Growth Model
2024-25 Technical Report

December 2025

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Introduction

This document describes the model used to measure student growth for institutional accountability in New York State. The accountability system progressed through three phases in its development: the Restart Phase, the Rebuild Phases, and the Reimagine Phase.

After a two-year pause of the accountability system in the 2019-20 and 2020-21 school years due to the impact of the COVID-19 pandemic, the Restart Phase was implemented in the 2022-23 school year based on 2021-22 school year results under an approved one-year ESSA Accountability State Plan Addendum. Following consultation with educational experts and extensive stakeholder input, the United States Department of Education (USDE) approved amendments to the accountability section of the New York State consolidated State plan to implement a two-year Rebuild Phase in the 2023-24 and 2024-25 school years based on results from the 2022-23 and 2023-24 school years. Student growth was provided for informational purposes only during the Restart and Rebuild phases.

For the Reimagine Phase, NYSED has reintroduced Student Growth as part of the accountability indicator roster for accountability determinations beginning in the 2025-26 school year. The approved New York State ESSA plan can be found on the NYSED Approved New York State ESSA Plan webpage.

The Growth models are implemented in Grades 4-8 ELA and mathematics (including Grade 8 Algebra I) and are based on assessing each student's change in performance between 2023-24 and 2024-25 on State assessments compared with students who have similar test histories. For further information on the accountability system for the 2025-26 school year under the federally approved New York State ESSA plan, please see the document titles *Understanding the New York State Accountability System under the Every Student Succeeds Act (ESSA) for 2025-26 Accountability Statuses Based on 2024-25 Results*, found on the NYSED School and District Accountability Resources and Data website.

Content and Organization of This Report

The information presented in this report is based on 2024 -25 and prior school years' student growth results used for Institutional Accountability.

This technical report contains four sections:

1. **Data** – Description of the data used to implement the student growth model, including data processing rules and relevant issues that arose during processing.
2. **Model** – Description of the statistical model.
3. **Results** – Overview of key model results aimed at providing information on model quality and characteristics.

4. **Reporting** – Description of reporting metrics.

Data

To measure student growth and attribute that growth to schools and districts, at least two sources of data are required:

1. Student test scores that can be observed across time.
2. Information describing how students are linked to schools and districts.

The following sections describe the data used for model estimation in New York in more detail.

Test Scores

New York's student growth model draws on test score data from statewide testing programs in Grades 3-8 in ELA and mathematics (including Grade 8 Algebra I) for students in Grades 4-8. The Grades 4-8 growth model is estimated separately by grade and subject using scores from each grade (e.g., Grade 5 mathematics) as the outcome.

State Tests in ELA and Mathematics (Grades 3-8) and Algebra I

The New York State tests at the elementary and middle school grade levels are administered in the spring and measure a range of knowledge and skills in ELA and mathematics.

New York's Grades 4-8 growth model includes up to three prior year test scores (depending on the grade) in the same subject area. If the immediate prior-year test score was missing from the immediate prior grade, the student was not included in the growth measure for that subject. Two examples of how students would not have growth scores computed for them are:

1. Students without a prior-year test score (e.g., a 6th grade student with a valid 6th grade ELA test score in 2024-25 who did not have a valid ELA test score in 2023-24); or

2. Students with a prior-year test score for the same grade as the current year test score (e.g., a 6th grade student with a valid 6th grade ELA test score in 2024-25 who also had a 6th grade ELA test score in 2023-24).

For the additional prior scores, missing data indicators were used. These missing indicator variables allow the model to include students who do not have the maximum possible test history and mean that the model results measure outcomes for students with and without the maximum possible assessment history. This approach was taken to include as many students as possible.

The specific tests used as predictors vary by grade and subject and are shown in Table 1.

Table 1. Pretests for student inclusion by grade

		Current Year Assessment				
		Grade 4	Grade 5	Grade 6	Grade 7	Grade 8/ Algebra I
Prior Years Assessment, Same Subject	Grade 3	REQUIRED	USE IF AVAILABLE	USE IF AVAILABLE	USE IF AVAILABLE	
	Grade 4		REQUIRED	USE IF AVAILABLE	USE IF AVAILABLE	USE IF AVAILABLE
	Grade 5			REQUIRED	USE IF AVAILABLE	USE IF AVAILABLE
	Grade 6				REQUIRED	USE IF AVAILABLE
	Grade 7					REQUIRED

In addition to test scores, the New York Grades 4-8 growth model also uses the conditional standard errors of measurement (CSEMs) of those test scores. All assessments contain some amount of measurement error, and the New York growth model accounts for this error (as described in more detail in the Model section of this report). The State's test vendor provides a table of CSEMs for each year's test scores. Appendix A provides an overview of data processing.

Model

New York State uses the Mean Growth Percentile (MGP) model to produce growth measures for Grades 4-8 using State assessments in ELA and mathematics. This section describes the statistical model used to measure student growth between two points in time on a single subject of a State assessment. The section begins with a description of the statistical model used to form the comparison point against which students are measured—based on similar students—and then describes how SGPs are derived from the comparison point. In addition, this section describes how MGPs and all variance estimates are produced.

At the core of the New York State growth model is the production of an SGP. This statistic characterizes each student's current year score relative to other students with similar prior test score histories. For example, an SGP equal to 75 denotes that a student's current year growth score is the same as or better than 75% of the students in the State with similar prior test score histories. It does *not* mean that the student's growth is better than that of 75% of all other students in the population.

One common approach to estimating SGPs is to use a quantile regression model (Betebenner, 2009). This approach models the current year score as a function of prior test scores and finds the SGP by comparing the current year score to the predicted values at various quantiles of the conditional distribution.

The methods described here do not rely on the quantile regression method for two reasons. First, the typical implementation of the quantile regression makes no correction for measurement error in the predictor variables or the outcome variable. Ignoring the measurement error in the predictor variables yields bias in the model coefficients (e.g., Wei and Carroll, 2009). Further complicating the issue, the measurement error in the outcome variable also adds to the bias in a quantile regression (Hausman, 2001), an issue that does not occur with linear regression.

A linear spline regression model is used to compute the SGPs each year for New York's growth model. The approach used to estimate the model is designed to account for measurement error in the predictor variables to yield unbiased estimates of the model coefficients. Subsequently, these model coefficients are used to form a predicted score, which is used to calculate the SGP. The next section describes this model in detail.

Specifications for the Growth Model

The statistical model implemented as the growth model is typically referred to as a *covariate adjustment model* (McCaffrey, Lockwood, Koretz, & Hamilton, 2004), as the current year observed score is conditioned on prior levels of student achievement as well as other possible covariates. The model is specified in a way that recognizes measurement error in measures of prior achievement. It is also specified in a way that allows for measures of prior achievement to have a nonlinear relationship with the posttest, with the nonlinearity specified as a linear spline.

The model takes the following form:

$$y = \xi + f(z^*)\lambda + W\beta + e^*$$

$$z = z^* + v$$

where y indicates the model's posttest; z indicates the model's measured pretests, which are made up of a "true" pretest z^* and measurement error v; $f(.)$ indicates a function that produces a spline transformation of the primary (same-subject, single-lag) pretest while leaving the non-primary pretests the same; W represents indicators for missing non-primary pretests; and e^* is the component of the model's posttest that cannot be explained with its pretests.

We estimate this model in a way that accounts for measurement error in the observed variables by constructing predictions of the "true" pretest scores z^* given measured pretest scores z and using those predictions as right-hand-side variables in the regression. Procedures are described in more detail in Appendix B.

The SGP models are implemented separately for each grade and subject. Special procedures are used to adjust standard errors of measurement. These procedures are described in Appendix C.

Student Growth Percentiles (SGPs)

The previously described regression models yield unbiased estimates of the coefficients by accounting for the measurement error in the observed scores. The resulting estimates are then used to form a student-level SGP statistic. For purposes of the growth model, a predicted value and the variance of its difference from the observed value for each student are required to compute the SGPs as follows:

$$SGP_i = \Phi\left(\frac{y_i - \hat{y}_i}{\sqrt{\sigma^2_{y_i - \hat{y}_i}}}\right)$$

where y_i is the observed value of the outcome variable; $\hat{y}_i$ is the predicted value of the outcome variable; $\sigma^2_{y_i - \hat{y}_i}$ is the variance of the difference between the observed and predicted values of the outcome; and $\Phi(.)$ is the standard normal cumulative distribution function. Details on how to calculate the SGP are provided in Appendix B.

