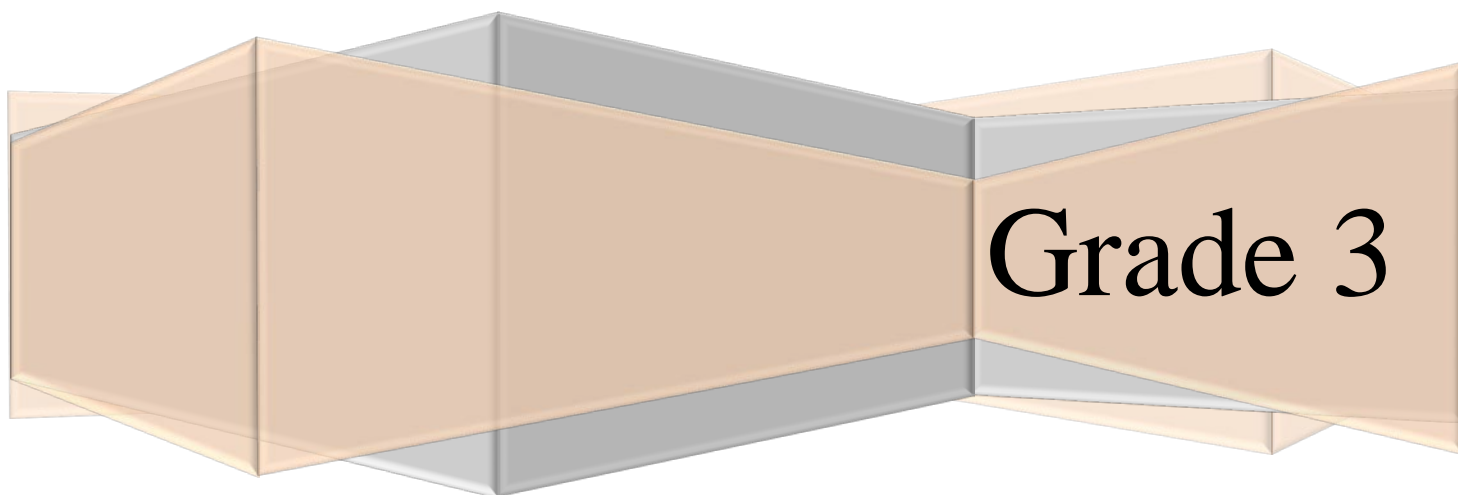


Scaffolding Instruction for All Students:

A Resource Guide for Mathematics



The University of the State of New York
State Education Department
Office of Curriculum and Instruction
and Office of Special Education
Albany, NY 12234



Scaffolding Instruction for All Students: A Resource Guide for Mathematics Grade 3

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Introduction

The Next Generation English Language Arts (ELA) and Mathematics Learning Standards intend to foster the 21st century skills needed for college and career readiness and to prepare students to become lifelong learners and thinkers. Learning standards provide the “destination” or expectation of what students should know and be able to do while teachers provide the “map” for getting there through high-quality instruction. **Lessons need to be designed to ensure accessibility to a general education curriculum designed around rigorous learning standards for all students, including students who learn differently (e.g., students with disabilities, English Language Learners (ELLs)/Multilingual Learners(MLLs), and other students who are struggling with the content).** It is vital that teachers utilize a variety of research-based instructional and learning strategies while structuring a student-centered learning environment that addresses individual learning styles, interests, and abilities present among the students in the class. Classrooms should be supportive and nurturing, and factors such as the age, academic development, English and home language proficiency, culture and background knowledge, and disability, should be considered when designing instruction. The principles of Universal Design for Learning should be incorporated into curricula to provide students with learning experiences that allow for multiple means of representation, multiple means of expression, and multiple means of engagement. These learning experiences will reduce learning barriers and foster equal learning opportunities for all students.

The purpose of this guide is to provide teachers with examples of scaffolds and strategies to supplement their instruction of ELA and mathematics curricula. Scaffolds are instructional supports teachers intentionally build into their lesson planning to provide students support that is “just right” and “just in time.” Scaffolds do not differentiate lessons in such a way that students are working on or with different ELA texts or mathematical problems. Instead, scaffolds are put in place to allow all students access to grade-level content within a lesson. Scaffolds allow students to develop the knowledge, skills, and language needed to support their own performance in the future and are intended to be gradually removed as students independently master skills.

The scaffolds contained in this guide are grounded in the elements of explicit instruction as outlined by Archer and Hughes (2011). Explicit instruction is a structured, systematic approach to teaching which guides students through the learning process and toward independent mastery through the inclusion of clear statements regarding the purpose and rationale for learning the new skill/content; explanations and demonstrations of the instructional target; and supported practice with embedded, specific feedback.

The scaffolds in this guide can be adapted for use in any curricula and across content areas. While the exemplars were all drawn from the ELA and mathematics [EngageNY](#) modules, teachers are encouraged to customize the scaffolds in any lesson they deem appropriate. **All teachers (e.g., general, special education, English as a New Language, and Bilingual Education teachers) can use these scaffolds in any classroom setting to support student learning and to make the general education curriculum more accessible to all students without interfering with the rigor of the grade-level content.**

How to Use This Guide

The provision of scaffolds should be thoughtfully planned as to not isolate or identify any student or group of students as being “different” or requiring additional support. Therefore, in the spirit of inclusive and culturally responsive classrooms, the following is suggested:

- Make scaffolded worksheets or activities available to all students.
- Heterogeneously group students for group activities when appropriate.
- Provide ELLs/MLLs with opportunities to utilize their home language knowledge and skills in the context of the learning environment.
- Make individualized supports or adapted materials available without emphasizing the difference.
- Consistently and thoughtfully use technology to make materials more accessible to all students.

