Scaffolding Instruction for All Students:

A Resource Guide for Mathematics

Grade 8

The University of the State of New York
State Education Department
Office of Curriculum and Instruction
and Office of Special Education
Albany, NY 12234

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Acknowledgements

The New York State Education Department Office of Curriculum and Instruction and Office of Special Education gratefully acknowledge the following individuals for their valuable contributions in the development of this guide:

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January 2020

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Introduction

The Next Generation English Language Arts (ELA) and Mathematics Learning Standards intend to foster the 21st century skills needed for college and career readiness and to prepare students to become lifelong learners and thinkers. Learning standards provide the "destination" or expectation of what students should know and be able to do while teachers provide the "map" for getting there through high-quality instruction. **Lessons need to be designed to ensure accessibility to a general education curriculum designed around rigorous learning standards for all students, including students who learn differently (e.g., students with disabilities, English Language Learners (ELLs)/Multilingual Learners(MLLs), and other students who are struggling with the content).** It is vital that teachers utilize a variety of research-based instructional and learning strategies while structuring a student-centered learning environment that addresses individual learning styles, interests, and abilities present among the students in the class. Classrooms should be supportive and nurturing, and factors such as the age, academic development, English and home language proficiency, culture and background knowledge, and disability, should be considered when designing instruction. The principles of Universal Design for Learning should be incorporated into curricula to provide students with learning experiences that allow for multiple means of representation, multiple means of expression, and multiple means of engagement. These learning experiences will reduce learning barriers and foster equal learning opportunities for all students.

The purpose of these guides is to provide teachers with examples of scaffolds and strategies to supplement their instruction of ELA and mathematics curricula. Scaffolds are instructional supports teachers intentionally build into their lesson planning to provide students support that is "just right" and "just in time." Scaffolds do not differentiate lessons in such a way that students are working on or with different ELA texts or mathematical problems. Instead, scaffolds are put in place to allow all students access to grade-level content within a lesson. Scaffolds allow students to develop the knowledge, skills, and language needed to support their own performance in the future and are intended to be gradually removed as students independently master skills.

The scaffolds contained in these guides are grounded in the elements of explicit instruction as outlined by Archer and Hughes (2011). Explicit instruction is a structured, systematic approach to teaching which guides students through the learning process and toward independent mastery through the inclusion of clear statements regarding the purpose and rationale for learning the new skill/content; explanations and demonstrations of the instructional target; and supported practice with embedded, specific feedback.

The scaffolds in these guides can be adapted for use in any curricula and across content areas. While the exemplars were all drawn from the ELA and mathematics EngageNY modules, teachers are encouraged to customize the scaffolds in any lesson they deem appropriate. **All teachers (e.g., general, special education, English as a New Language, and Bilingual Education teachers) can use these scaffolds in any classroom setting to support student learning and to make the general education curriculum more accessible to all students without interfering with the rigor of the grade-level content.**

How to Use These Guides

The provision of scaffolds should be thoughtfully planned as to not isolate or identify any student or group of students as being “different” or requiring additional support. Therefore, in the spirit of inclusive and culturally responsive classrooms, the following is suggested:

- Make scaffolded worksheets or activities available to all students.
- Heterogeneously group students for group activities when appropriate.
- Provide ELLs/MLLs with opportunities to utilize their home language knowledge and skills in the context of the learning environment.
- Make individualized supports or adapted materials available without emphasizing the difference.
- Consistently and thoughtfully use technology to make materials more accessible to all students.

In the ELA guides, the *Table of Contents* is organized to allow teachers to access strategies based on the instructional focus (reading, writing, speaking and listening, and language) and includes a list of scaffolds that can be used to address those needs. In the mathematics guides, the *Table of Contents* is organized around the scaffolds themselves.

Each scaffold includes a description of what the scaffold is, who may benefit, and how it can be implemented in a lesson-specific model (see graphic below). Teachers are encouraged to make changes to presentation and language to best support the learning needs of their students. While lessons from the EngageNY modules are used to illustrate how each scaffold can be applied, **the main purpose of the exemplars is to show how teachers can incorporate these scaffolds into their lessons as appropriate.**

Title of Scaffold
Module: Unit: Lesson:
Explanation of scaffold: This section provides a deeper explanation of the scaffold itself including what it is and how it can and should be used. This section is helpful when implementing the scaffold in other lessons.
Teacher actions/instructions: This section provides specific instructions for the teacher regarding successful implementation of the scaffold.
Student actions: This section describes what the students are doing during the scaffolded portion of the lesson.
Student handouts/materials: This section indicates any student-facing materials that must be created to successfully use this scaffold.