Figure 1 illustrates an SGP for a hypothetical student, using the approach described above. The illustration considers only a single predictor variable that enters linearly and does not necessarily reflect the shape of the actual model, although the concept can be generalized to multiple predictor variables that enter in other ways (e.g. spline). For each student, we find a predicted value conditional on his or her observed prior score(s) and the model coefficients. We would form a conditional distribution around the predicted value and find the portion of the normal distribution that falls below the student's observed score. This is equivalent to

$$SGP_i = \int_{-\infty}^{y_i} f(x)dx$$

with $f(x)$ being the normal probability density function with a mean of $\hat{y}$ and a variance of $\sigma^2_{y_i - \hat{y}_i}$. This is readily accomplished using the standard normal cumulative distribution function $\Phi(.)$. In the case of the student in Figure 1, the SGP is 90, suggesting that the student's observed score is in the 90th percentile of the distribution of observed scores conditional on their predicted score.

Figure 1. Sample Growth Percentile from Model

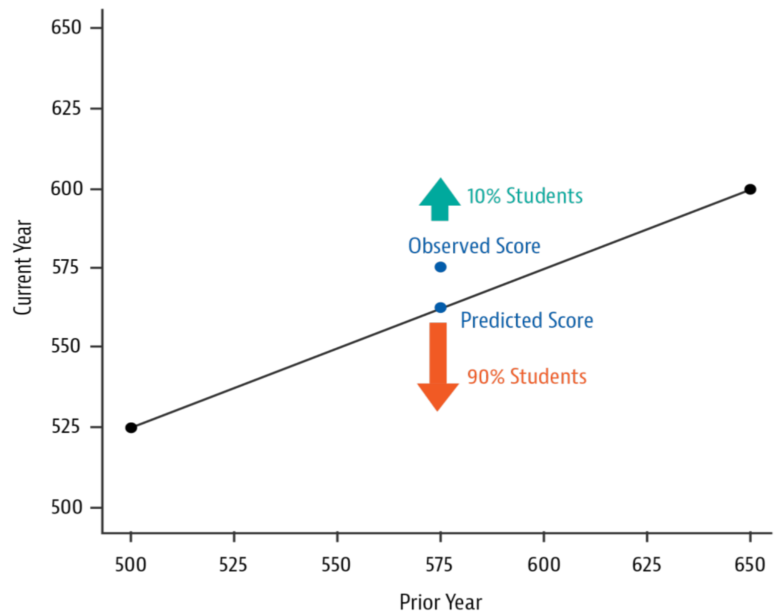
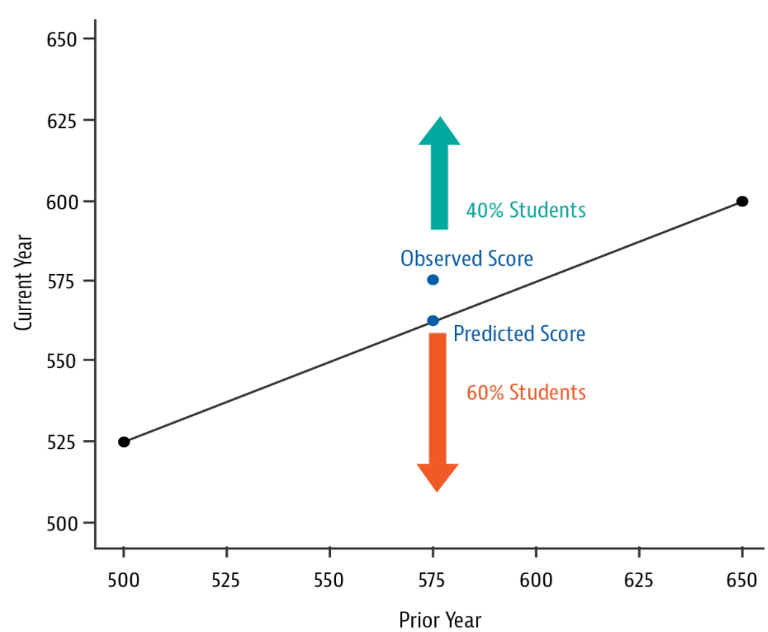


Figure 2 illustrates the same hypothetical student shown in Figure 1. Note that the observed score and predicted value are identical to what we see in Figure 1. However, the prediction variance is larger than in Figure 1. As a result, when we integrate over the normal distribution from $-\infty$ to y_i, the SGP is 60, not 90 as in the previous example. This difference occurs because the conditional density curve has become more spread out, reflecting less precision in the prediction.

Figure 2. Sample Growth Percentile from Model



Mean Growth Percentiles (MGPs)

Once SGPs are estimated for each student, they are averaged to generate the Mean Growth Percentile (MGP) or Growth Index. Note that these MGPs may be slightly different from the MGPs that NYSED calculates internally as the underlying datasets used are slightly different. For each aggregate unit j, such as a school, the statistic of interest is a summary measure of growth for students within this group. Within group j, there are SGPs for N students associated with group j $\{SGP_{j(1)}, SGP_{j(2)}, \ldots, SGP_{j(N)}\}$. That is, there is an observed SGP for each student for each year within group j. The Growth Index for unit j is produced as the simple mean

$$\theta_j = mean(SGP_{j(\mathrm{i})})$$

As with all statistics, the MGP is an estimate, and it has a variance term. The following measures of variance are produced for the MGP. The analytic standard error of the unweighted MGP is computed within unit j as

$$se(\theta_j) = \frac{sd(SGP_{ij})}{\sqrt{N_j}}$$

Because the SGPs are expressed as percentiles, they are free from scale-specific inferences and can be combined. Therefore, for the MGPs, all SGPs of relevant students are pooled and the mean of the pooled SGPs is calculated.

Attributing Student Growth Percentiles

Student growth scores are attributed to schools and districts for students who were continuously enrolled. To be attributed, a student must have been enrolled in the same school on BEDS Day (i.e., the first Wednesday of October) and during the State test administration in the spring. For district attribution, students attending charter schools are not included. Tables 2 and 3 show attribution rates for schools and districts.

Table 2. School Attribution Rates

Grade	Valid Student Records	Valid Student Records Attributed to a School	Attribution Rate
4	293,582	287,498	97.9%
5	295,628	289,430	97.9%
6	284,550	278,239	97.8%
7	282,026	275,930	97.8%
8	278,572	262,876	94.4%
Total	1,434,358	1,393,973	97.2%

Table 3. District Attribution Rates

Grade	Valid Student Records	Valid Student Records Attributed to a District	Attribution Rate
4	260,745	259,994	99.7%
5	263,576	262,716	99.7%
6	250,575	249,676	99.6%
7	249,804	248,538	99.5%
8	249,875	239,543	95.9%
Total	1,274,575	1,260,467	98.9%

MGPs are calculated for the All Students group and each of the accountability subgroups for which the count of SGPs is greater than or equal to 30: American Indian or Alaska Native, Black or African American, Hispanic or Latino, Asian/Pacific Islander, White, Multiracial, Students with Disabilities, English Language Learners, and Economically Disadvantaged.[1] The Growth Index is then rounded to the nearest tenth decimal place and assigned to one of four Growth Levels based on the cut points described in Table 4.

Table 4. Growth Index to Growth Level

Growth Index	Growth Level
45 or less	1
45.1 to 50	2
50.1 to 54	3
Greater than 54	4

Results

This section provides an overview of the results for the 2024-25 growth model estimation. A pseudo R-squared statistic and summary statistics characterizing the SGPs, MGPs, and their precision provide information about model fit.

Model Fit Statistics

The *R*-square value is a statistic commonly used to describe the goodness-of-fit for a regression model. Because the model implemented here is a linear spline model corrected for

[3] When calculating the Growth Index for English language learners (ELLs), SGPs for former ELLs are included if the total number of SGPs for former ELLs is less than 50% of the sum of SGPs for ELLs in the current year. Former ELLs are students who are not ELLs in the current reporting year but were ELLs in one or more of the previous four reporting years. If there are 20 or more SGPs for the Students with Disabilities (SWD) subgroup, then former students with disabilities are included in the Growth Index. Former students with disabilities are students that are not SWDs in the current reporting year but were reported in at least one of the two previous reporting years.

measurement error, not a least squares regression, we refer to this as a *pseudo R*-square. (See Appendix B for more information on the linear spline model.) Table 5 presents the pseudo *R*-square values for each grade and subject, computed as the squared correlation between the fitted values and the outcome variable.

Table 5. Pseudo R-Squared Values by Grade and Subject

Grade	ELA	Mathematics
4	0.641	0.663
5	0.659	0.720
6	0.669	0.719
7	0.702	0.751
8	0.698	0.669
Algebra I	N/A	0.599

Student Growth Percentiles

SGPs describe a student's current year score relative to those of other students in the data with similar prior academic histories. A student's SGP should not be expected to be higher or lower based on his or her prior-year score. Table 6 shows the correlation between the prior-year scale score and SGP for each grade and subject. These correlations are generally small; they need not be zero. Squaring these values gives the percentage of variation in SGPs explained by prior-year scores for any grade and subject. Although prior-year test scores are generally good predictors of current year test scores, the prior-year test score is a poor predictor of current year SGPs. As shown in Table 6, prior-year test scores by grade level explain less than three percent of the variation in SGPs. Because SGPs are intended to allow students to show low or high growth no matter their prior performance, this result is as expected.