In the ELA guides, the [document](#) is organized to allow teachers to access strategies based on the instructional focus (reading, writing, speaking and listening, and language) and includes a list of scaffolds that can be used to address those needs. In the mathematics guides, the [document](#) is organized around the scaffolds themselves.

Each scaffold includes a description of what the scaffold is, who may benefit, and how it can be implemented in a lesson-specific model (see graphic below). **The scripts provided are only for demonstrating what a scaffold *might* look like in action.** Teachers are encouraged to make changes to presentation and language to best support the learning needs of their students. While lessons from the [EngageNY](#) modules are used to illustrate how each scaffold can be applied, the main purpose of the exemplars is to show how teachers can incorporate these scaffolds into their lessons as appropriate.

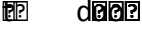
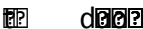
Title of Scaffold Module: Unit: Lesson:
Explanation of scaffold: This section provides a deeper explanation of the scaffold itself, including what it is and how it can and should be used. This section is helpful when implementing the scaffold in other lessons.
Teacher actions/instructions: This section provides specific instructions for the teacher regarding successful implementation of the scaffold.
Student actions: This section describes what the students are doing during the scaffolded portion of the lesson.
Student handouts/materials: This section indicates any student-facing materials that must be created to successfully use this scaffold.

Graphic Organizer (*RDW (Read, Draw, Write) Template*)

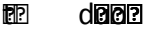
Exemplar from:

[Module 1: Topic A: Lesson 1](#): Application Problem

Explanation of scaffold:

The  is a graphic organizer that can be used to support students who have difficulty organizing information and recalling multistep, problem-solving strategies. This template is intended to help students keep track of the steps involved in the RDW process that can be used to solve real world/application word problems. Some students may need additional scaffolding and explicit instruction to use this tool to structure their work to solve a mathematical problem. The following example shows one way to instruct students who need modeling and guided practice to use this tool and learn this new problem-solving strategy. Although the application problem in this lesson is used as an exemplar, graphic organizers such as the  can be used in any lesson to support students while learning a multistep problem-solving strategy without changing the rigor of the content.

Teacher actions/instructions:

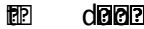
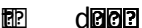
Instruct students in the use of the RDW process and completion of the  to solve a mathematical problem as follows:

1. Read the problem.
2. Draw and label. Use a tape diagram, number bond, or array to make your drawing. Ask yourself, “What do I know? What do I need to find? How can I draw what I’m looking for?” Label your drawing.
3. Read the problem again.
4. Write an equation. Look at the evidence in your drawing, write an equation, and solve the problem.
5. Write a word sentence.

For students who require explicit instruction on how to use the RDW process to solve a mathematical problem and complete the *RDW Template*, the following sample *script* is provided to demonstrate one way instruction *might* look like:

Step 1: Read the problem.

T (teacher): We are going to learn a strategy to help us solve problems. This strategy is called Read, Draw, Write, or RDW. We are going to use the RDW Template to gather the evidence, or important parts, from the word problem that we need to answer the question and solve the problem.

Display a large version of the  on chart paper or use a document camera to project your work. Hand out student copies, and direct students to complete their  to solve the problem as demonstrated.

Display the word problem:

There are 83 girls and 76 boys in third grade. How many total students are in third grade?

T: The first step is read.

S (student): Read.

T: The problem says, There are 83 girls and 76 boys in third grade. How many total students are in third grade? I read the problem, so I can put a check in the box on my PDW Template.

Step 2: Draw and label.

T: Step 2 is draw and label.

S: Draw and label.

T: /

Step 3: Read the problem again.

T: What is step 3?

S: Read again.

T: Let's read the problem together. [Chorally read the problem with students.]

Step 4: Write an equation.

T: What is Step 4?

S: Write an equation.

T:

S:

T: Look at your drawings again. How many boys are in third grade?

S: There are 76 boys.

T: $83 + 76 = 159$

S: $83 + 76 = 159$

T: $83 + 76 = 159$ *total number of students*

S: $83 + 76 = 159$

T: $83 + 76 = 159$ [Write $T = 83 + 76$ on the $83 + 76 = 159$ while chorally reading it aloud with students.]

S: $83 + 76 = 159$

T: $83 + 76 = 159$ [Write $83 + 76 = 159$ on the $83 + 76 = 159$]

Step 5: Write a word sentence.

T: $83 + 76 = 159$

S: *Write a word sentence.*

T: Finally, we have to write a sentence to answer the question. We have to remember to include all the information to tell the whole story. To tell how many total students are in third grade we will write, There are _____.

S: 159 students in third grade.

T: Did we answer the question?

S: Yes

T: Remember, we are going to use RDW and the RDW Template when we need to solve word problems.

As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Once students demonstrate the ability to use the RDW process with limited prompting, provide multiple, independent practice opportunities to ensure success.

Additional practice problems:

The garage has 98 cars and 42 motorcycles waiting for repair. How many total vehicles at the garage need to be fixed?

The bus took 52 students and 29 workers on its morning route. How many total people rode the bus this morning?

Our grocery store sells 41 gallons of chocolate ice cream and 87 gallons of vanilla ice cream every day. How many total gallons of chocolate and vanilla ice cream does our grocery store sell each day?

Student actions:

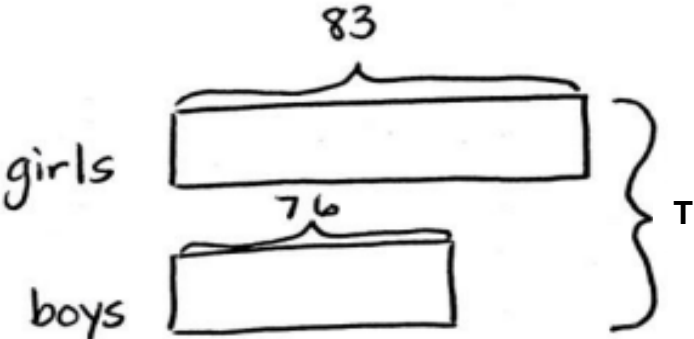
Students chorally respond and complete the $52 + 29 = 81$. Students may work in pairs or small groups to complete additional practice problems if needed.