Warm-up Review

Exemplar from:
Module 1: Topic A: Lesson 2: Multiplication of Numbers in Exponential Form

Explanation of scaffold:
This scaffold provides students with the opportunity to review previously learned skills and concepts that are needed to build a strong foundation for new lesson material. Establishing a warm-up review routine at the beginning of class allows students to connect with prior knowledge and allows teachers to quickly assess student understanding of key concepts, build automaticity and fluency of important skills and concepts, and give targeted corrective feedback.

Teacher actions/instructions:
A warm-up review can be used at the beginning of class to engage students and activate prior knowledge, before introducing a new lesson, when reteaching skills and concepts to small groups of students, and as homework for struggling students. The procedures of the routine for completion of a warm-up review should be explicitly taught to students at the beginning of the school year.

The following is a model of how a warm-up review could be used to focus on the prerequisite skills needed for this lesson:

1. Display a large version of the *Warm-up Review* sheet on chart paper, or use a document camera to project your work, and hand out student copies.
2. Give students five minutes to complete warm-up.
   - Walk about the classroom, and monitor student work.
   - Give corrective feedback to individual students as needed.
   - Give struggling students the option to work with a partner.
   - Review the definitions of exponent and base:
     - The **exponent,** or power, tells us how many times to use a number as a factor.
     - The **base** is the number to be multiplied.
   - Remind students of information from previous lessons:
     - Exponents are used to express repeated multiplication of the same factor.
     - When a negative number is raised to an even power, the sign of the answer is positive.
     - When a negative number is raised to an odd power, the sign of the answer is negative.
3. Review answers as a class.
   - Have students explain steps;
   - Review steps, but involve students by eliciting unison responses; or
   - Have students use thumbs up/thumbs down to indicate agreement/disagreement with answers. Have them explain why.

Answers to Problem 1:

a. 3^5
b. $5^2 \times 7^2 \times 11$

Answers to Problem 2:

a. $2^2 \times 3^2 \times 5^2 = 900$
b. $(\frac{1}{5})^4 = \frac{1}{625}$

Answer to Problem 3:

- When a negative number is raised to an even power, the sign of the answer is positive.
  - Student uses an equation to demonstrate. For example:
    - $(-5)^2 = (-5)\,(-5)$
      $= 25$
    - $(-2)^4 = (-2\,)(-2)\,(-2)\,(-2)$
      $= (4)\,(4)$
      $= 16$

Answer to Problem 4:

- When a negative number is raised to an odd power, the sign of the answer is negative.
  - Student uses an equation to demonstrate. For example:
    - $(-5)^3 = (-5)\,(-5)\,(-5)$
      $= (25)(-5)$
      -125
    - $(-2)^5 = (-2\,)(-2)\,(-2)\,(-2)(-2)$
      $= (4)\,(4)\,(-2)$
      $= (16)\,(-2)$
      $= -32$

Student actions:
Students complete the *Warm-up Review* sheet and participate in the warm-up review routine as directed.

Student handouts/materials:
Warm-up Review sheet (found on the next page)

Warm-up Review
Module 1, Topic A, Lesson 2: Multiplication of Numbers in Exponential Form

Name ______________________________ **Date** ____________________

1. Rewrite using exponents.

   a. $3 \times 3 \times 3 \times 3 \times 3$ ______________

   b. $5 \times 5 \times 7 \times 7 \times 11$ ______________

2. Rewrite using exponents, and solve the problem.

   a. $2 \times 2 \times 3 \times 3 \times 5 \times 5$ ______________________

   b. $(-\frac{1}{5}) \times (-\frac{1}{5}) \times (-\frac{1}{5}) \times (-\frac{1}{5})$______________________

3. When a negative number is raised to an even power, what is the sign of the result? Use an equation to demonstrate why.