Table 6. Correlation Between SGP and Prior-Year Scale Score

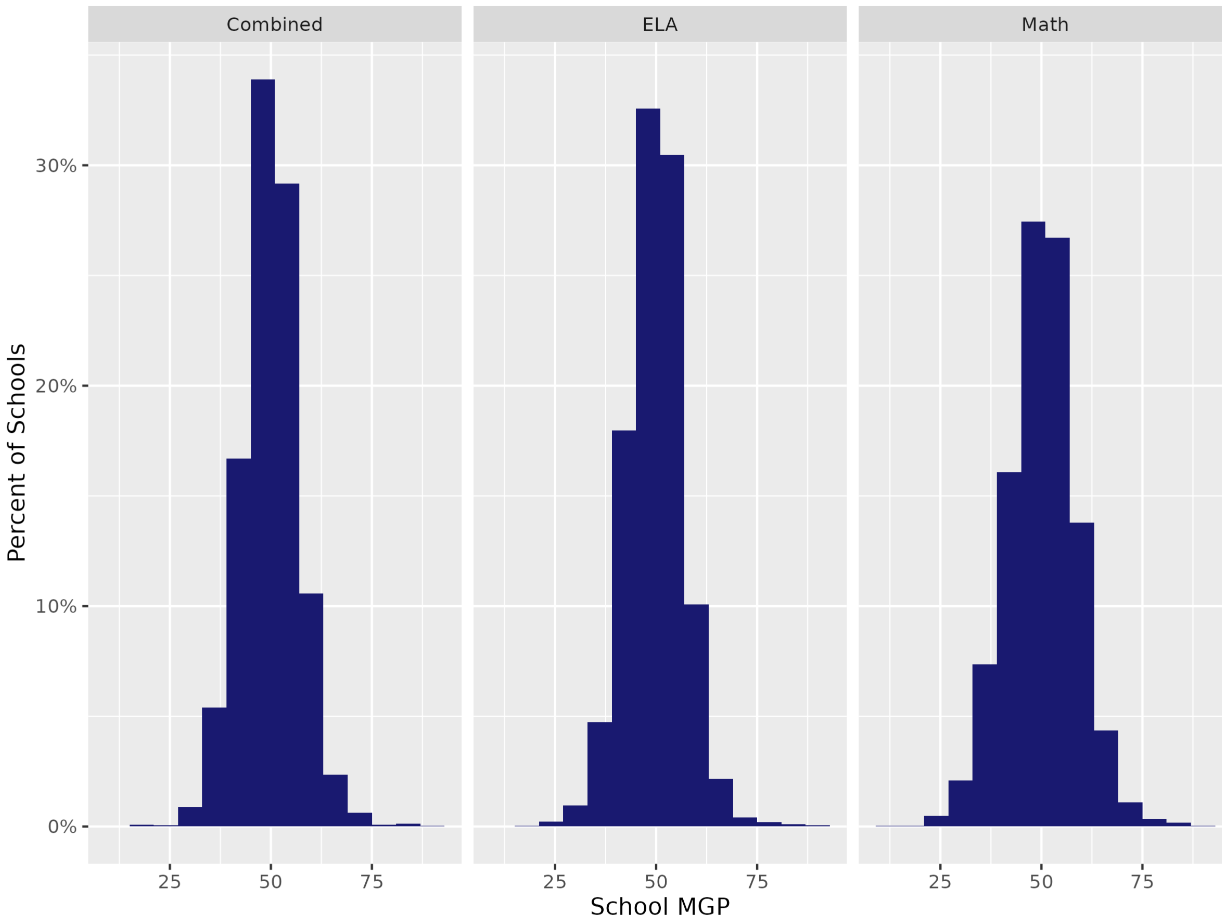
Grade	ELA	Mathematics
4	-0.168	-0.131
5	-0.136	-0.107
6	-0.141	-0.121
7	-0.157	-0.114
8	-0.173	-0.126
Algebra I	N/A	-0.075

Mean Growth Percentiles

As described earlier in this report, MGPs are aggregate statistics, computed as the mean of SGPs for all students associated with a particular school or district. In this section, we provide descriptive statistics on overall (combined) MGPs.

Figure 3 shows the distribution of school MGPs, which center around the mean of 50.

Figure 3. Distribution of School MGPs



Reliability of MGPs

It is useful to examine the reliability statistic to assess the precision of the school-level MGPs, specified here as ρ:

$$\rho = 1 - \left(\frac{\bar{\sigma}}{sd(\hat{\theta}_j)}\right)^2$$

where $\bar{\sigma}$ is the mean standard error of the MGP, and $sd(\hat{\theta}_j)$ is the standard deviation between school MGPs. In theory, the highest possible value is one, which would represent complete precision in the measure. When the ratio is zero, the variation in MGPs is explained entirely by sampling variation. Larger values of ρ are associated with more precisely measured MGPs.

Table 7 provides the weighted mean standard errors, the weighted standard deviations, and the values of weighted ρ for the model for schools, using the number of SGPs as weights. These results are based upon the one-year MGPs for the 2024-25 model. The values shown below are very similar to what was reported for prior year models.

Table 7. Weighted Mean Standard Errors, Standard Deviation, and Value of ρ for Schools, Weighted by Number of SGPs

	Weighted Mean Standard Error	Weighted Standard Deviation	Weighted Reliability Statistic (ρ)
Schools	1.314	6.075	0.944

Table 8 provides the share of schools with combined MGPs significantly above or below the State mean, using the 95% confidence intervals. The percentage exceeding the mean is larger than what would be expected by chance alone, indicating the model distinguishes between schools (i.e., 2.5% of schools would be expected to be above or below the mean by chance alone).

Table 8. Percentage of Combined MGPs Above or Below the Mean at the 95% Confidence Level

	Below Mean		Above Mean	
Level	N	%	N	%
School	1,055	29%	1,122	31%

Neutrality of Growth Measures

It is helpful to consider the relationship between the growth measures and school characteristics to identify any relationships that might suggest non-neutrality. Large correlations between MGP and school characteristics would indicate systematic relationships between scores and the types of students schools serve. A value of 0.10 or less indicates that 1% or less of the variance in MGPs can be predicted with that demographic variable and, therefore, represents results that are essentially zero. In 2024-25, all correlations of MGPs with classroom characteristics have absolute values of 0.279 or lower.

The scatter plots in Figures 4 through 8 provide visual representations of the correlation between school-level MGPs and school characteristics: the percent of students who are ELL, the percent of SWD, the percent of students in poverty or with economic disadvantage (ED), and the mean prior ELA or mathematics score of the students. The scatter plots show that the school MGPs have a low correlation with respect to school demographic and pretest characteristics. The low correlation means that the growth measures can be considered to be neutral with respect to these school characteristics and this neutrality means that schools can demonstrate growth, regardless of the academic starting point or characteristics of their students.

Figure 4. School MGP Scores by Percentage of English Language Learners in School

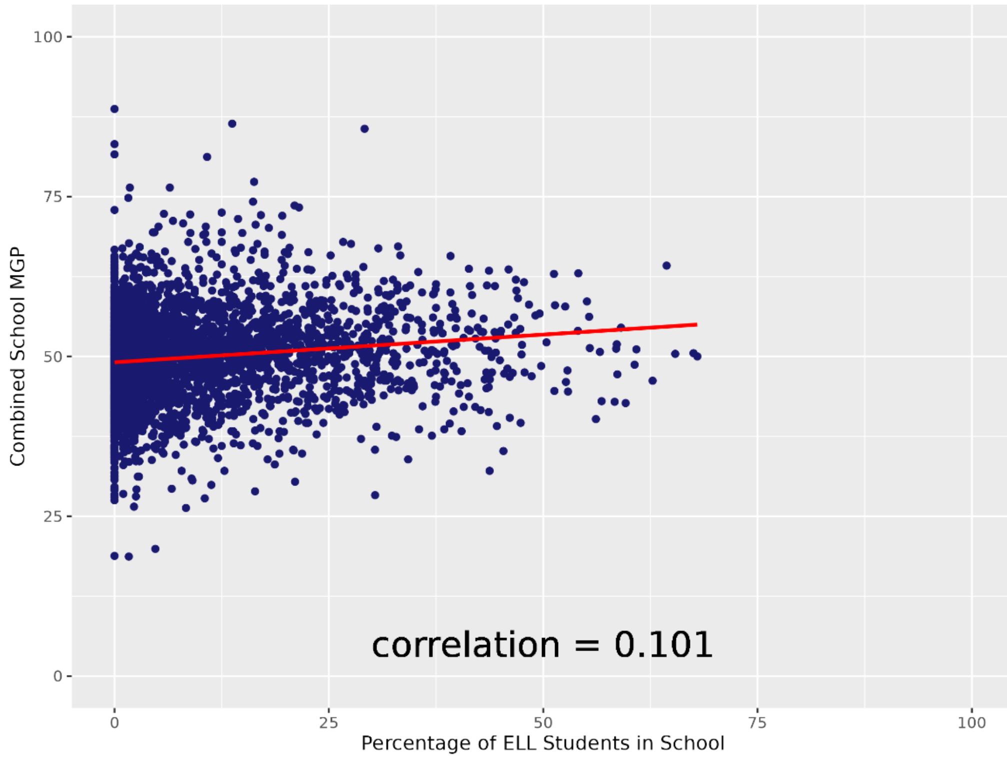


Figure 5. School MGP Scores by Percentage of Students with Disabilities in School