Student handouts/materials:

$52 + 29 = 81$ (found on page 6)

NAME: _____

RDW Template (example)

Read	Make a ✓ after you read the problem. <input checked="" type="checkbox"/>
Draw and label	Draw a picture and label it. 
Read again	Make a ✓ after you read the problem again. <input checked="" type="checkbox"/>
Write	Write an equation. $\begin{array}{r} T=83+76 \\ 83 \\ +76 \\ \hline 159 \\ T=159 \end{array}$
Write	Write a sentence. <p>There are 159 students in third grade.</p>

NAME: _____

RDW Template

Read	Make a ✓ after you read the problem. <input data-bbox="865 365 922 422" type="checkbox"/>
Draw and label	Draw a picture and label it.
Read again	Make a ✓ after you read the problem again. <input data-bbox="865 1180 922 1236" type="checkbox"/>
Write	Write an equation.
Write	Write a sentence.

Frayer Model

Exemplar from:

[Module 1: Topic A: Lesson 3](#): Concept Development

Explanation of scaffold:

The Frayer model is a four-square graphic organizer that includes a student-friendly definition, a description of important characteristics, examples, and nonexamples. It provides a format to organize information and visual representations of the mathematical term being defined. Developing vocabulary skills is essential for students as they learn to **analyze** and develop their abstract reasoning and problem-solving skills. The following example demonstrates how to provide explicit instruction for those students who need information broken down into smaller, more manageable chunks as well as modeling and guided practice to effectively use this tool to learn new vocabulary words. The term **analyze** is used as an exemplar. However, the Frayer model can be used in any lesson to help students strengthen their conceptual knowledge and develop their understanding of unfamiliar vocabulary.

Teacher actions/instructions:

Select key mathematical terms. These terms should be limited in number and essential to developing a deeper understanding of the mathematical concepts or skills in the lesson.

Instruct students to complete Frayer models as follows:

1. Write the mathematical term in the middle circle.
2. Define the term, using student-friendly language, in the **Definition** box. Use your own words.
3. Write words to describe the term in the **Characteristics** box. Again, use your own words.
4. List examples of the definition in the **Examples** box. Draw a picture and/or write an equation to help you understand the term if needed.
5. List nonexamples of the definition in the **Nonexamples** box. Again, draw a picture and/or write an equation if needed.
6. Test yourself.

For students who require explicit instruction on how to use the Frayer model, the following sample script is provided to demonstrate one way instruction might look like :

Step 1: Write the mathematical term.

T (teacher): We are going to use a graphic organizer called a Frayer model to help us understand what certain math terms, or vocabulary words, mean. It is very important we understand what a term means so we can understand what a math problem is asking us to find and can talk about math with others. Understanding vocabulary will make us better mathematicians

Display a large version of the Frayer model on chart paper, or use a document camera to project your work. Hand out student copies, and direct students to complete their Frayer models as demonstrated.

T: t factor. t

?

S (student): Factor.

?

T: When we use the Frayer model, the first thing we do is write the vocabulary word in the middle circle. Let's write factor.

Step 2: Define the term.

T: You can see there are also 4 boxes. The first box is labeled Definition.

Factor

?

?

[Chorally say the definition with students.]

Definition

Step 3: Describe the word in terms of its characteristics.

T: The next box is Characteristics.

factor

[Draw two groups of

three apples.]

?

?

?

?

?

S: $2 \times 3 = 6$.

?

T: That's right. There are 2 groups of 3 apples, so we can write $2 \times 3 = 6$. In this equation, 2 is one factor.

factor

?

S: 3.

T: Right again. We multiplied 2, the number of groups, and 3, the number of apples in each group, to find 6, the product, or total number of apples. The numbers 2 and 3 are factors.

?

?

?

[Circle or highlight the factors, 2 and 3, and write down

additional information as needed to describe factor.]

Step 4: List examples.

T: The third box is Examples.

factors. t

factors

?

?

S: 4 and 2.

?

T: Correct. 4 and 2 are factors.

?

factor

factors

?

S: 1 and 8.

T: Why are 1 and 8 factors?

S: 8

?

T: You got it! Let's write down this equation in the **Examples** box. $8 = 2 \times 4$ **factor**

Step 5: List nonexamples.

T: The last box is **Nonexamples**.

$8 = 2 \times 2 \times 2$

$8 = 3 \times 3$

$8 = 2 \times 2 \times 2$

factors

$8 = 2 \times 2 \times 2$

$8 = 2 \times 2 \times 2$

$8 = 2 \times 4$

?

?

?

S: 3.

?

T: Why isn't 3 a **factor**?

?

S: Because there's not a number you can multiply 3 by to get 8.

?

T: Excellent! You remembered that **factor**.

$8 = 2 \times 4$

$8 = 2 \times 4$

$8 = 2 \times 4$

$8 = 2 \times 4$

factor

$8 = 2 \times 4$

factor

[Write down any reasonable answers.]

Step 6: Test yourself.

The study step is critical to student success in using vocabulary strategies such as the Frayer model. Students need to study the terms to internalize them for later use. Students can quiz each other during "down times," or the models/cards can be used as part of a center activity.

Instruct students to study their Frayer models as follows:

1. Cover each box of the Frayer model with a sticky note. Do not cover the math term in the middle circle.
2. Say the term in the middle and try to say the definition.
3. If you do not know the definition, uncover the **Characteristics** box, and try to say the definition.
4. If you do not know the definition, uncover the **Examples** box, and try to say the definition.
5. If you do not know the definition, uncover the **Nonexamples** box, and try to say the definition.
6. If you do not know the definition, uncover the **Definition** box.