____.

Equation:

4. When a negative number is raised to an odd power, what is the sign of the result? Use an equation to demonstrate why.

   ____.

   Equation:

Guided Notes with Partially Completed Problems

Exemplar from:
Module 2: Topic C: Lesson 11: Definition of Congruence and Some Basic Properties

Explanation of scaffold:
This scaffold supports students who require new information to be presented in smaller steps and increased opportunities to respond. It provides a structure in which difficult tasks are broken down and student practice is guided. When completed, guided notes with partially completed problems serve as a useful reference tool.

Teacher actions/instructions:
Guided notes with partially completed problems can be used with individuals, small groups, or the whole class when introducing a new skill or concept. It is best to use following a review of prerequisite skills or an opening activity and should be used in combination with teacher materials. Monitor student responses, adjust instruction as needed, and fade to independent practice of the skill being taught.

The following is a model of how guided notes with partially completed problems could be used to provide structure and guidance for introducing the definition of congruence as "a sequence of basic rigid motions that maps one figure onto another":

1. Hand out student copies of the *Guided Notes*.
2. Proceed through *Example 1* found on pages 130-131 of the *Teacher Version* of the lesson. Provide students with manipulatives (e.g., plastic tiles or paper cut-outs of two scalene triangles of equal size and shape) if appropriate.
3. Guide students in completing the notes and partially completed problems from the classwork and discussion questions.
4. Monitor for accuracy and adjust instruction as needed.

Student actions:
Students participate in class discussion and complete the *Guided Notes* as directed.

Student handouts/materials:
Guided Notes (found on the next page)

Guided Notes
Module 2, Topic C, Lesson 11: Definition of Congruence and Some Basic Properties

Name ____ **Date** ________________________

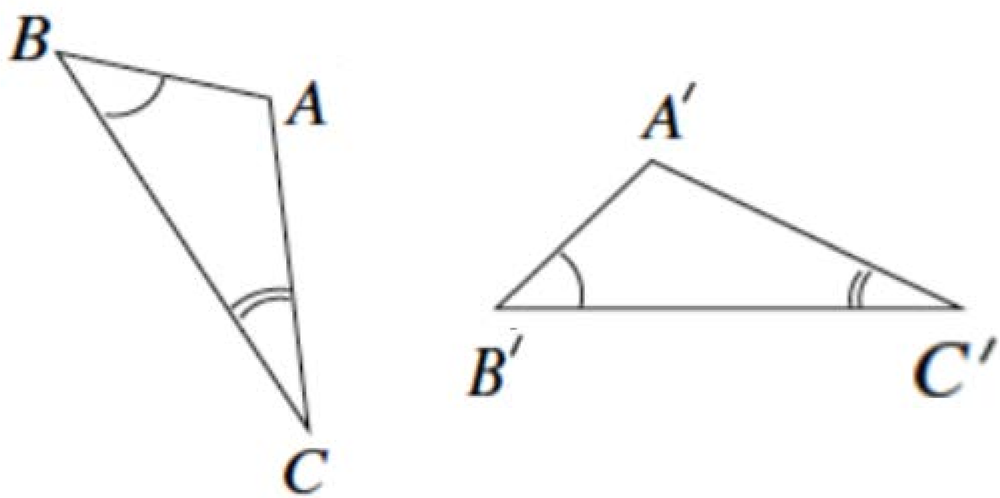
Example 1

- To prove two geometric figures congruent, there must be a sequence of basic

  ______________________ ______________________ that maps one figure onto another figure.

- The notation related to congruence is the symbol ______.

- Demonstrate the two triangles below are congruent.



Choose one rigid motion that would bring together at least one pair of corresponding points (vertices). Be specific.

____

What rigid motion would bring together one pair of sides? Be specific.

____

After these two rigid motions, we have shown that $\triangle ABC \cong \triangle A'B'C'$ through the sequence of a

________________________followed by a ________________________

How many rigid motions were needed for this sequence? ____________

Cooperative Learning

Exemplar from:

Module 4: Topic D: Lesson 25: Geometric Interpretation of the Solutions of a Linear System

Explanation of scaffold:

Cooperative learning includes those strategies where small groups of students contribute equally toward shared learning goals. This scaffold provides students of different ability levels an opportunity to engage with, assist, and learn from their peers. It motivates students to take responsibility for their own learning and can be used in any lesson to support students while they improve their understanding of a concept or skill without changing the rigor of the content.