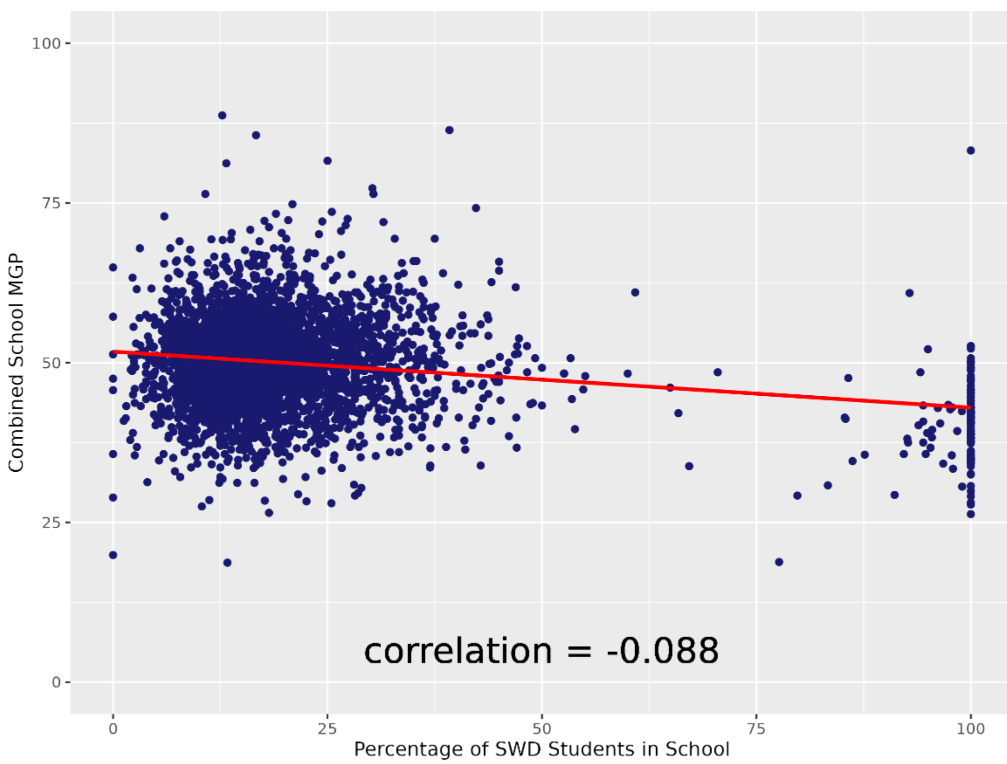


Figure 6. School MGP Scores by Percentage of Economically Disadvantaged Students in School

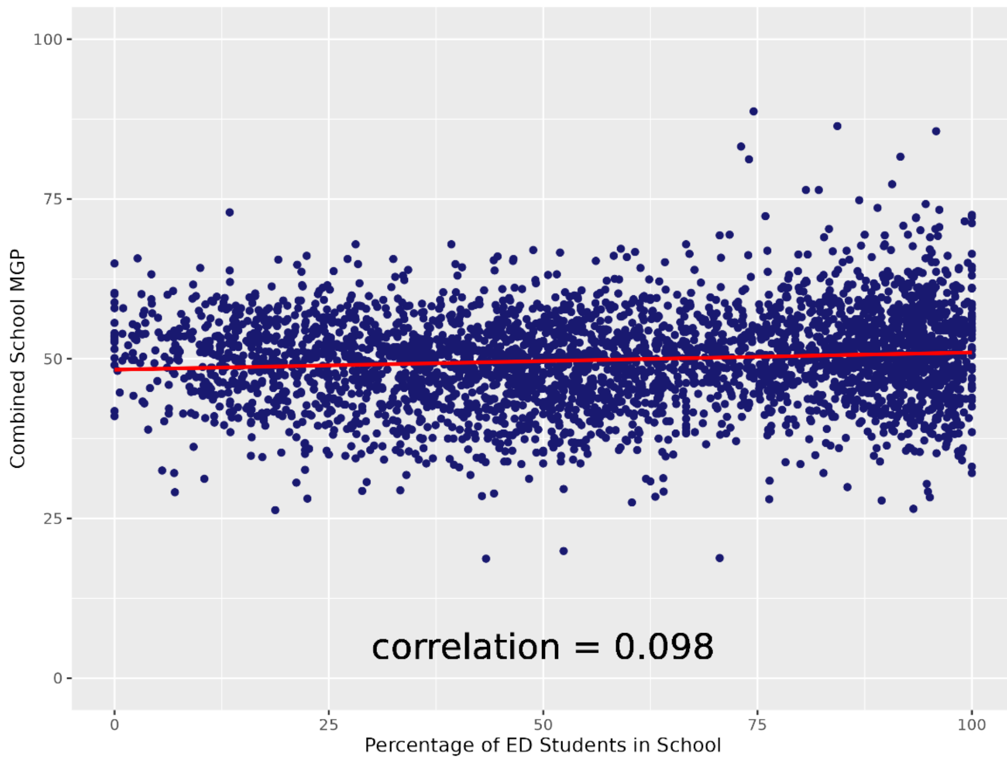


Figure 7. School MGP Scores by Mean Prior ELA Z-Score for Students in School

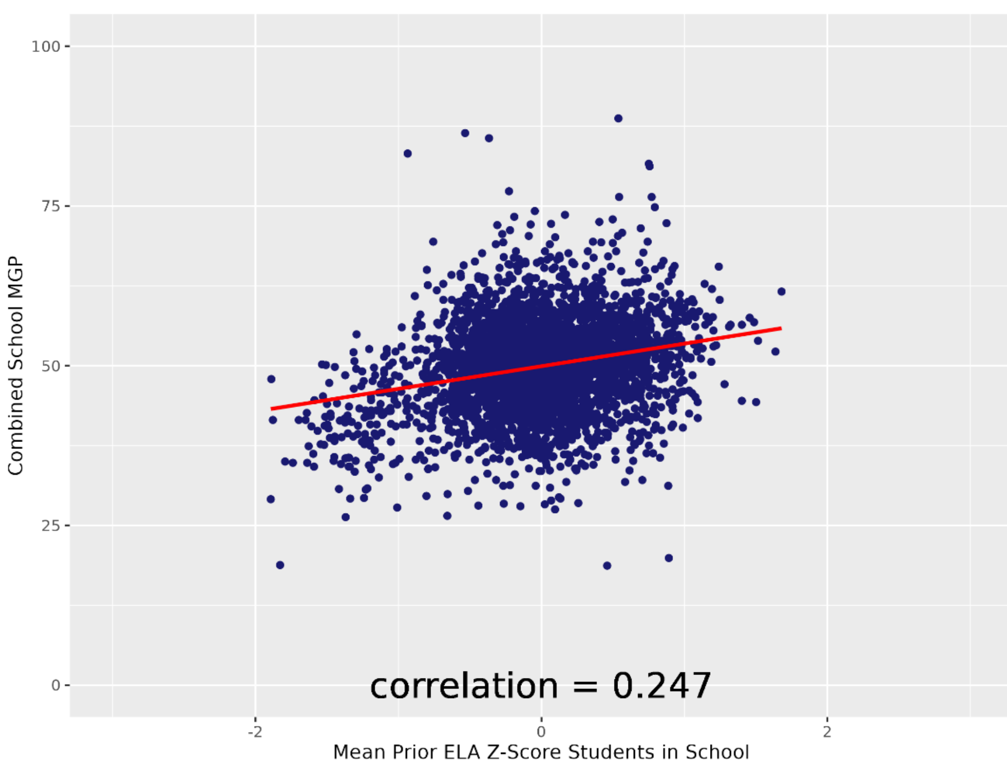


Figure 8. School MGP Scores by Mean Prior Mathematics Z-Score for Students in School