Repeat steps 1-6 for each Frayer model.

Student actions:

Students work either individually or in pairs to make and study Frayer models.

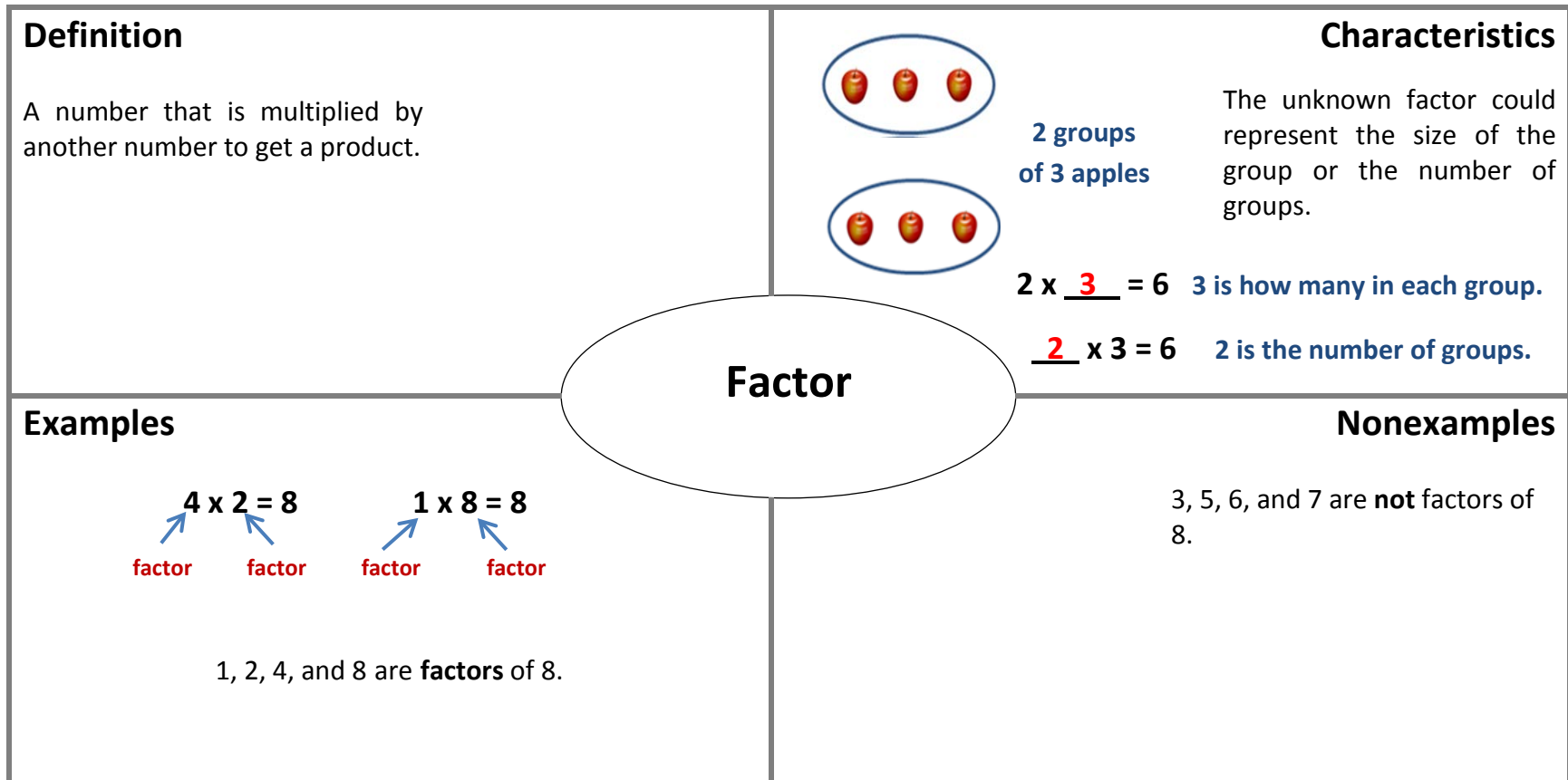
Student handouts/materials:

Frayer model template (found on page 11)

