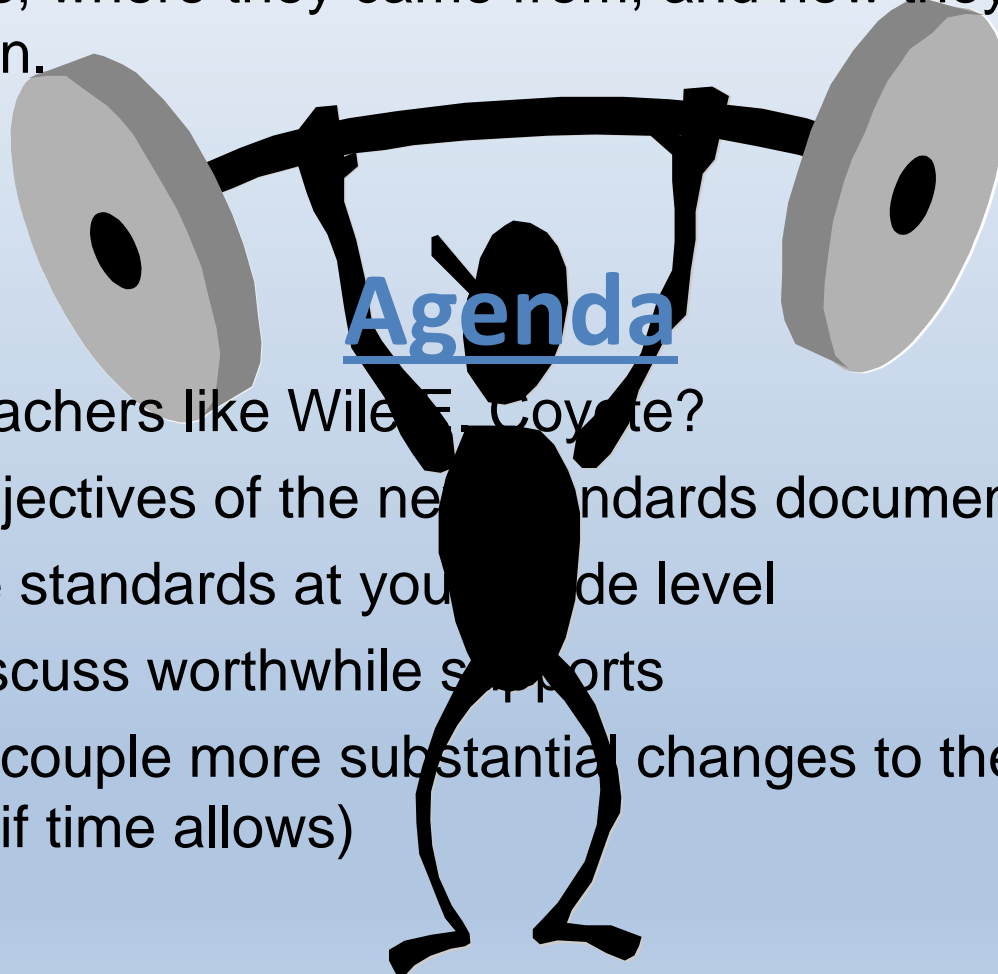pvolkert@btboces.org
@VolkertMath

SCDN Math Team

Goal

Gain more insight into the types of support features built into the new standards, where they came from, and how they support implementation.

Agenda

1. How are teachers like Wile E. Coyote?
2. Vision & objectives of the new standards document
3. Dig into the standards at your grade level
4. Share & discuss worthwhile supports
5. Highlight a couple more substantial changes to the *content* of the standards (if time allows)

In 2012, at UC Berkeley, Bill McCallum talks about the worthwhile residue left behind when the CCSS-M collapses. He offers, for example, that well developed research-based curricula may endure and benefit students long after the CCSS-M.

As NYS moves forward from the CCSS, we wanted to:

- Keep some the good parts.
- Leave behind what didn't work for us.
- Make improvements and upgrades wherever possible.

Counting & Cardinality   K.CC

Know number names and the count sequence.
1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.
4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
   d. *Develop understanding of ordinal numbers (first through tenth) to describe the relative position and magnitude of whole numbers.*
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.
6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

___
[1] Include groups with up to ten objects.

PARCC Model Content Frameworks

Mathematics

Grades 3–11

Version 4.0
December 2014

PARCC Model Content Frameworks

A Companion to the Common Core State Standards

Mathematics:

Kindergarten through Grade 2

September 2014

Examples of Key Advances from Kindergarten to Grade 1

- Students gradually come to employ mental strategies (such as counting on and making ten) that make use of embedded concepts of number and the properties of addition and subtraction; by contrast, kindergarten students determine sums and differences primarily by representing problems with objects or drawings.

## Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

Examples of Major Within-Grade Dependencies

- 1.NBT.B.2 describes the place-value foundations for 1.NBT.B.3 and 1.NBT.C.4. Comparing numbers (1.NBT.B.3) involves thinking about the sizes of tens and ones, and adding two-digit numbers (1.NBT.C.4) involves adding tens with tens and ones with ones, and sometimes composing a ten. These ideas and methods rest on an understanding of the place-value units and the use of visual models of these units in solving and explaining problems using these standards.