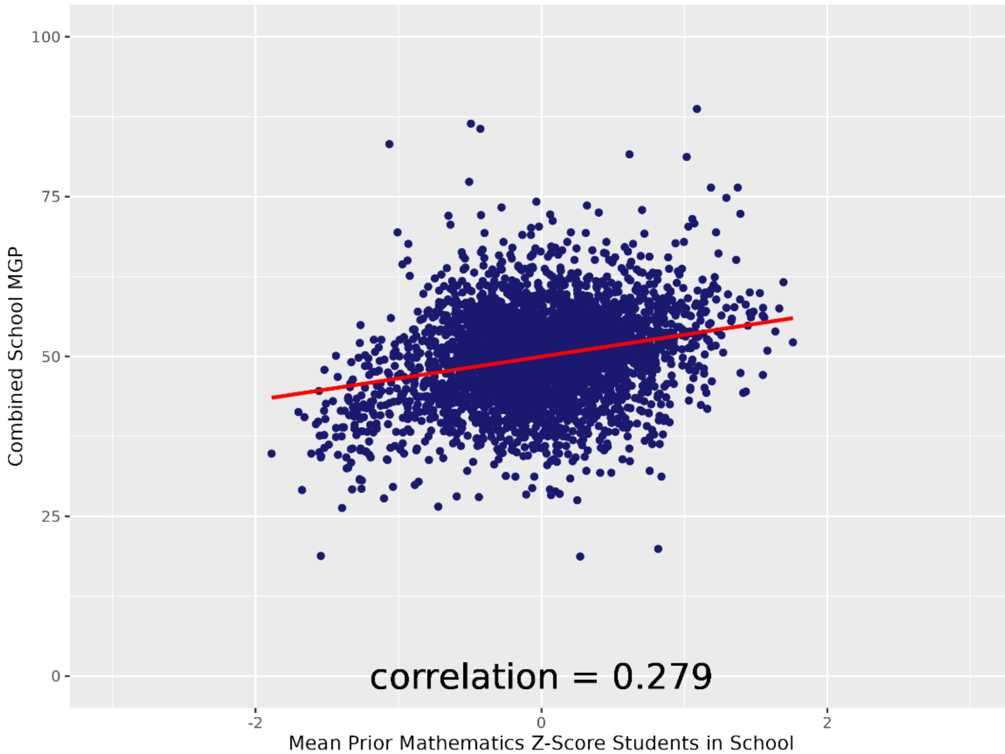


Table 9 provides the observed correlations of school MGPs by student body characteristic, summarized from the figures above.

Table 9. School MGP Correlations by School Characteristics

School Characteristics	2023-24	2024-25
ELL students in school	0.04	0.10
Students with disabilities in school	-0.08	-0.09
Economically disadvantaged students in school	0.03	0.10
Mean prior ELA Z-score	0.22	0.25
Mean prior mathematics Z-score	0.24	0.28

Given that a primary claim for the use of MGPs is that all educators and schools can demonstrate growth, regardless of the academic starting point of students, it is necessary to further disaggregate the prior achievement data by grade level to determine if a strong relationship exists between MGPs and average prior achievement. To that end, Table 10 shows the correlations between MGPs and average prior achievement, which are low across all grades and subjects. These correlations illustrate that the MGPs are substantially neutral to prior achievement.

Table 10. Correlation Between School MGPs and Average Prior Achievement by Grade Level

Grade	ELA	Mathematics
4	0.041	0.041
5	0.015	-0.039
6	-0.047	-0.008
7	0.018	-0.079
8	0.008	0.079

Informational Teacher and Principal Growth Results

To assist with continuous improvement and planning, the Department also links SGPs to teachers and principals. Teachers and principals with 20or more SGPs attributed to them receive an MGP. These results are provided for informational purposes only.

New York's growth model uses district-reported staff assignment data to link student growth scores for continuously enrolled students to principals (see Table 11).

Table 11. Principal Attribution Rates

Grade	Valid Student Records	Valid Student Records Attributed to at Least One Principal	Attribution Rate
4	293,582	273,573	93.2%
5	295,628	276,489	93.5%
6	284,550	265,013	93.1%
7	282,026	263,828	93.5%
8	278,572	263,641	94.6%
Total	1,434,358	1,342,544	93.6%

A critical element of growth analyses is the accurate identification of the courses students are taking in which they learn the content and skills covered on the tests used to measure their learning. Another critical element is identifying who is teaching those courses. A first step is to identify which courses are considered "relevant"—that is, courses in which instruction is provided that is aligned to the test being used to measure student growth. New York has developed a common set of course codes across the State, and these are used to identify courses as relevant for analysis. Appendix D provides a list of the item descriptions (grade and subject of relevant courses) used in analysis.

The methodology used to link continuously enrolled students to teachers consists of using two existing collections: Course Instructor Assignment and Student Class Entry Exit.

Students enrolled in relevant courses are attributed to the teacher(s) identified as a teacher of record for that course (see Table 12).

Table 12. Teacher Attribution Rates

Grade	Valid Student Records	Valid Student Records Attributed to at Least One Principal	Attribution Rate
4	293,582	264,531	90.10%
5	295,628	269,541	91.20%
6	284,550	257,803	90.60%
7	282,026	255,758	90.70%
8	278,572	248,904	89.30%
Total	1,434,358	1,296,537	90.40%

Reporting

Results of the New York growth models are reported to districts in a series of data files.

Institutional Accountability Results

Institutional accountability growth results are provided for all students and disaggregated by subgroup using a minimum n-size of 20.

The main reporting metrics for schools are as follows:

- **Sum of SGPs –** The sum of the SGP results in ELA and in math.
- **Number of Student Scores** – The number of SGP results in ELA and in math for continuously enrolled students.
- **Growth Index** – The mean of the SGPs for students attributed to the subgroup.
- **Growth Level** – The Growth Level associated with the reported Growth Index.

Results are presented at an aggregate level for the district and its schools separately and are available to authorized users in the Student Information Repository System (SIRS) 112 Student Growth Report for Accountability. Growth levels are only provided at the school level. For information on how to access these reports, please see the Information and Reporting Services Verification and Certification webpage. See also the Student Growth for Accountability Report Guide for information regarding the contents of the SIRS report. Additional resources about the Student Growth measure are available on the School and District Accountability webpage.

Growth Levels

As noted above, for accountability purposes, the Growth Index is translated to a Growth Level. Table 13 describes the observed distribution of Growth Levels for schools for the All Students subgroup based on their 2024-25 Growth Index.

Table 13. School and District Level Distributions

Output Level	Level 1	Level 2	Level 3	Level 4
School	21%	27%	25%	27%

Note: Because of rounding, percentages may not add to 100 percent.

Informational Teacher and Principal Results

Teacher and principal growth results are also provided for informational and improvement planning purposes. These results are not disaggregated by subgroup and use a minimum n-size of 20.

The main reporting metrics generated are as follows:

- **Number of Student Scores** – The number of SGP results in ELA and in math for continuously enrolled students.
- **MGP** – The mean of the SGPs for students attributed to the teacher or principal.

MGPs disaggregated by subject student roster files are available to authorized users on the secure Information and Reporting Services Portal (IRSP). For information about accessing the IRSP, see the IRS Portal Resources and Information webpage.

References

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Wei, Y., & Carroll, R. J. (2009). Quantile regression with measurement error. *Journal of the American Statistical Association, 104,* 1129–1143.

Appendix A. Data Processing Overview

The process used to convert the raw data to results runs through six standardized processes. The process and raw data files used to produce the results are explained in detail below.

Raw Data

All historical and current data files are transferred from NYSED. In addition to EA's standard raw data QC process, we conducted an additional quality control check this year where EA and NYSED separately confirmed the file size and number of rows in each file transferred. This ensured that the files were complete and there would be no missing data. The raw data files that were used in the production of growth results this year include:

1. **Assessment and CSEM (2024-25, 2023-24, 2022-23, and 2021-22)** – Student-level results on the State Grades 3-8 assessments and CSEMs.
2. **Directory** – Listing of all New York State Public and Nonpublic Schools.
3. **Teacher Student Course** – Students linked to each teachers' classroom used to attribute students to teachers.
4. **Staff Assignment** – Students linked to programs that principals oversee, including the start and end dates.
5. **Enrollment (Grade 8 Algebra I Continuous Enrollment, and BOCES Enrollment)** – Students who were enrolled on Basic Educational Data System (BEDS) day and the last day of the test administration period.