Sticky notes

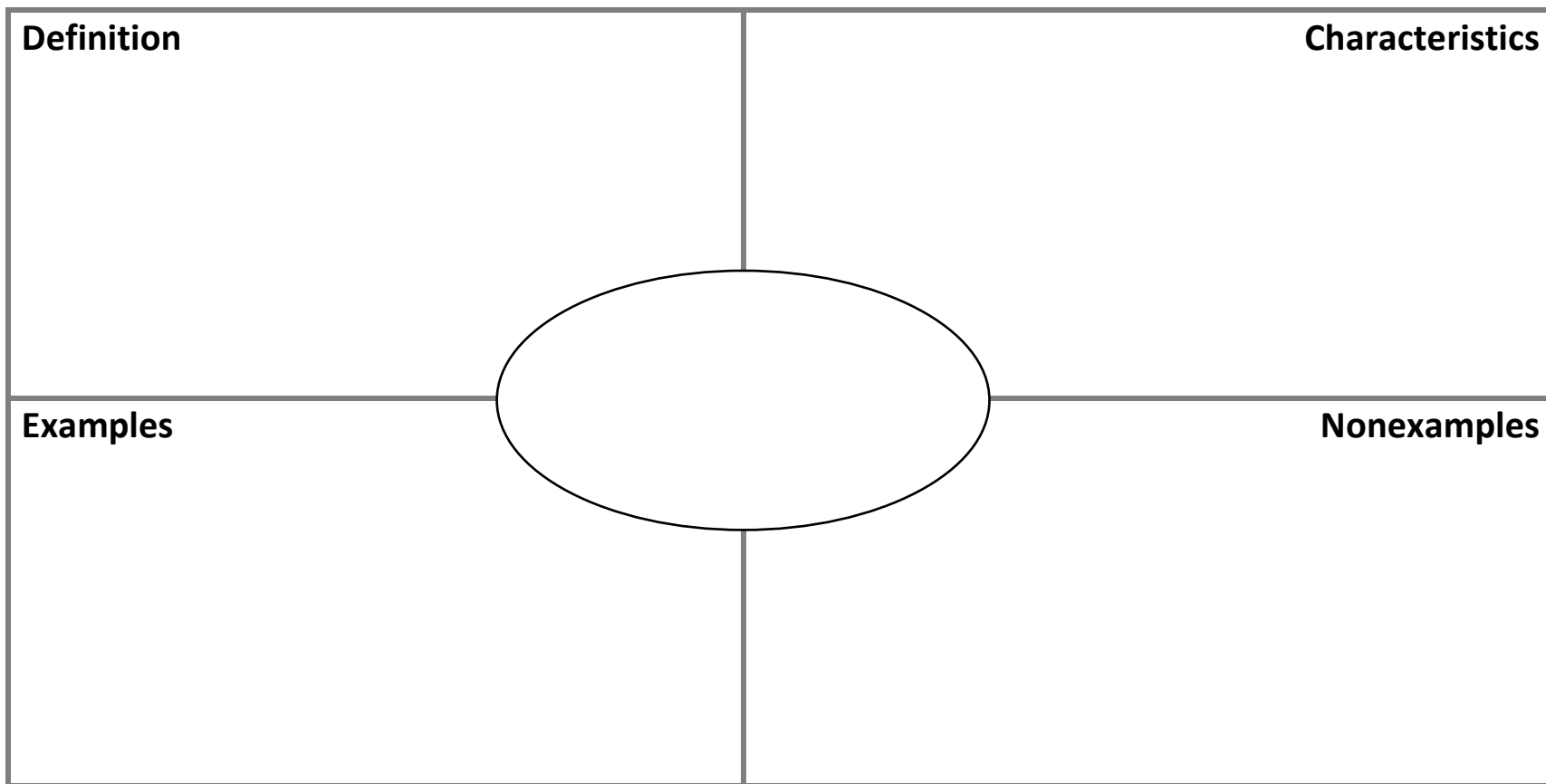
NAME: _____

Frayer Model (example)



NAME: _____

Fruyer Model



Desk Reference Sheet (*How to Round Numbers*)

Exemplar from:

[Module 2: Topic C: Lesson 12](#): Concept Development

Explanation of scaffold:

The [How to Round Numbers](#) desk reference sheet supports students who have difficulty understanding the concept of rounding and recalling the multistep process involved. An explicit rule for rounding numbers provides additional support to students who need to develop a concrete understanding of this mathematical process which is important to the content of the lesson and will be necessary to use in the future. Although the [How to Round Numbers](#) desk reference sheet connects to and uses the concept development section in this lesson as an exemplar, desk reference sheets can be used as a visual support in any lesson until students build fluency remembering key mathematical terms, concepts, processes, and/or skills.

Teacher actions/instructions:

Instruct students in the process for rounding numbers and use of the [How to Round Numbers](#) desk reference sheet as follows:

1. Find and label the benchmarks/endpoints.
2. Find and label the midpoint/halfway point.
3. Plot and label the number you need to round.
4. Round using the rule: $\begin{cases} \text{if } n < \frac{1}{2} & \text{round down} \\ \text{if } n \geq \frac{1}{2} & \text{round up} \end{cases}$

For students who require explicit instruction on how to use the *How to Round Numbers* desk reference sheet to round a number, the following sample *script* (based on the language found in the concept development section of Module 2, Topic C, Lesson 12) is provided to demonstrate one way instruction *might* look like:

Step 1: Find and label the benchmarks/endpoints.

T (teacher): We are going to learn to round twodigit numbers to the nearest ten. We are going to use a vertical number line to help us do that. A vertical number line goes up and down instead of across, or horizontal. Objects such as thermometers and measuring cups use vertical number lines.


Display a large version of the [How to Round Numbers](#) desk reference sheet on chart paper, or use a document camera to project your work. Hand out student copies (these should be placed in plastic sleeves, so students can write on them and erase during lessons), and direct students to use their number lines to round numbers as demonstrated.

T: This beaker has 73 milliliters of water in it. [Show a beaker holding 73 milliliters of water.] 

~~73 milliliters of water~~
~~73 milliliters of water~~



S (student): 7 tens.

T: That's right. Follow along with me on the number line on your How to Round Numbers sheet. [Write $70 = 7$ tens to the right of the lowest benchmark.] 

S: 8 tens.

T: Right again. [Write $80 = 8$ tens to the right of the highest benchmark.]

Step 2: Find and label the midpoint.

T: The second thing we need to do is find and label the midpoint on our number line. A midpoint is a point that is exactly in the middle between two points. This point will be used to help see where 73 falls with respect to the 7 tens (70) or the 8 tens (80). What number is halfway between 7 tens and 8 tens?

S: 75.

T: Correct. The number 75 is halfway between 7 tens and 8 tens. $75 = 7$ tens and 5 _____.

S: Ones.

T: Let's write $75 = 7$ tens 5 ones to the right of the tick mark showing the midpoint or halfway point on the number line.

Step 3: Plot and label the number you need to round.

T: The next step is to plot 73 on our number line. Remind me, what unit of measurement are we plotting on our number line?

S: Milliliters.

T: You got it. Say, Stop when my finger points to where 73 milliliters should be. [Move your finger up the number line from 70 toward 75.]

S: Stop

T: Now, let's put a tick mark on the number line where you said to stop. To label, we will write $73 = 7$ tens _____.

S: 7 tens 3 ones.

