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NYS Plus Mathematics Learning Standards					
Number and Quantity The Complex Number System (N-CN)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Perform arithmetic operations with complex numbers.	**N-CN.A.3+**	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	NO CHANGE	

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<table>
<tr><th colspan="6">NYS Plus Mathematics Learning Standards</th></tr>
<tr><th colspan="6">Number and Quantity
The Complex Number System (N-CN)</th></tr>
<tr><th colspan="2"></th><th>Standard Code</th><th>Current Standard</th><th>Revised Standard Recommendation for 2018-19</th><th>Additional Information/Notes</th></tr>
<tr><td rowspan="5">Cluster</td><td rowspan="5">B. Represent complex numbers and their operations on the complex plane.</td><td rowspan="2">N-CN.B.4+</td><td rowspan="2">Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</td><td>4a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and convert between rectangular and polar forms of a given complex number.</td><td rowspan="2">Removed "Explain why". Also, unpacked standard into two separate parts.</td></tr>
<tr><td>4b. Determine whether rectangular or polar form is more efficient given the context.</td></tr>
<tr><td>N-CN.B.5+</td><td>Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
For example, $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.</td><td>5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°. For example: DeMoivre's Theorem</td><td>Included one of the main applications as an example otherwise left unchanged.</td></tr>
<tr><td rowspan="2">N-CN.B.6+</td><td rowspan="2">Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</td><td>6a. Calculate the distance between two points in the complex plane.</td><td rowspan="2">Separated the standard since it addresses two different concepts.</td></tr>
<tr><td>6b. Find the midpoint of the segment whose endpoints are in the complex plane.</td></tr>
</table>

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NYS Plus Mathematics Learning Standards					
Number and Quantity **The Complex Number System (N-CN)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	C. Use complex numbers in polynomial identities and equations.	**N-CN.C.8+**	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.	NO CHANGE.	
		N-CN.C.9+	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	State and apply the Fundamental Theorem of Algebra.	"Know" is ambiguous and Fundamental Theorem of Algebra applies to all polynomials not just quadratics.

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NYS Plus Mathematics Learning Standards					
Number and Quantity **Vector and Matrix Quantities (N-VM)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Represent and model with vector quantities.	**N-VM.A.1+**	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).	Represent a vector analytically and geometrically. For example: rectangular form, polar form, unit form.	Vocabulary should be defined in a glossary. Rewrote as a measurable application.
		N-VM.A.2+	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Find the magnitude and direction of a given vector.	Used clear and concise language.
		N-VM.A.3+	Solve problems involving velocity and other quantities that can be represented by vectors.	Solve problems using vectors analytically and geometrically (e.g. velocity and forces).	Used clear and concise language.

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NYS Plus Mathematics Learning Standards					
Number and Quantity **Vector and Matrix Quantities (N-VM)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B. Perform operations on vectors.	**N-VM.B.4+**	Add and subtract vectors.	Add and subtract vectors analytically and geometrically.	Used clear and concise language.
		N-VM.B.4a+	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	REMOVE STANDARD	By adding analytically and geometrically in previous standard **N-VM.B.4+** .
		N-VM.B.4b+	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	REMOVE STANDARD	By adding analytically and geometrically in previous standard **N-VM.B.4+**.
		N-VM.B.4c+	Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	REMOVE STANDARD	By adding analytically and geometrically in previous standard **N-VM.B.4+**.

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NYS Plus Mathematics Learning Standards					
Number and Quantity Vector and Matrix Quantities (N-VM)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B.Perform operations on vectors.	**N-VM.B.5+**	Multiply a vector by a scalar.	Multiply a vector by a scalar analytically and geometrically.	Used clear and concise language
		N-VM.B.5a+	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	REMOVE STANDARD	By adding analytically and geometrically in previous standard **N-VM.B.5+**.
		N-VM.B.5b+	Compute the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).	REMOVE STANDARD	By adding analytically and geometrically in previous standard **N-VM.B.5+**.

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NYS Plus Mathematics Learning Standards

Number and Quantity
Vector and Matrix Quantities (N-VM)

Cluster		Standard Code	Current Standard	Revised Standard Recommendation for 2018-19	Additional Information/Notes
Cluster	C. Perform operations on matrices and use matrices in applications.	**N-VM.C.6+**	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	Use matrices to represent and model real world situations. For example: networks.	"Data" is too vague and gaming implies gambling.
		N-VM.C.7+	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Multiply matrices by scalars.	Used clear and concise language.
		N-VM.C.8+	Add, subtract, and multiply matrices of appropriate dimensions.	Add, subtract, and multiply matrices.	Used clear and concise language.
		N-VM.C.9+	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Determine if matrices are a group under addition and multiplication.	Had been formally named as a concept (2005 AN1 standards). Use concise, mathematical language where appropriate.
		N-VM.C.10+	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	REMOVE STANDARD	Standard is redundant with rewording of N-VM.C.9+
		N-VM.C.11+	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Use matrices to perform linear transformations in the plane. For example: multiplying a vector by 2x2 matrix.	Used clear and concise language.
		N-VM.C.12+	Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Calculate and interpret the determinant of a matrix. For example: calculating area.	Used clear and concise language.