Teacher actions/instructions:

Cooperative learning can be used at almost any point of instruction, but in mathematics, it is most beneficial after material has been presented by the teacher. This means it is best used when students are reviewing and practicing concepts or skills to reinforce their learning. It can also be used to assess student learning in the form of group or team projects and tests.

Although instructions will vary depending on the cooperative learning strategy being used, specific directions and explicit expectations should always be provided to minimize off-topic conversations and other distracting behaviors. Student groups or teams should be thoughtfully assigned and mixed heterogeneously by ability.

The following is a model of how cooperative learning could be used in this lesson:

1. Explain to students that they will be working first with a partner to complete *Exercises 1-5* found on pages 397-402 of the *Teacher Version* of the lesson. Then they will choose an exercise with their partner and present their work to a small group. After their presentations, student partners will answer any questions posed by other group members.
2. Assign student partners by matching struggling students with those who have a better understanding of the material.
3. Give students approximately 25 minutes to work with their partners to complete *Exercises 1-5*. Walk about the classroom to answer students' questions and provide guidance as needed.
4. When work time is up, join three sets of partners to form small, mixed-ability groups of six. Have each pair of partners choose an exercise or assign each pair an exercise to present to the group. Explain to students that both partners should have an opportunity to speak. Have additional graph paper available if students want to use it during their presentations.
5. Give students approximately 20 minutes to explain how they completed the exercise to the group and to answer any questions. Circulate around the classroom, and monitor student work.

Student actions:

Students work with their partners to complete the lesson exercises and present information to a small group of their classmates.

Student handouts/materials:

Graph paper

Instruction with Computer Technology

Exemplar from:

Module 3: Topic A: Lesson 1: What Lies Behind "Same Shape"?

Explanation of scaffold:

Instruction with computer technology involves using computer programs and websites to increase academic engagement and reinforce understanding of concepts. This scaffold provides visual and conceptual support for students who need additional models and practice opportunities to learn new information. Videos and game applications are an engaging way for students to interact with new information, practice skills, and receive immediate feedback. Guided notes or checkout activities can be used to assist students in attending and allow teachers to check for understanding. These notes or activities can also serve as reference tools for students.

Teacher actions/instructions:

Instruction with computer technology is beneficial when introducing or reteaching a concept or skill. It can be used as a homework assignment, during whole class instruction with a smart board, or during small group or individual instruction using computers. Ensure students have the prerequisite skills for operating computers and navigating the internet, websites are accessible to all students, and assistive technology needs are satisfied.

The following is a model of how instruction with computer technology can be used to reinforce students' understanding of dilation and its importance to precisely defining similarity:

1. Hand out student copies of the *Guided Notes*.
2. Access the Khan Academy video, *Dilating Points.*
3. Pause the video as needed to allow students to fill in their guided notes.
4. Carefully monitor students' activities to confirm on-task behaviors.

Student actions:

Students view the video and complete the *Guided Notes* as directed.

Student handouts/materials:

Computer access
Guided Notes (found on the following pages)