T: Excellent. Write the label to the right of the tick mark you drew on your number line now.

Step 4: Round using the rule.

T: Now that we know where 73 milliliters is, we can round the measurement to the nearest 10 milliliters. Look at your vertical number line. Is 73 milliliters more than halfway or less than halfway between 70 milliliters and 80 milliliters?

S: Less than halfway.

T: We know 73 milliliters is less than halfway between 70 milliliters and 80 milliliters because 3 is less than 5, and 5 marks the mid-or halfway point. Looking at your number line, what is 73 milliliters rounded to the nearest ten. Remember to use the rule: *If the number is below the midpoint/halfway point, then round down. If the number is at the midpoint/halfway point or above, then round up.*

S: 73 milliliters rounded to the nearest ten is 70 milliliters.



T: Correct. Another way to say it is that 73 milliliters is about 70 milliliters.

As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Once students demonstrate the ability to round numbers with limited prompting, provide multiple, independent practice opportunities to ensure success. Explain that the vertical number line can be used even when the units of measurement change.

Additional practice for rounding to the nearest ten:

- 61 centimeters
- 38 minutes
- 25 grams

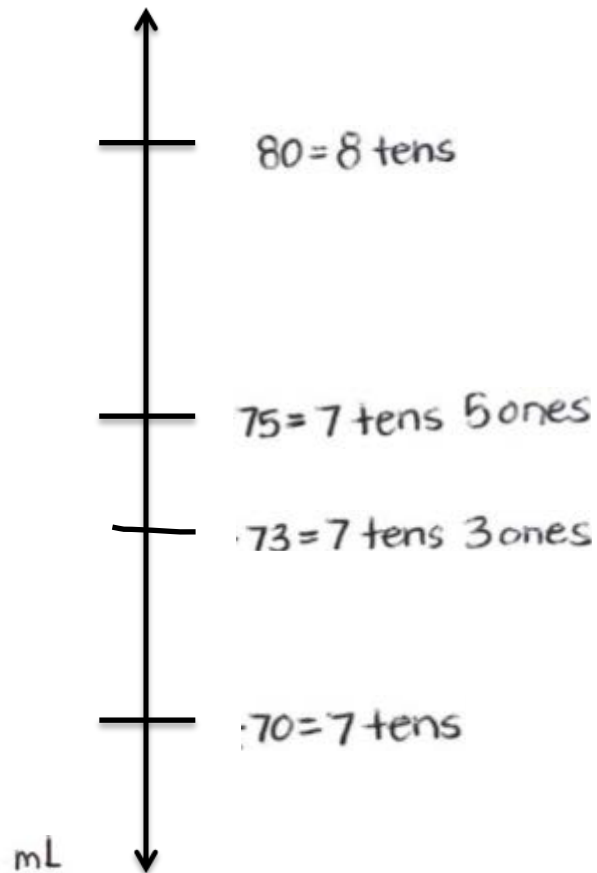
Student actions:

Students chorally respond and use the  desk reference sheet to follow along with the teacher's model. Students then work in pairs, small groups, or independently as appropriate to round numbers using the  desk reference sheet as needed.

Student handouts/materials:

 desk reference sheet (found on the page 16)

How to Round Numbers (example)



1. Find and label the benchmarks/endpoints.
2. Find and label the midpoint/halfway point.
3. Label the number you need to round.
4. Round using the rule: **below the midpoint/halfway point round down**
at the midpoint/halfway point or above round up

How to Round Numbers



1. Find and label the benchmarks/endpoints.
2. Find and label the midpoint/halfway point.
3. Label the number you need to round.
4. Round using the rule: **below the midpoint/halfway point round down**
at the midpoint/halfway point or above round up

Concrete-Representational-Abstract (CRA)

Exemplar from:

[Module 3: Topic A: Lesson 2](#): Concept Development

Explanation of scaffold:

CRA is a three-part instructional strategy in which the teacher begins by modeling and thinking aloud with **concrete** objects (e.g., blocks, disks, etc.), and then progresses to **representing** the concrete objects with drawings. The final level is the **abstract** level, where only numbers and mathematical symbols are used to complete the algorithm. Although the following exemplar connects to and uses the concept development section in this lesson as an exemplar, CRA is a method that can be used in any lesson when teaching abstract concepts that are difficult for students to understand.

The Concept Development for this lesson centers on expanding students' study of factors, specifically working with factors of seven. CRA provides additional support for students to see the groups of five that are within a factor of seven.

Teacher actions/instructions:

In this module lesson, it is suggested that students requiring additional concrete support start out by using cubes or a Rekenrek. Provide student partners with linking cubes of two different colors or a Rekenrek to allow opportunities for guided practice with concrete materials before transitioning to the representational/pictorial, and eventually to the abstract examples. Instruct students to share the linking cubes or Rekenrek with their partners, work together, and follow along as you model how to use these manipulatives to solve the problem. Use a document camera to project your work.

For students who require explicit instruction on how to use the materials provided to use familiar five facts to solve multiplication problems and apply the commutative and distributive properties, the following sample script is provided to demonstrate one way instruction might look like:

Concrete

T (teacher): 

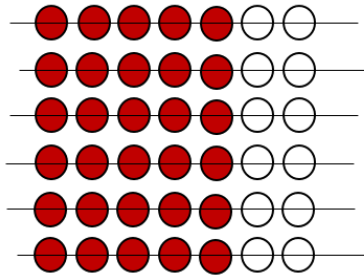
  

S (student): 

T:   

  [If a Rekenrek is unavailable, students can use linking cubes to show the six groups of seven, highlighting groups of five within the seven.]



T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $\frac{1}{2}$

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $\frac{1}{2}$

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $\frac{1}{2}$

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $\frac{1}{2}$

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $\frac{1}{2}$

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

S: $6 \times 2 = 12$.