The importance of specifying the whole
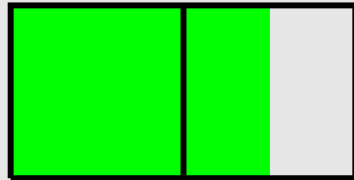
Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded [...] *entire rectangle is the* [...]

ore

draft)

The word *fluent* is used in the Standards to mean "fast and accurate." Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students. The extensive work relating addition and subtraction means that subtraction can frequently be solved by thinking of the related addition, especially for smaller numbers. It is also important that these patterns, strategies and decomposi-

Draft, 5/29/2011, comment at commoncoretools.wordpress.com.

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Mathematics

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K.CC

begin at 1).
representing a

with one and only

c. Understand that each suc

d. *Develop understanding* ==*magnitude of whole num*==

5. Count to answer "how many?"
circle, or as many as 10 things in

Compare numbers.

6. Identify whether the number of
another group, e.g., by using mato
7. Compare two numbers between

Progressions for the Common Core
State Standards in Mathematics (draft)

©The Common Core Standards Writing Team

7 April 2011

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[1] Include groups with up to ten objects.

- Next Generation Math Learning Standards Doc

- Compare how page 37 in the NGMS is different from page 18 in the CCLS (handout)
  - What looks different?
  - Was anything added or taken away?
  - What do you like?
  - What do you have questions about?

URL for the NGMS document

goo.gl/Ukh3Bq

How is this different from page 37 in the NGMS document?
-Was anything added or taken away?
-What do you like? What questions you have?

Operations & Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.
1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.
2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.
3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

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[1] See Glossary, Table 1.
[2] See standard 1.OA.6 for a list of mental strategies.

Standards Document – A better way!

NY-2.OA

Operations and Algebraic Thinking

NY-2.OA

Standards Document – A better way!

NY-2.OA   **Operations and Algebraic Thinking**

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

Coherence: NY-1.OA.6 → NY-2.OA.2

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Standards Document – A better way!

NY-2.OA — Operations and Algebraic Thinking

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

- counting on;

Coherence: NY-1.OA.6 → NY-2.OA.2

Levels	8 + 6 = 14	14 − 8 = 6
Level 1: Count all	Count All a 1 2 3 4 5 6 7 8 1 2 3 4 5 6 ○○○○○ ○○○ ○○○○○ ○ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ○○○○○ ○○○○○ ○○○○ 1 2 3 4 5 6 7 8 1 2 3 4 5 6 b c
Level 2: Count on	Count On 8 (○○○○○ ○○○) ○○○○○ ○ →8 9 10 11 12 13 14	To solve 14 − 8 I count on 8 + ? = 14 10 11 12 9 13 I took away 8 14 8 to 14 is 6 so 14 − 8 = 6

Standards Document – A better way!

NY-2.OA — Operations and Algebraic Thinking

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

- counting on;

- making ten;

- decomposing a number leading to a ten;

- using the relationship between addition and subtraction; and

- creating equivalent but easier or known sums.

2b. Know from memory all sums within 20 of two one-digit numbers.

Coherence: NY-1.OA.6 → NY-2.OA.2

Levels	$8 + 6 = 14$	$14 - 8 = 6$
Level 1: Count all	Count All	Take Away
Level 2: Count on	Count On	To solve $14 - 8$ I count on $8 + ? = 14$ I took away 8 8 to 14 is 6 so $14 - 8 = 6$

e.g., $8 + 6 =$
$8 + 2 + 4 =$
$10 + 4 = 14$

e.g., $13 - 4 =$
$13 - 3 - 1 =$
$10 - 1 = 9$

e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$

e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$

e.g., $13 - 4 =$
$13 - 3 - 1 =$
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Standards Document – A better way!

NY-2.OA — Operations and Algebraic Thinking

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

- counting on;

Coherence: NY-1.OA.6 → NY-2.OA.2

Note on *Fluency* with Facts:
- *Fluently* adding and subtracting within 20 (NY-2.OA.2) means students can find sums and differences within 20 reasonably quickly, and say or write it. Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.[10] Reaching fluency will take much of the year for many students. For more on how children develop fluency, see *K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*, pp. 18-19 and *Adding it Up*, pp. 182-195.

Note on *Fluency* vs. *Knowing from Memory*:
- The standards intentionally distinguish between asking for *fluency* with addition and subtraction (NY-2.OA.2a) and asking students to *know from memory* addition facts (NY-2.OA.2b). *Fluency* means students are fast, accurate, flexible, and have understanding. They use strategies efficiently.[12] By the end of the K–2 grade span, students have sufficient experience with these strategies to *know from memory* all single-digit sums.[10]

Standards Document – A better way!

Note on *Fluency* with Procedures:

- Fluency with procedures (procedural fluency) means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure. [12] [13] For more on developing procedural fluency, see *Adding it Up*, pp. 121-124.