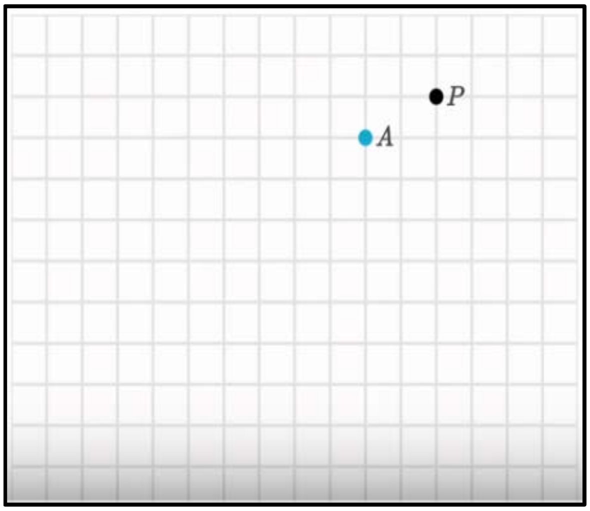
Guided Notes

Dilating Points

Name ____ Date ______________________

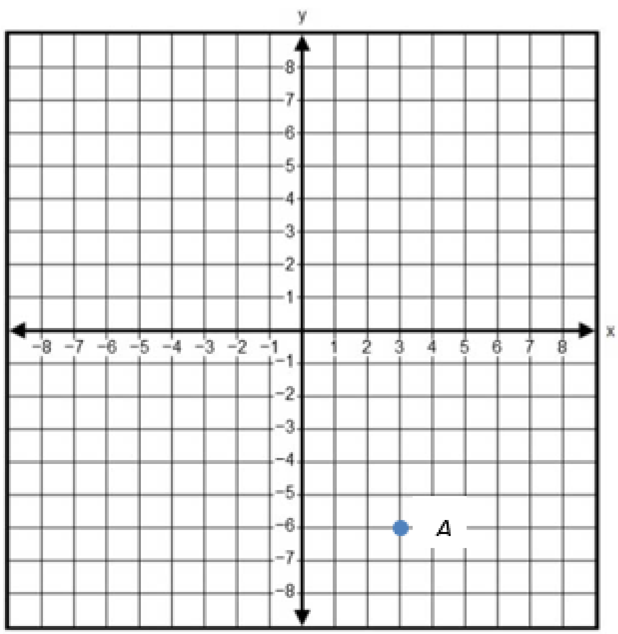
Plot the image of point *A* under a dilation about point *P* with a scale factor of 3.

- If we dilate point *A* about point *P* with a scale factor of 3, it's going to be ___________ times further in the _____________ direction.
- The first thing we need to do is think about how far point *A* is from point *P*.
- To go from *P* to *A*, we have to go _______ down and _________ to the left, so minus 1 and minus 2.
- Under a dilation about point *P* with a scale factor of 3, A' will be plotted _____ down and _________ to the left, so minus _______ and minus_________.



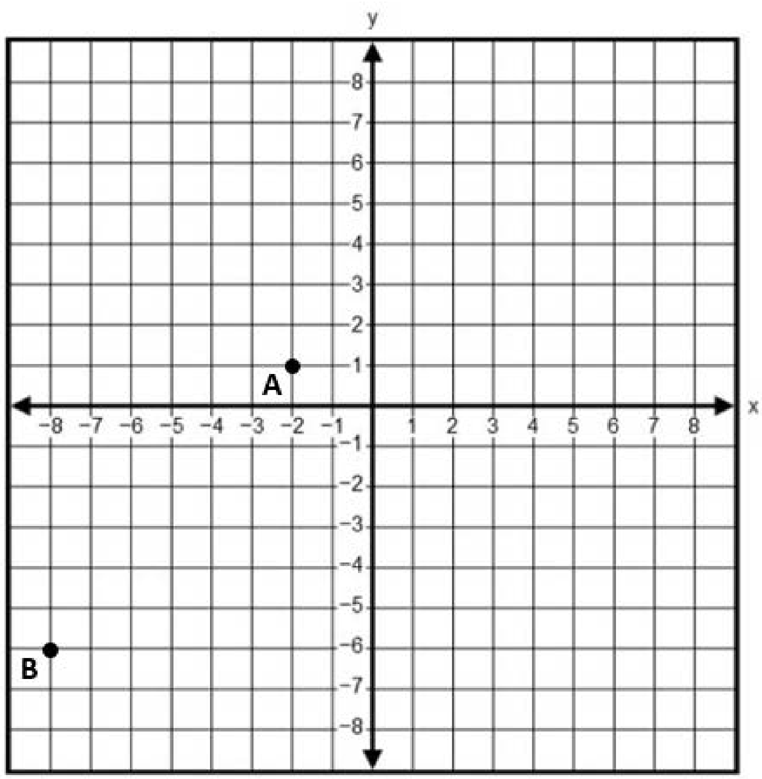
Plot the image of point *A* under a dilation about the origin (*0, 0*) with a scale factor of $\frac{1}{3}$.