T: $\frac{1}{2}$ of the circles are red. $\frac{1}{2}$ of the circles are white.

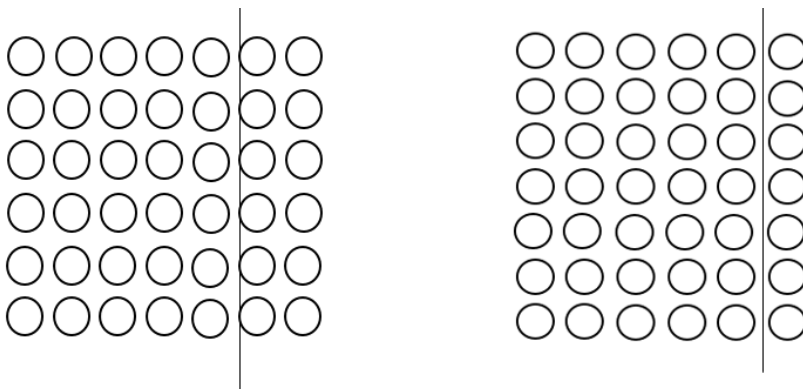
S: [redacted]

T: d [redacted]

[redacted]

Representational

Students can be asked to draw an array that represents 6 x 7. Diagrams will look like the concrete models. Students may draw an array that highlights 6 groups of 7, or they may draw an array that highlights 7 groups of 6. Based on previous work with the commutative property, students should understand that both represent the same product of 42.



T: [redacted] / [redacted]

[redacted]

[redacted]

[redacted]

S: [redacted]

T: d [redacted]

[redacted]

S: [redacted]

T: d [redacted] (Students can skip count if necessary. They can also color in the groups of 5 as well.)

S: [redacted]

T: d [redacted]

[redacted]

S: [redacted]

T: 1000000
by two if necessary.)

(Students can skip count

S: 2

T: 1000000 1000000 1000000
 1000000 1000000

S: 1000000 1000000

T: 1000000
 1000000

T: Now, let's look at the second array. It shows 7 groups of 6 (7×6), which I know from the commutative property is the same as 6×7 . In this array, I would need to count by sixes, seven times, to get my answer. Maybe, sixes are challenging to count by 1000000

1000000 1000000 1000000 1000000 1000000 1000000 1000000 1000000
 1000000 1000000 1000000 1000000 1000000 1000000 1000000 1000000

S: 42 42

T: 1000000 1000000
 1000000 1000000

S: Seven.

T: 1000000 1000000 (Students can skip count if necessary and color in the groups of five.)

S: 2

T: 1000000 1000000 1000000 1000000 1000000
 1000000 1000000 1000000 1000000 1000000

S: 10 10

T: 1000000 1000000 (Students can count if necessary.)

S: 2

T: 1000000 1000000 1000000 1000000 1000000 1000000
 1000000 1000000 1000000 1000000 1000000 1000000

S: 1000000 $35 + 7$ is the same as $35 + 5 + 2$

Following from the second array, a similar approach can be used to translate the breaking down of 7×6 into two easier facts, using five facts.

Seven groups of six is the same as seven groups of five plus seven groups of one.

$$7 \times 6 = (7 \times 5) + (7 \times 1)$$

$$7 \times 6 = 35 + 7$$

$$7 \times 6 = 42$$

Provide additional opportunities for students to reinforce their understanding of the commutative and distributive properties while using their familiar multiplication five facts to complete problems using concrete, representational, and/or abstract means. As students become more familiar with the strategy, fade the use of modeling and guided practice.

Additional problems from the module lesson:

$$6 \times 9$$

One group of 9 is the same as a group of five plus a group of _____.

Six groups of 9 is the same as six groups of five plus six groups of _____.

$$6 \times 9 = (6 \times 5) + (\quad)$$

$$9 \times 6$$

One group of 6 is the same as a group of five plus a group of _____.

Nine groups of 6 is the same as nine groups of five plus nine groups of _____.

$$9 \times 6 = (9 \times 5) + (\quad)$$

$$6 \times 6 = ?$$

Student actions:

Students chorally respond, arrange Rekenrek or linking cubes, and work in pairs to complete problems.

Student handouts/materials:

Rekenrek or linking cubes (two different colors)

Personal white boards

*****Note: Students may choose to use other pictorial representations, such as number bonds, to represent problems and distribute differently depending on their use of the commutative property.**

Worked Problems

Exemplar from:

[Module 7: Topic A](#): Lesson 2: Homework

Explanation of scaffold:

Worked problems provide support as students go through the learning stages of acquisition to proficiency to fluency to generalization and can be used to build learners' momentum and self-efficacy. Students are provided models of completed and/or partially completed problems while they work on developing and applying a newly learned skill without corrective feedback from the teacher.

The use of worked problems is based on the [van Lehn model](#) (/t) and involves alternating between fully worked and/or partially completed examples that students can use as a reference and practice problems for students to complete independently. A high level of scaffolding can be provided by giving worked examples and practice problems that are very similar in structure and by providing annotations or problem-solving steps alongside worked examples. Support can be faded by providing fewer worked examples or providing practice problems that are less similar to worked examples. Although the homework problems in this lesson are used as an exemplar, worked problems can be used in any lesson to support students who understand the mathematical concept involved but need examples as they practice what they have learned during the day's lesson to complete homework assignments.

Teacher actions/instructions:

Add worked problems to homework sheets. Provide the adapted sheets as needed to students, and direct them to complete the assigned problems. Instruct students to fill in the information needed to solve partially completed problems wherever they see a blank line. You may consider additional scaffolding by assigning specific problems on a homework sheet for those students who are likely to benefit from a shorter period of practice in which they are able to complete the task, rather than a longer period of practice in which they are unsuccessful.

Student actions:

Students complete problems on the adapted sheets as assigned.