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NYS Plus Mathematics Learning Standards					
Algebra Arithmetic with Polynomials and Rational Expressions (A-APR)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	C. Use polynomial identities to solve problelms.	**A-APR.C.5+**	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.	Use the Binomial Theorem for the expansion of $(x + y)^n$ for a positive integer n.	Proof by mathematical induction does not exist anywhere in the standards. "Know" is ambiguous.

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NYS Plus Mathematics Learning Standards					
Algebra **Arithmetic with Polynomials and Rational Expressions (A-APR)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	D. Rewrite rational expressions.	**A-APR.D.7+**	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	NO CHANGE.	

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NYS Plus Mathematics Learning Standards					
Algebra Reasoning with Equations and Inequalities (A-REI)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	C. Solve systems of equations.	**A-REI.C.6b+**	ADDITION Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (Shared with Algebra I) PARCC/NYSED: i) Tasks are limited to 3x3 systems only. Systems of 3 linear equations with 3 variables only.	6b. Solve systems of linear equations in three variables.	Moved from A2. This part of the standard fits better with matrix standards. See revised A-REI.C.6a for Algebra I. A-REI.C.6 is currently shared with Algebra I and II.
		A-REI.C.8+	Represent a system of linear equations as a single matrix equation in a vector variable.	NO CHANGE.	
		A-REI.C.9+	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	NO CHANGE.	

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NYS Plus Mathematics Learning Standards					
Functions **Interpreting Functions (F-IF)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	C. Analyze functions using different representations.	**F-IF.C.7d+**	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★	NO CHANGE.	

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NYS Plus Mathematics Learning Standards					
Functions **Building Functions (F-BF)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Build a function that models a relationship between two quantities.	**F-BF.A.1c+**	Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. ★	NO CHANGE.	

NYS Plus Mathematics Learning Standards					
Functions Building Functions (F-BF)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B. Build new functions from existing functions.	**F-BF.B.3c+**	ADDITION 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (Shared with AI). PARCC: i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions ii) Tasks may involve recognizing even and odd functions.	3c. Determine algebraically whether or not a function is even or odd.	Added this concept from Algebra II, **F-BF.B.3,** to the plus standards. See revised F-BF.B.3a (Algebra I) and F-BF.B.3b (Algebra II)
		F-BF.B.4b+	Verify by composition that one function is the inverse of another.	NO CHANGE.	
		F-BF.B.4c+	Read values of an inverse function from a graph or a table, given that the function has an inverse.	Given the graph or table of an invertible function, determine coordinates of its inverse.	Used clear and concise language.
		F-BF.B.4d+	Produce an invertible function from a non-invertible function by restricting the domain.	Determine an invertible function from a non-invertible function by restricting the domain. For example: F.TF.B.6(+)	"Produce" not commonly used in the standards.
		F-BF.B.5+	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	5b. Use inverse relationships to solve problems involving logarithms and exponents (+). 5c. Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms and solve logarithmic equations. (+)	See **F-BF.B.5a** addition to Algebra II. Needed to solve exponential modeling equations.

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NYS Plus Mathematics Learning Standards					
Functions **Trigonometric Functions (F-TF)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Extend the domain of trigonometric functions using the unit circle.	**F-TF.A.3+**	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number.	Move standard to Geometry (limited to degrees and quadrants 1 and 2 on the unit circle). The rest of the standard is removed.	Since we are including the Laws of Sines and Cosines in Geometry to include all triangles (instead of limited to right triangles), we need to address using trigonometric ratios of obtuse angles. It's also a logical introduction of the unit circle, which is built upon in Algebra II in F.TF.A.2. The angles are specified here to restrict angle measurement to degrees, and to focus on the special triangles for the introduction of the unit circle.
		F-TF.A.4+	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Move standard to Algebra II.	

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NYS Plus Mathematics Learning Standards					
Functions **Trigonometric Functions (F-TF)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B. Model periodic phenomena with trigonometric functions.	**F-TF.B.6+**	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	NO CHANGE.	
		F-TF.B.7+	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	Use inverse functions to solve trigonometric equations that arise in modeling contexts, evaluate the solutions using technology, and interpret them in terms of the context. ★	Replaced semicolon with comma.