- If we dilate point *A* about the origin point (*0, 0*) with a scale factor of $\frac{1}{3}$, it's going to be _____ as far in the _____________ direction.
- To go from the origin to *A*, we have to go ______ down and _______ to the right.
- Under a dilation about the origin of point A with a scale factor of $\frac{1}{3}$, A' will be plotted __________ down and _____________ to the right.



Check for Understanding

Plot the images of point *A* and point *B* under a dilation about the origin. Use a scale factor of 4 to dilate point *A* and a scale factor of $\frac{1}{2}$ to dilate point *B*. Label the dilated images A' and B'. Show your work.



How does a scale factor greater than 1 affect the image of the point being dilated?

It moves the image____.

How does a scale factor less than 1 (but greater than 0) affect the image of the point being dilated?

It moves the image____.

Concrete-Representational-Abstract (CRA)

Exemplar from:
Module 2: Topic A: Lesson 4: Definition of Reflection and Basic Properties

Explanation of scaffold:
CRA is a method used when teaching abstract, mathematical concepts that are difficult for students to understand. This scaffold employs a combination of a representation in the form of physical objects, a representation written on paper, and a carefully constructed arrangement of an idea or representation in one's mind. The teacher begins by modeling and thinking aloud with concrete objects (e.g., blocks, disks, etc.), then progresses to representing the concrete objects with drawings. The final level is the abstract level, where only numbers, notations, and mathematical symbols are used to complete the algorithm. Each phase of instruction builds on the previous phase to promote student learning and can help students better apply mathematical concepts to real world situations.

Teacher actions/instructions:
CRA can be used with individuals, small groups, or during whole class instruction when introducing a concept or teaching word problems. When using CRA, model the strategies, and provide multiple opportunities for student practice. Verbal explanations, visual demonstrations, and time for questions should be provided during each phase.

The following is a model of how CRA could be used to deepen students' understanding of reflection across a line prior to or during this lesson:

Concrete

1. Provide each student with a copy of *Practice with Reflection*, four scalene triangles of equal size and shape (e.g., plastic tiles or paper cut-outs), and a reflection mirror.
2. Direct students to position a triangle tile or paper cut-out on top of each triangle that is on the practice sheet; use a reflection mirror to reflect each triangle across the line of reflection (*L*); then "flip" the tile or cut-out across *L* so it lands on the reflected image.
3. Walk about the classroom to answer students' questions and provide guidance as needed.
4. Discuss how each point, side, and angle of the triangle is reflected across *L* as the position of the triangle changes and how it affects the position of the other triangle representing its reflection.

Representational

1. Provide each student with a transparency that is the same size as one section of the practice sheet.
2. For each section of the practice sheet, direct students to trace the line of reflection (*L*) and the triangle exactly, using a red marker.
3. Direct students to "flip" the transparency while keeping *L* on top of its black image.
4. Walk about the classroom to answer students' questions and provide guidance as needed.
5. Discuss how the position of the red triangle on the transparency represents the reflection of the original figure. Have students remove the transparency and draw the image on the grid. Fade the use of the transparency if possible.