Student handouts/materials:

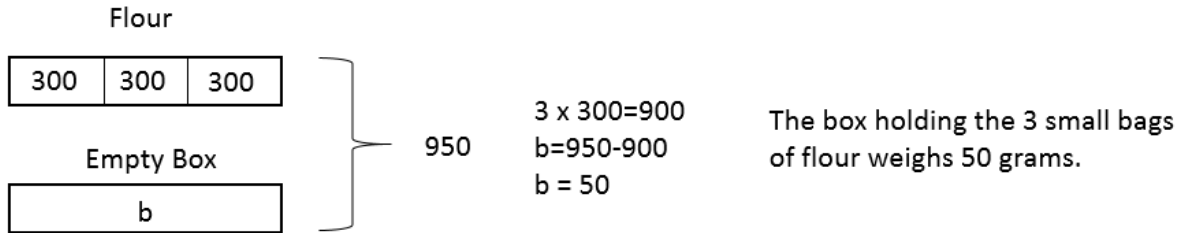
[HW](#), [HW](#) sheets (found on the following pages)

*****Note: The module lesson homework sheet included word problems 1-6 only. Supporting information, including tape diagrams, was added to scaffold student learning.**

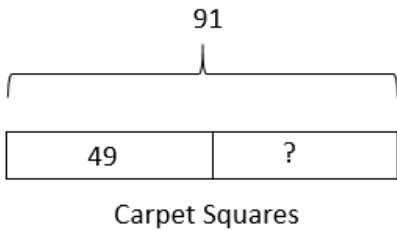
Name _____ Date _____

Use the RDW process to solve. Use a letter to represent the unknown in each problem.

1. A box containing 3 small bags of flour weighs 950 grams. Each bag of flour weighs 300 grams. How much does the empty box weigh?

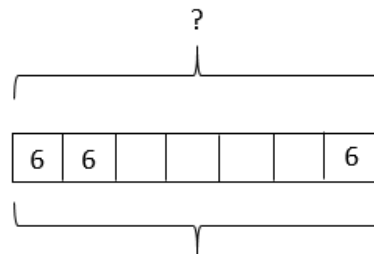


2. Mr. Cullen needs 91 carpet squares. He has 49 carpet squares. If the squares are sold in boxes of 6, how many more boxes of carpet squares does Mr. Cullen need to buy?



$$\begin{array}{r} 91 \\ - 49 \\ \hline ? \end{array}$$

? is the number of **carpet squares** Mr. Cullen needs to buy.



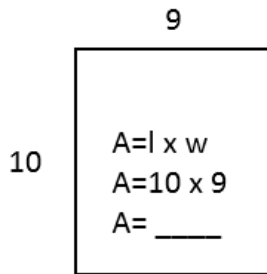
b is the number of **boxes** of 6 that Mr. Cullen needs to buy

$b = \underline{\hspace{2cm}}$

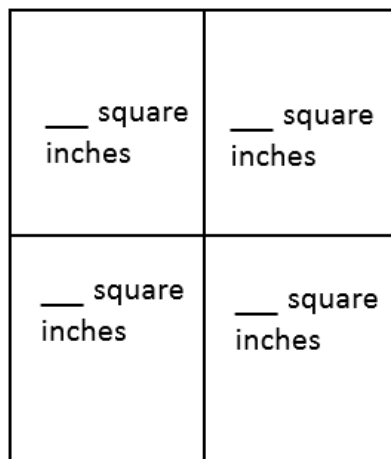
$b = ? \div 6$

Mr. Cullen needs to buy **boxes** of carpet squares.

3. Erica makes a banner using 4 sheets of paper. Each paper measures 9 inches by 10 inches. What is the total area of Erica's banner?

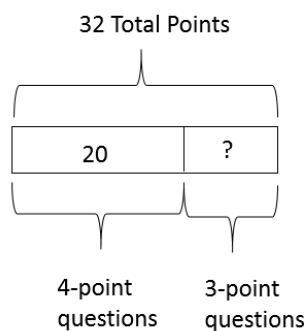
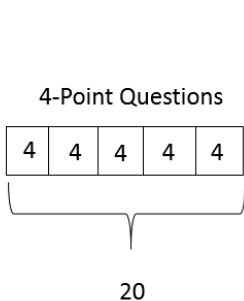


Each sheet of paper has an area of square inches.

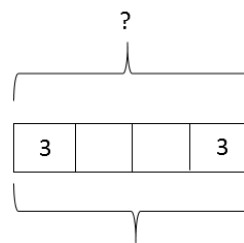


The total area of Erica's banner is square inches.

4. Monica scored 32 points for her team at the Science Bowl. She got 5 four-point questions correct, and the rest of her points came from answering three-point questions. How many three-point questions did she get correct?



? is the total amount of points for the 3-point questions.



The number of 3-point questions that Monica got correct.

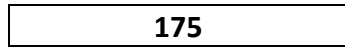
q is the number of 3-point questions Monica got correct.

$q = \underline{\quad}$

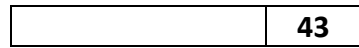
$q = ? \div 3$

Monica got 3-point questions correct.

5. Kim's black kitten weighs 175 grams. Her gray kitten weighs 43 grams less than the black kitten. What is the total weight of the two kittens?

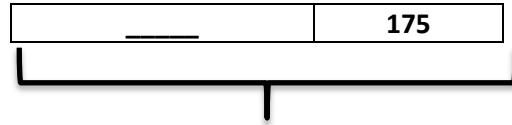


Black kitten



Gray kitten

$$175 - 43 = \underline{\quad}$$



Total weight (w)

$$w = 175 + \underline{\quad}$$

$$w = \underline{\quad}$$

The total weight of the two kittens is _____.

6. Cassias and Javier's combined height is 267 centimeters. Cassias is 128 centimeters tall. How much taller is Javier than Cassias?