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NYS Plus Mathematics Learning Standards					
Functions **Trigonometric Functions (F-TF)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	C. Prove and apply trigonometric identities.	**F-TF.C.9+**	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	NO CHANGE.	

NYS Plus Mathematics Learning Standards					
Geometry Similarity, Right Triangles and Trigonometry (G-SRT)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	D. Apply trigonometry to general triangles.	**G-SRT.D.9+**	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Moved standard to Geometry.	Including this standard in the Geometry course is consistent with the goal of achieving mastery of the trigonometric ratios sine, cosine and tangent. This is a natural progression from right triangles to all triangles. This standard was differentiated into an assessable and non-assessable part with the utilization of the word "explore", specifically the derivation is important for understanding but not appropriate for summative assessment. However, the application is appropriate for assessment in Geometry.
		G-SRT.D.10+	Prove the Laws of Sines and Cosines and use them to solve problems.	Moved standard to Geometry.	Same as the rationale above.
		G-SRT.D.11+	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	11a +. Prove the Law of Sines and the Law of Cosines and apply in all cases including the ambiguous case and resultant forces. ★	Although the application of this standard is addressed in Geometry, the special case is reserved for the plus standards. Forces naturally align with vector representation.

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NYS Plus Mathematics Learning Standards					
Geometry **Circles (G-C)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Understand and apply theorems about circles.	**G-C.A.4+**	Construct a tangent line from a point outside a given circle to the circle.	Moved to Geometry.	This construction ties together standards with other construction standards lending coherence. It should be in geometry, not considered an additional or "+" standard.

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NYS Plus Mathematics Learning Standards					
Geometry **Expressing Geometric Properties with Equations (G-GPE)**					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Translate between the geometric description and the equation for a conic section.	**G-GPE.A.2**	ADDITION Derive the equation of a parabola given a focus and directrix.	2+. Explore the relationship among the parabola, focus, and directrix and use the equation to model a real life situation. ★	Moved from A2 and included real life applications. This naturally fits with conic sections.
		G-GPE.A.3+	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Derive the equations of ellipses and hyperbolas given the foci and use the equations to model real life situations. ★	Needed real world context.

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NYS Plus Mathematics Learning Standards					
Geometry Geometric Measurement and Dimension (G-GMD)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	A. Explain volume formulas and use them to solve problems.	**G-GMD.2+**	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	REMOVE STANDARD	Concept belongs in integral calculus.

NYS Plus Mathematics Learning Standards					
Statistics and Probability ★ Conditional Probability and the Rules of Probability (S-CP)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**S-CP.B.8+**	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	REMOVE STANDARD	Merged with **S.CP.A.3** including the symbolic form. (Algebra II). This standard is redundant.
		S-CP.B.9+	Use permutations and combinations to compute probabilities of compound events and solve problems.	Solve problems using permutations and combinations to compute probabilities of compound events.	Used clear and concise language.

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NYS Plus Mathematics Learning Standards					
Statistics and Probability ★ Using Probability to Make Decisions (S-MD)					
		Standard Code	Current Standard	Revised Standard Recommendation for 2018-19	Additional Information/Notes
Cluster	A. Calculate expected values and use them to solve problems.	**S-MD.A.1+**	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	Graph a probability distribution for a discrete random variable based on either empirical or theoretical probabilities.	Used clear and concise language.
		S-MD.A.2+	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	Calculate and interpret the expected value of a random variable.	Used clear and concise language.
		S-MD.A.3+	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.	REMOVE STANDARD.	This standard is addressed in S.MD.A.1 (+) and S.MD.A2 (+).
		S-MD.A.4+	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?	REMOVE STANDARD.	This standard is addressed in S.MD.A.1 (+) and S.MD.A2 (+).

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NYS Plus Mathematics Learning Standards					
Statistics and Probability Using Probability to Make Decisions (S-MD)					
		Standard Code	**Current Standard**	**Revised Standard Recommendation for 2018-19**	**Additional Information/Notes**
Cluster	B. Use probability to evaluate outcomes of decisions.	**S-MD.B.5+**	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	Use expected values from probability distributions to evaluate and compare the outcomes of decisions.	Used clear and concise language.
		S-MD.B.5a+	Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.	REMOVE STANDARD.	This standard is already addressed included S.MD.B.5+.
		S-MD.B.5b+	Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.	REMOVE STANDARD.	This standard is already addressed included S.MD.B.5+.
		S-MD.B.6+	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Use probabilities to make fair decisions. Such as, determine if a decision making strategy produces equally probable outcomes.	Used clear and concise language.
		S-MD.B.7+	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Using probability concepts, evaluate decisions and strategies. Such as, make decisions based on the most favorable outcome.	Used clear and concise language.

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