Standard Data

Raw data are transformed into a standardized format that 1) facilitates the processing of raw data through business rules and 2) can be interpreted by other analysts. Throughout this process, raw data modifications are catalogued, all observations are maintained, and variable names are standardized.

Input Sets

Most of the business rules in data processing are applied in transition from standard data to input sets. Input sets are the data sets that are used to estimate the regression models. Students who will ultimately be excluded from the model are retained in the input sets with an exclusion reason flag activated. These exclusion reasons, which describe students excluded from the growth results for teachers, schools, and principals, are investigated as part of the process of producing input sets.

Modeling

The statistical models are computed using the input sets in the modeling phase and the output is analyzed using a diagnostics tool that examines coefficients, residual mean squared error, student predictions, highest observable scale score, lowest observable scale score, and other key metrics.

Aggregation

Results from the modeling phase are combined to create teacher, principal, and school level metrics, such as Mean Growth Percentile, for each level. This step also includes examining aggregate diagnostic measures such as neutralities, reliability, and sample size.

Output

After the aggregation step, the final files are created for NYSED and parsed for each district.

Appendix B. Model Specification

The value-added model is based on a statistical model that explains current student achievement, measured using a posttest, using prior student achievement, measured using pretests. The primary (same-subject, prior-year) pretest enters the model via a spline function, while other pretests enter the model linearly. The model acknowledges that the pretests, being of finite length, are necessarily measured with error.

The model takes the following form:

$$y = \xi + f(z^*)\lambda + W\beta + e^*$$

$$z = z^* + v$$

Where y is posttest score; z is a vector of measured pretest scores; z^* is a vector of unobserved "true" pretest scores, equal to what z would be in the absence of measurement error; $f(.)$ is a function that produces a spline transformation of the primary (same-subject, single-lag) pretest and leaves the non-primary pretests untransformed; v is measurement error in the measured pretests z; W is a vector of other covariates; and e^* is the component of the posttest score y not explained by z or W.

In the growth model, the only variables in W are indicators for missing non-primary pretests. In observations with missing pretests, the elements in z and z^* corresponding to the missing pretests are set to zero, and the indicators corresponding to the missing pretests in W are activated. No observations in which the primary pretest is missing are included in the analysis.

The steps below describe how this model is estimated. Steps 1 through 5 illustrate steps for producing predictions of the "true" pretest scores z^* from the measured pretest scores z.

Step 1: The first step in predicting z^* from z is to break out the component of z that can be explained by W. This is useful because this component, being predictable from other variables, is not attributable to measurement error. We execute this step by regressing, using OLS, the measured pretest variables z on W. If the version of the model estimated does not include the variables W, then this regression includes only an intercept. Denote the residual from this regression u_2. Note that this residual includes pretest measurement error v. Denote as u_1 the part of the residual that is not measurement error. The important difference between u_1 and v is that u_1 is a component of the "true" pretest z^*, while v is not.

$$z = W\pi + u_2$$

$$u_2 = u_1 + v$$

Step 2: To understand the extent to which variance in u_2 is and is not generated by measurement error in z, we estimate the variance-covariance matrix of u_1, the component of u_2 that is not pretest measurement error. We estimate this as follows:

$$\Omega = V - \Sigma_{vv}$$

$$V = \frac{u_2{}' u_2}{N - k - 1}$$

where V is the variance-covariance matrix of u_2; Σ_{vv} is an estimate of the variance-covariance matrix of v; and *N* is the number of observations and *k* the number of regressors in the regression in step 1. The matrix Σ_{vv} is a diagonal matrix in which each diagonal entry is equal to the average squared conditional standard error of measurement (CSEM) across students for each pretest in z. In observations for which some pretests in z are missing, the CSEMs for those pretests are set to zero.

Step 3: We use V and Ω to estimate coefficients from a regression of the unobserved u_1 on the observed u_2. Denote these coefficients γ_1.

$$\gamma_1 = V^{-1}\Omega$$

Step 4: Using the coefficients γ_1, we produce a prediction, $\tilde{z}$, of the "true" pretest scores z^* from the measured pretest scores z.

$$\tilde{z} = z - u_2(I - \gamma_1)$$

Step 5: To measure the extent to which the prediction $\tilde{z}$ differs from the "true" pretest scores z^*, we estimate the variance-covariance matrix of r = $\tilde{z}$ - z^*, which we denote Σ_{rr}.

$$\Sigma_{rr} = \Sigma_{vv}\gamma_1$$

In steps 6 through 8, we produce variables that can be used to estimate a regression with a spline function in the primary pretest that accounts for measurement error in that primary pretest.

Step 6: Before producing pretest variables that are transformed in a way that will allow us to estimate a spline function, we need to place the spline knots. We are interested in setting knots based on the distribution of the "true" primary pretest score z_1^* rather than the measured primary pretest score z_1. The first step toward this goal is to estimate the variance of z_1^*, using the difference between the variance of z_1 and the variance of its measurement error v_1.

$$\sigma^2_{z(1)*} = \sigma^2_{z(1)} - \sigma^2_{v(1)}$$

We estimate the variance of v_1 using the mean of the squared CSEMs of z_1 across students.

Step 7: We place the spline function's knots at approximately the 5th, 25th, 75th, and 95th percentiles of the "true" primary pretest score z_1^*. We approximate these percentiles using the following steps:

a) First, we compute what these percentiles would be for the *measured* pretest z_1 under an assumption that z_1 is normally distributed. For the *p*th percentile, this is equal to $d^*(p) = \mu_{z(1)} + \sigma_{z(1)} \times \Phi^{-1}(p/100)$, where $\mu_{z(1)}$ and $\sigma_{z(1)}$ are the mean and standard deviation of z_1 and $\Phi^{-1}(.)$ is the inverse standard normal cumulative distribution function.
b) Second, we compute the empirically observed percentiles for the *measured* pretest z_1. For the *p*th percentile, denote this *d*(*p*). The difference between $d^*(p)$ and *d*(*p*) is that the former assumes normality, while the latter does not. In practice, the test scores may not be normally distributed, which may lead to discrepancies between $d^*(p)$ and *d*(*p*).
c) Third, we compute what these percentiles would be for the *true* pretest z_1^* under an assumption that z_1^* is normally distributed. For the *p*th percentile, this is equal to $c^*(p) = \mu_{z(1)} + \sigma_{z(1)*} \times \Phi^{-1}(p/100)$, where $\sigma_{z(1)*}$ is the standard deviation of z_1^*, computed by taking the square root of the variance $\sigma^2_{z(1)*}$ computed in step 6.
d) Finally, approximate the *p*th percentile of z_1^* as $c(p) = c^*(p) + d(p) - d^*(p)$.

Step 8: A spline function in the "true" primary pretest score with knots at *c*(5), *c*(25), *c*(75), and *c*(95) would be estimated by including the following variables on the right-hand-side of a regression:

$$z^*_{11i} = (z^*_{1i} - c(5))I(z^*_{1i} < c(5))$$

$$z^*_{12i} = (z^*_{1i} - c(25))I(z^*_{1i} < c(25))$$

$$z^*_{13i} = (z^*_{1i} - c(75))I(z^*_{1i} > c(75))$$

$$z^*_{14i} = (z^*_{1i} - c(95))I(z^*_{1i} > c(95))$$

alongside the student-level "true" primary pretest score z^*_{1i} for student *i*, where $I(.)$ is an indicator variable that equals 1 if the statement inside the parentheses is true and 0 otherwise. However, given that z^*_{1i} is not observed, we instead produce predictions of the spline variables using the following formulas:

$$\tilde{z}_{11i} = (\tilde{z}_{1i} - c(5))P_{1i} - \sigma_r \phi\left(\frac{c(5) - \tilde{z}_{1i}}{\sigma_r}\right)$$

$$\tilde{z}_{12i} = (\tilde{z}_{1i} - c(25))P_{2i} - \sigma_r \phi\left(\frac{c(25) - \tilde{z}_{1i}}{\sigma_r}\right)$$

$$\tilde{z}_{13i} = (\tilde{z}_{1i} - c(75))P_{3i} + \sigma_r \phi\left(\frac{\tilde{z}_{1i} - c(75)}{\sigma_r}\right)$$

$$\tilde{z}_{14i} = (\tilde{z}_{1i} - c(95))P_{4i} + \sigma_r \phi\left(\frac{\tilde{z}_{1i} - c(95)}{\sigma_r}\right)$$

where σ_r is the square root of the diagonal element of Σ_{rr} corresponding to the primary pretest, $\phi(.)$ is the standard normal probability density function, and the P terms are equal to

$$P_{1i} = \Phi\left(\frac{c(5)-\tilde{z}_{1i}}{\sigma_r}\right) \qquad P_{2i} = \Phi\left(\frac{c(25)-\tilde{z}_{1i}}{\sigma_r}\right) \quad P_{3i} = \Phi\left(\frac{\tilde{z}_{1i}-c(75)}{\sigma_r}\right) \quad P_{4i} = \Phi\left(\frac{\tilde{z}_{1i}-c(95)}{\sigma_r}\right)$$

which are estimates of the probabilities that the "true" pretest z_{1i}^* is below its 5th percentile, below its 25th percentile, above its 75th percentile, and above its 95th percentile, respectively.