Abstract

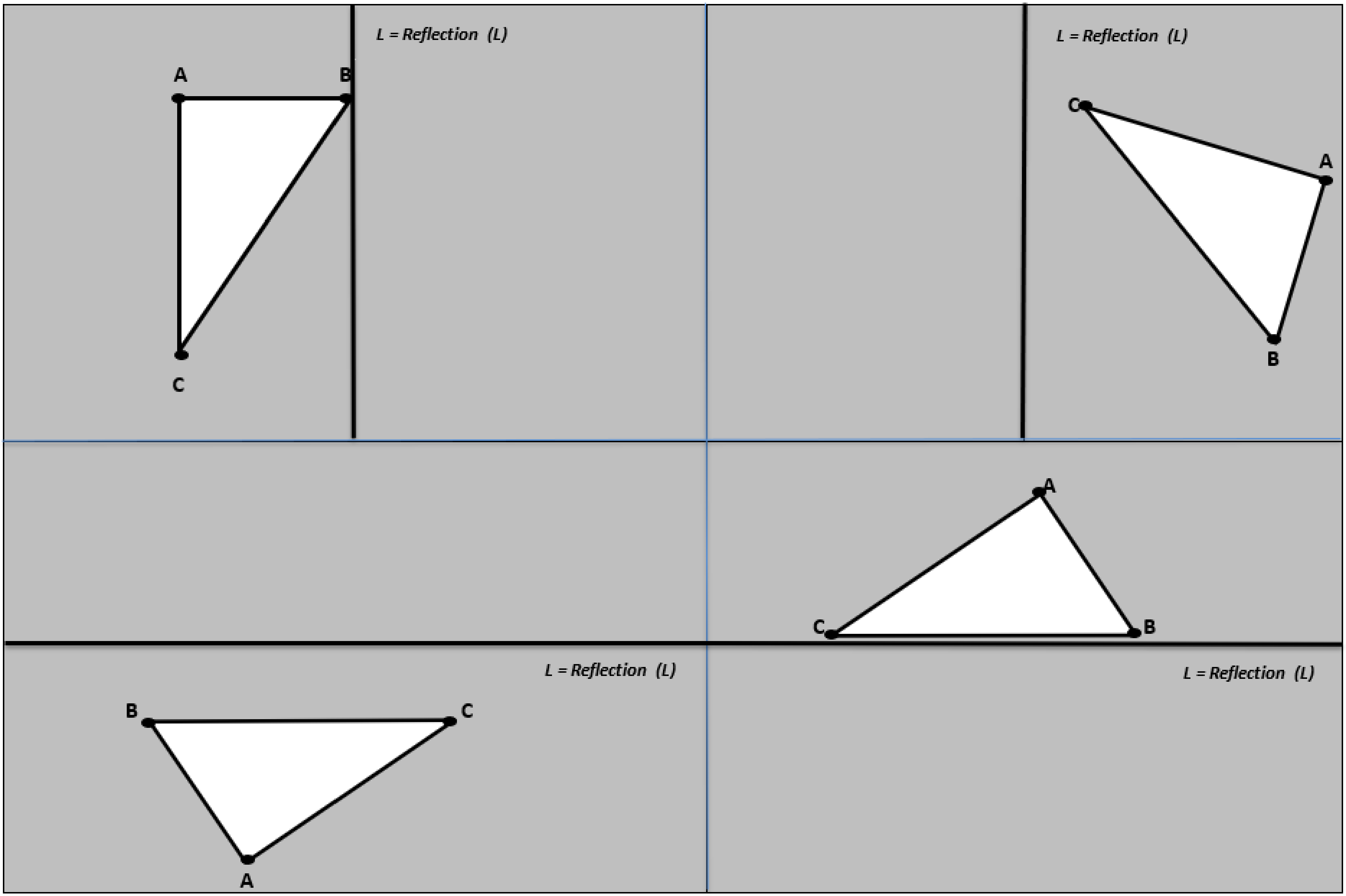
1. Direct students to label the points of each reflected image using A', B', and C'.
2. Discuss how each point is reflected across the line.
3. Discuss what is true about the corresponding angle measures of triangle ABC and triangle $A'B'C'$.
4. Discuss what is true about the corresponding side lengths of triangle ABC and triangle $A'B'C'$.
5. Discuss what is true about the line segment that connects a point to its image and the line of reflection.
6. Proceed as indicated on pages 39 - 51 of the *Teacher Version* of the lesson.

Student actions:
Students use manipulatives to facilitate their learning of reflection. Reflection mirrors and transparencies may continue to be used as needed by students when completing problems.

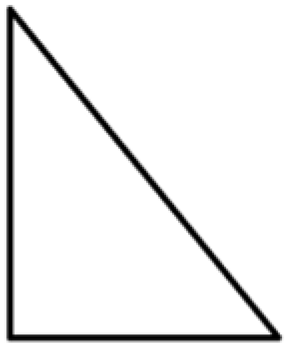
Student handouts/materials:
Practice with Reflection sheet (found on the next page)
Scalene triangle manipulatives (*Triangle Cut-out Templates* found on page 15)
Reflection mirror
Transparency
Red dry erase marker