Note on Manipulatives in Grades K-2:

- A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[9]

Within-Grade Connections:

- When students use the making ten strategy (NY-1.OA.6), they are applying the Associative property of addition (NY-1.OA.3). [9] For example, when solving 8 + 3 by "making a ten," a student decomposes the 3 into 2 + 1 in order to re-associate the 2 with the 8, making ten + 1.

Connecting the Standards for Mathematical Practice to Mathematical Content:

- Grade 1 students work with some sophisticated addition and subtraction situations (NY-1.OA.1), such as "Lucy has 8 fewer apples than Julie. Julie has 12 apples. How many apples does Lucy have?" Making a math drawing or using objects to model the situation is very helpful for students (MP.5). The equations 12 – 8 = ?, 8 + ? = 12, and ? + 8 = 12 are all mathematical models of this situation (MP.4). [9]

Standards Document – A better way!

Add and subtract within 20.

2a. Fluently add and subtract within 20 using mental strategies. Strategies could include:

- counting on;

- making ten;

- decomposing a number leading to a ten;

Coherence: NY-1.OA.6   →   NY-2.OA.2

Levels	$8 + 6 = 14$	$14 - 8 = 6$
Level 1: Count all	Count All	Take Away
Level 2: Count on	Count On	To solve $14 - 8$ I count on $8 + ? = 14$

e.g., $8 + 6 =$
$8 + 2 + 4 =$
$10 + 4 = 14$

e.g., $13 - 4 =$
$13 - 3 - 1 =$
$10 - 1 = 9$

Standards Document – A better way!

AI-F.BF

Algebra I

AI-F.BF	Functions Building Functions	Algebra I

Build new functions from existing functions.

3a. Using $f(x) + k$, $k\,f(x)$, and $f(x + k)$:

   i)   identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative);

   ii)   find the value of k given the graphs;

   iii) write a new function using the value of k; and

   iv) use technology to experiment with cases and explore the effects on the graph.

(Shared standard with Algebra II)

Coherence: AI-F.BF.3a → AII-F.BF.3b

Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ $(b \neq 1)$.[14]

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

(Shared standard with Algebra II)

Standards Document – A better way!
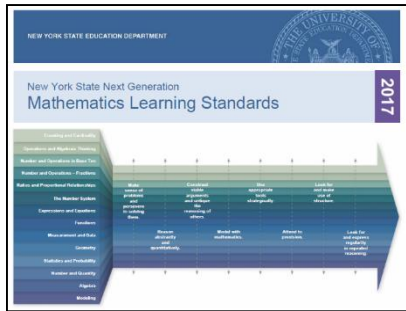
- Standards tagging is distinct from, but connected to CCSS
- Embed support at point-of-use:
  - "Coherence Links" to show the vertical coherence of the standards and help teachers differentiate
  - Notes and illustrations to clarify individual standards, answer FAQs, or otherwise support implementation
  - "Within-Grade Connections" to show horizontal coherence
  - Notes highlighting connections between the Standards for Mathematical Practice and content standards
  - footnotes from the original standards
- Linked navigation
- HS standards organized by course (not by Conceptual Category) and tagged to make the course clear
- *Algebra I* and *Algebra II* shared standards clearly marked

Treasure Hunt

Utilizing the New Teacher-Support Features Built Into the NYS Next Generation
Mathematics Learning Standards Document

Recording Template, p. 1

Number Reversals PK - K	
Manipulatives K - 2	
Note on Explore PK - K, A1 - (+)	
Fluency with Facts K - 3	
Fluency vs Memory 2 - 3	
Fluency with Procedures 2 - A2	
Illustration merged from CCSS appendix PK - 5	
Illustration merged from outside source or new K - 4	

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Recording Template, p. 1

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Fluency with Procedures 2 - A2	
Illustration merged from CCSS appendix PK - 5	
Illustration merged from outside source or new K - 4	

Share & Discuss

BT BOCES ENHANCED DOCUMENT
btboces.org/MathPD.aspx

Thank You!

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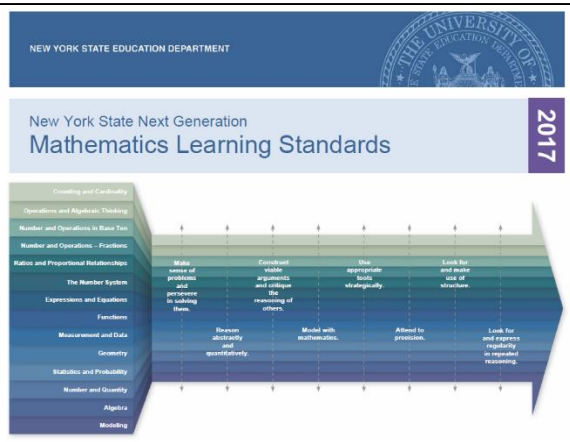
John Svendsen (Math Associate)
John.Svendsen@nysed.gov

Sue Brockley (Math Associate)
Susan.Brockley@nysed.gov

Andrew Mitchell
Andrew.Mitchell@wflboces.org
@gadfly1974

Paul Volkert
pvolkert@btboces.org
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