In steps 9 through 14, we estimate the growth model and produce student growth percentiles.

Step 9: We estimate the growth model's coefficients by regressing, by OLS, the posttest y on the predictions that correspond to the spline function of the primary pretest, $\tilde{z}_{1i}$, $\tilde{z}_{11i}$, $\tilde{z}_{12i}$, $\tilde{z}_{13i}$, and $\tilde{z}_{14i}$; all predictions of non-primary pretest variables in $\tilde{z}$; and all indicators for missing non-primary pretest variables W. This is estimating the growth model described at the beginning of this section, except that all right-hand-side variables affected by measurement error in pretests are replaced with predictions of what those variables would be in the absence of measurement error. The use of these predictions adjusts for pretest measurement error when estimating the coefficients.

Step 10: The coefficients on the spline variables are often difficult to interpret. It is much easier to interpret the slopes of the segments of the spline function instead. We calculate these slopes for the five segments of the slope function as below:

Slope of 1st segment (below 5th percentile): sum of coefficients on $\tilde{z}_{1i}$, $\tilde{z}_{11i}$, and $\tilde{z}_{12i}$

Slope of 2nd segment (5th to 25th percentile): sum of coefficients on $\tilde{z}_{1i}$ and $\tilde{z}_{12i}$

Slope of 3rd segment (25th to 75th percentile): coefficient on $\tilde{z}_{1i}$

Slope of 4th segment (75th to 95th percentile): sum of coefficients on $\tilde{z}_{1i}$ and $\tilde{z}_{13i}$

Slope of 5th segment (above 95th percentile): sum of coefficients on $\tilde{z}_{1i}$, $\tilde{z}_{13i}$, and $\tilde{z}_{14i}$

If any of these slopes are negative, we replace the coefficient on the appropriate spline variable with a new coefficient equal to the difference between the coefficient and the slope, forcing the slope of this segment to be zero. For example, suppose the slope of the 5th segment of the spline is -0.014, and coefficient on $\tilde{z}_{14i}$ is -1.536. In this situation, we replace the coefficient on $\tilde{z}_{14i}$ with -1.536 - (-0.014) = -1.522, which sets the slope of the 5th segment to zero.

Step 11: Using the coefficients estimated above, we compute student growth residuals equal to the difference between actual post achievement y and predicted achievement $\hat{y}$. When computing predicted achievement $\hat{y}$, the coefficients on the predicted pretests $\tilde{z}$ are multiplied by the *measured* pretests z, not the predicted pretests $\tilde{z}$. Similarly, the coefficients on the predicted spline variables for the primary pretest $\tilde{z}_{11i}$, $\tilde{z}_{12i}$, etc. are multiplied by spline variables for the *measured* primary pretest z_1, with the knots for the spline set at *d*(*p*) rather than *c*(*p*).

Step 12**:** To compute SGPs, we need to know not only the difference between actual and predicted post achievement, but also the variance of this difference. We allow this variance to differ by the level of predicted achievement $\hat{y}$. This is computed using the following steps:

- Demean the student growth residuals computed in step 11.
- Divide predicted achievement $\hat{y}$ into 20 bins, each containing an equal number of observations. (The bins may not be exactly equal, depending on the data.)
- Calculate the standard deviation (SD) of the growth residuals in each of the predicted achievement bins.
- Compute a moving average standard deviation (MASD) by computing the average SD across three adjacent bins, starting from the 2nd bin to the 19th bin. For example, the MASD for the 2nd bin is the mean SD across the 1st, 2nd, and 3rd bins.
- Set the MASD of the 1st bin to the average of the SD of the 1st bin and the MASD of the 2nd bin, and the MASD of the 20th bin to the average of the MASD of the 19th bin and the SD of the 20th bin.

Step 13**:** The SGP is calculated for any given student by dividing the student growth residual (obtained from step 11) by the moving average standard deviation (MASD) of residuals (obtained in step 12) for the appropriate bin; transforming the quotient using the standard normal cumulative distribution function; and multiplying the result by 100 and rounding.

Appendix C. Interpolating Standard Errors of Measurement at the Lowest and Highest Obtainable Scale Scores

Sometimes, the model used to produce student-level predictions $\hat{y}_i$ can cause these predictions to fall outside the boundaries of the defined scale score. Let the floor and ceiling in the data be denoted as η_f and η_c, respectively. It is, therefore, possible that $\hat{y}_i < \eta_f$ or $\eta_c < \hat{y}_i$. However, the observed score can never fall outside these bounds.

When a prediction falls outside the boundaries of the scale score, it can cause bias in the statistics used to characterize a student, teacher, principal, or school. This phenomenon seems to occur as a result of the large conditional standard errors of measurement at the extreme scores, $csem(\hat{\theta}_i)$. The following procedure is implemented to deal with these large standard errors.

Interpolation Procedure for Conditional Standard Errors of Lowest and Highest Obtainable Scale Scores

Interpolate new conditional standard errors of measurement as the “nearest neighbor” of any extreme value. Thus, at an $M = 2$ cutoff, for the highest obtainable scale score (HOSS) and the score immediately below the HOSS, the SEM associated with the score two below the HOSS would be used. Similarly, the lowest obtainable scale score (LOSS) and the score immediately above the LOSS would have the SEM associated with the score two above the LOSS. As M increases more points are included, and the point they are set to moves toward the middle of the scale score distribution.

Adjustment of out-of-boundary predictions is implemented using the following steps:

Step 1. Run the regression without modification.

Step 2. Verify that $\eta_f \leq \hat{y}_i \leq \eta_c$ for all i.

Step 3. If the inequality in Step 2 is true, stop; the run is complete. Otherwise, continue to Step 4.

Step 4. Set $M = 1$ and update the SEMs of the exact HOSS and LOSS scores.

Step 5. Use the updated $csem(\hat{\theta}_i)$ in lieu of the standard error of the LOSS and HOSS in the test score data.

Step 6. Run the growth model.

Step 7. Verify the inequality in Step 2; if it holds, stop updating. If it does not hold, increase M by 1 and return to Step 5.

If this method does not result in the inequality in Step 2 being met after M = 7 (i.e., after running with M = 7), then simply take the most recent run that did converge, set $\hat{y}_i = \eta_c$ where $\hat{y}_i > \eta_c$, and $\hat{y}_i = \eta_f$ where $\hat{y}_i < \eta_f$. For the predicted variance, use the predicted variance of the closest estimate where the inequality in Step 6 does hold.

Appendix D. Item Description Used in Analysis

The teacher-student-course linkage file includes information about courses taught to students. The item description provides information about which courses are relevant to State tests.

Table D1 shows the records used for growth model analysis. Students enrolled in Algebra I (course codes 02050), Geometry (course code 02072CC and 02072), or Algebra II (course code 02056CC) who take Grades 6-8 mathematics assessments or Grade 8 students who take the Algebra I Regents examination are included in the analysis.

Table D1. Relevant Item Descriptions

Item Description
Grade 3 ELA
Grade 3 Mathematics
Grade 4 ELA
Grade 4 Mathematics
Grade 5 ELA
Grade 5 Mathematics
Grade 6 ELA
Grade 6 Mathematics
Grade 7 ELA
Grade 7 Mathematics
Grade 8 ELA
Grade 8 Mathematics
Grade 8 Algebra I

Appendix E: Model Coefficients

The tables that follow display regression model coefficients (labeled as "Effects") for the New York growth model in each grade and subject. These model coefficients represent the predicted change in current year test scores for one unit of change in each variable shown in the table, holding other variables constant. For example, in Table E1, the coefficient on the pretest for the central segment is equal to the predicted change in a student's current-year test score given a one-point increase in a student's prior-grade test score when that prior-grade test score is in the central segment of the spline function. For example, according to Table E1, a one-point increase in a student's prior-grade ELA score within the range of the central segment of the spline is associated with a 1.077 increase in a student's current-year ELA score.

The coefficient on the pretest in the central-left or central-right segment is equal to the difference in slope between that segment and the central segment. To find out the predicted change in a student's current-year test score given a one-point increase in a student's prior-grade test score within the central-left or central-right segment, add the coefficient for that segment to the coefficient for the central segment. For example, according to Table E1, a one-point increase in a student's prior-grade ELA score within the range of the central-right segment is associated with a -0.535 + 1.077 = 0.542 increase in a student's current-year ELA score.