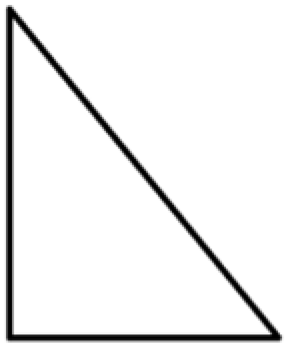
Practice with Reflection

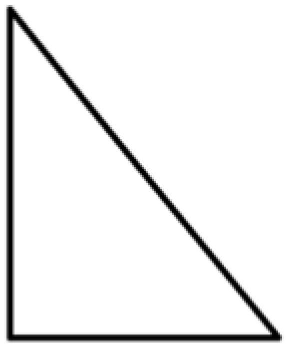
Name ______________________________ **Date** ______________________________

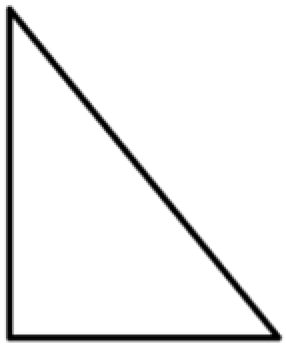


Triangle Cut-out Templates









Frayer Model

Exemplar from:
Module 2: Topic A: Lesson 2: Definition of Translation and Three Basic Properties

Explanation of scaffold:
The Frayer model is a graphic organizer that can be used in any lesson to help students understand unfamiliar vocabulary, including mathematical terms. This four-square model includes a student-friendly definition, a description of important characteristics, examples, and nonexamples. It provides a format to organize information and visual representations of the mathematical term being defined. Developing vocabulary skills is essential for students as they learn to "speak mathematically" and develop their abstract reasoning and problem-solving skills. The term *translation* is used to demonstrate how to apply this strategy when working with students.

Teacher actions/instructions:
Select key mathematical terms. These terms should be limited in number and essential to developing a deeper understanding of the mathematical concepts or skills in the lesson.

Instruct students to complete Frayer models as follows:

1. Write the mathematical term in the middle circle.
2. Define the term, using student-friendly language, in the **Definition** box. Use your own words.
3. Write words to describe the term in the **Characteristics** box. Again, use your own words.
4. List examples of the definition in the **Examples** box. Draw a picture and/or write an equation to help you understand the term if needed.
5. List nonexamples of the definition in the **Nonexamples** box. Again, draw a picture and/or write an equation if needed.
6. Test yourself.
    - The study step is critical to student success in using vocabulary strategies such as the Frayer model. Students need to study the terms to internalize them for later use.
    - Students can quiz each other during "down times," or the models/cards can be used as part of a center activity.

Student actions:
Students work either individually or in pairs to make and study Frayer models.

Student handouts/materials:
Frayer Model template (found on page 18)

NAME: ____

Frayer Model (example)

Translation

Definition

A transformation that moves every point of a figure in the same direction and the same distance within the plane.

Informally known as a "slide."

Characteristics

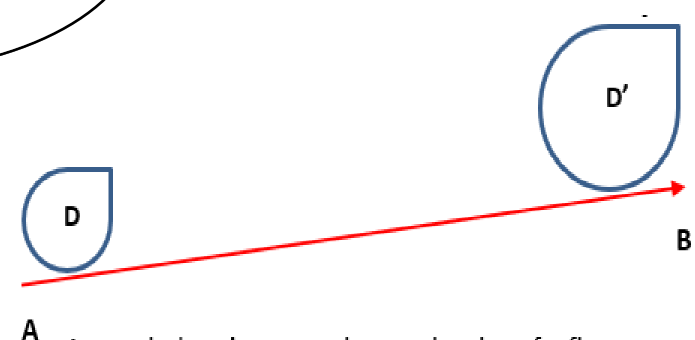
Three basic properties:

- Maps lines to lines, rays to rays, segments to segments, and angles to angles
- Preserves lengths of segments
- Preserves degrees of angles

Examples

Figure D translated along vector $\overrightarrow{AB}$

Nonexamples



A translation **does not** change the size of a figure.

NAME: ____

Frayer Model

Definition	**Characteristics**
Examples	**Nonexamples**

Evidence of Effectiveness

van Garderen, D. (2007). Teaching students with LD to use diagrams to solve mathematical work problems. *Journal of Learning Disabilities*, 40(6), 540-553.

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