The coefficient on the pretest in the top or bottom segment is equal to the difference in slope between that segment and the adjoining (if bottom, central-left; if top, central-right) segment. To find out the predicted change in a student's current-year test score given a one-point increase in a student's prior-grade test score within the top or bottom segment, add the coefficients for that segment, the adjoining segment, and the central segment. For example, according to Table E1, a one-point increase in a student's prior-grade ELA score within the range of the top segment is associated with a 0.243 - 0.535 + 1.077 = 0.785 increase in a student's current-year ELA score.

For yes/no variables, model coefficients represent the predicted change in current year test scores given a change from no to yes. Missing flags are yes/no variables set to yes if the noted variable is missing and no otherwise.

Because of the differences in model and variable types, it is important to keep in mind that effect sizes cannot be compared directly across different types of variables.

Table E1. Grade 4 ELA Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-25.309	2.820	0.000
Prior-Grade ELA Scale Score - Bottom Segment	-0.985	0.050	0.000
Prior-Grade ELA Scale Score - Central-left Segment	-0.093	0.020	0.000
Prior-Grade ELA Scale Score - Central Segment	1.077	0.006	0.000
Prior-Grade ELA Scale Score - Central-right Segment	-0.535	0.019	0.000
Prior-Grade ELA Scale Score - Top Segment	0.243	0.074	0.001

Table E2. Grade 5 ELA Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	36.547	2.545	0.000
Prior-Grade ELA Scale Score - Bottom Segment	-0.806	0.040	0.000
Prior-Grade ELA Scale Score - Central-left Segment	0.324	0.018	0.000
Prior-Grade ELA Scale Score - Central Segment	0.779	0.007	0.000
Prior-Grade ELA Scale Score - Central-right Segment	-0.322	0.018	0.000
Prior-Grade ELA Scale Score - Top Segment	0.448	0.075	0.000
Two-Grades-Prior ELA Scale Score	0.157	0.005	0.000
Missing Flag: Two-Grades-Prior ELA Scale Score	68.504	2.001	0.000

Table E3. Grade 6 ELA Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-7.656	2.663	0.004
Prior-Grade ELA Scale Score - Bottom Segment	-0.742	0.042	0.000
Prior-Grade ELA Scale Score - Central-left Segment	0.095	0.019	0.000
Prior-Grade ELA Scale Score - Central Segment	0.713	0.007	0.000
Prior-Grade ELA Scale Score - Central-right Segment	-0.249	0.017	0.000
Prior-Grade ELA Scale Score - Top Segment	-0.097	0.053	0.071
Two-Grades-Prior ELA Scale Score	0.231	0.006	0.000
Three-Grades-Prior ELA Scale Score	0.062	0.005	0.000
Missing Flag: Two-Grades-Prior ELA Scale Score	101.661	2.576	0.000
Missing Flag: Three-Grades-Prior ELA Scale Score	37.443	2.765	0.000

Table E4. Grade 7 ELA Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-19.153	2.584	0.000
Prior-Grade ELA Scale Score - Bottom Segment	-0.890	0.038	0.000
Prior-Grade ELA Scale Score - Central-left Segment	0.122	0.015	0.000
Prior-Grade ELA Scale Score - Central Segment	0.829	0.007	0.000
Prior-Grade ELA Scale Score - Central-right Segment	-0.369	0.017	0.000
Prior-Grade ELA Scale Score - Top Segment	0.042	0.049	0.384
Two-Grades-Prior ELA Scale Score	0.159	0.005	0.000
Three-Grades-Prior ELA Scale Score	0.052	0.004	0.000
Missing Flag: Two-Grades-Prior ELA Scale Score	69.889	2.245	0.000
Missing Flag: Three-Grades-Prior ELA Scale Score	31.852	2.593	0.000

Table E5. Grade 8 ELA Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-38.530	2.864	0.000
Prior-Grade ELA Scale Score - Bottom Segment	-0.541	0.037	0.000
Prior-Grade ELA Scale Score - Central-left Segment	-0.089	0.014	0.000
Prior-Grade ELA Scale Score - Central Segment	0.917	0.008	0.000
Prior-Grade ELA Scale Score - Central-right Segment	-0.283	0.017	0.000
Prior-Grade ELA Scale Score - Top Segment	-0.160	0.053	0.003
Two-Grades-Prior ELA Scale Score	0.106	0.007	0.000
Three-Grades-Prior ELA Scale Score	0.053	0.005	0.000
Missing Flag: Two-Grades-Prior ELA Scale Score	46.608	2.875	0.000
Missing Flag: Three-Grades-Prior ELA Scale Score	32.900	2.938	0.000

Table E6. Grade 4 Mathematics Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-38.898	2.812	0.000
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.893	0.047	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	-0.125	0.019	0.000
Prior-Grade Mathematics Scale Score - Central Segment	1.101	0.006	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.513	0.022	0.000
Prior-Grade Mathematics Scale Score - Top Segment	0.962	0.085	0.000

Table E7. Grade 5 Mathematics Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	41.329	1.958	0.000
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.684	0.041	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	-0.033	0.014	0.019
Prior-Grade Mathematics Scale Score - Central Segment	0.717	0.006	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.008	0.016	0.607
Prior-Grade Mathematics Scale Score - Top Segment	0.726	0.064	0.000
Two-Grades-Prior Mathematics Scale Score	0.187	0.004	0.000
Missing Flag: Two-Grades-Prior Mathematics Scale Score	84.930	1.984	0.000

Table E8. Grade 6 Mathematics Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-30.247	2.646	0.000
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.626	0.048	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	-0.087	0.019	0.000
Prior-Grade Mathematics Scale Score - Central Segment	0.713	0.006	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.091	0.011	0.000
Prior-Grade Mathematics Scale Score - Top Segment	0.168	0.051	0.001
Two-Grades-Prior Mathematics Scale Score	0.188	0.005	0.000
Three-Grades-Prior Mathematics Scale Score	0.127	0.005	0.000
Missing Flag: Two-Grades-Prior Mathematics Scale Score	83.525	2.303	0.000
Missing Flag: Three-Grades-Prior Mathematics Scale Score	76.218	2.924	0.000

Table E9. Grade 7 Mathematics Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-60.400	2.477	0.000
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.830	0.051	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	-0.057	0.019	0.003
Prior-Grade Mathematics Scale Score - Central Segment	0.887	0.006	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.184	0.011	0.000
Prior-Grade Mathematics Scale Score - Top Segment	-0.568	0.054	0.000
Two-Grades-Prior Mathematics Scale Score	0.133	0.005	0.000
Three-Grades-Prior Mathematics Scale Score	0.097	0.005	0.000.
Missing Flag: Two-Grades-Prior Mathematics Scale Score	58.530	2.174	0.000
Missing Flag: Three-Grades-Prior Mathematics Scale Score	57.964	2.897	0.000

Table E10. Grade 8 Mathematics Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-54.063	4.054	0.000
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.958	0.073	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	0.023	0.032	0.476
Prior-Grade Mathematics Scale Score - Central Segment	0.982	0.010	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.285	0.016	0.000
Prior-Grade Mathematics Scale Score - Top Segment	-0.052	0.038	0.177
Two-Grades-Prior Mathematics Scale Score	0.081	0.007	0.000
Three-Grades-Prior Mathematics Scale Score	0.045	0.007	0.000.
Missing Flag: Two-Grades-Prior Mathematics Scale Score	35.424	3.197	0.000
Missing Flag: Three-Grades-Prior Mathematics Scale Score	27.294	4.131	0.000

Table E11. Grade 8 Algebra I Model Coefficients

Effect Name	Effect	Standard Error	*p*-value
Constant Term	-169.826	3.002	0.000.
Prior-Grade Mathematics Scale Score - Bottom Segment	-0.430	0.035	0.000
Prior-Grade Mathematics Scale Score - Central-left Segment	0.236	0.014	0.000
Prior-Grade Mathematics Scale Score - Central Segment	0.354	0.007	0.000
Prior-Grade Mathematics Scale Score - Central-right Segment	-0.260	0.022	0.000
Prior-Grade Mathematics Scale Score - Top Segment	0.487	0.073	0.000
Two-Grades-Prior Mathematics Scale Score	0.068	0.006	0.000
Three-Grades-Prior Mathematics Scale Score	0.081	0.005	0.000
Missing Flag: Two-Grades-Prior Mathematics Scale Score	32.605	2.752	0.000
Missing Flag: Three-Grades-Prior Mathematics Scale Score	48.241	3.238	0.000