

Our Students. Their Moment.

Regents Examination in Geometry (Common Core)

> Sample Items Spring 2014



THE STATE EDUCATION DEPARTMENT / THE UNIVERSITY OF THE STATE OF NEW YORK / ALBANY, NY 12234

New York State Common Core Sample Questions: Regents Examination in Geometry (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Geometry (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Geometry (Common Core) we have provided fourteen questions. These questions include multiple-choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Geometry Regents Examination in Geometry measuring CCLS beginning in June 2015. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standards are assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.

- Notice the application of mathematical ways of thinking to real-world issues and challenges.
- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standards will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, all distractors will be based on plausible missteps.

Constructed Response Questions

Math constructed response questions are similar to past questions, asking students to show their work in completing one or more tasks or more extensive problems. Constructed response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings appear below each item to provide information for teacher use to help interpret the item, understand measurement with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice items, the correct option.

- **Measures CCLS:** This item measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.
- **Mathematical Practices:** If applicable, this is a list of mathematical practices associated with the item.
- **Commentary:** This is an explanation of how the item measures the knowledge, skills, and proficiencies characterized by the identified cluster.
- **Rationale:** For multiple-choice items, this section provides the correct option and demonstrates one method for arriving at that response. For constructed response items, one possible approach to solving the item is shown followed by the scoring rubric that is specific to the item. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

- 1 What are the coordinates of the point on the directed line segment from K(-5,-4) to L(5,1) that partitions the segment into a ratio of 3 to 2?
 - (1) (-3,-3)
 - (2) (-1,-2)
 - (3) $(0, -\frac{3}{2})$
 - (4) (1,-1)

Key: 4

Measures CCLS: G-GPE.B

Mathematical Practice: 2, 7

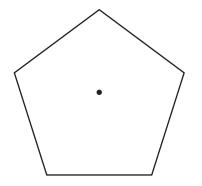
Commentary: This question measures G-GPE.B because the student needs to find the coordinates of a point dividing a directed line segment into the ratio of 3 to 2.

Rationale: Option 4 is correct. Since \overline{KL} is a directed line segment, the point dividing \overline{KL} into a ratio of 3 to 2 is $\frac{3}{5}$ the distance from point K to point L.

$$\begin{array}{cccc} x & y \\ -5 + \frac{3}{5}(5 - 5) & -4 + \frac{3}{5}(1 - 4) \\ -5 + \frac{3}{5}(10) & -4 + \frac{3}{5}(5) \\ -5 + 6 & -4 + 3 \\ 1 & -1 \end{array}$$

(1, -1)

2 A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

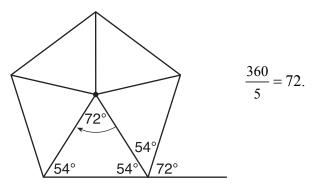
- (1) 54°
- (2) 72°
- (3) 108°
- (4) 360°

Key: 2

Measures CCLS: G-CO.A

Mathematical Practice: 2, 7

- **Commentary:** This question measures G-CO.A because it requires the student to describe a rotation that carries a regular pentagon onto itself.
- **Rationale:** Option 2 is correct. Segments drawn from the center of the regular pentagon bisect each angle of the pentagon, and create five isosceles triangles as shown in the diagram below. Since each exterior angle equals the angles formed by the segments drawn from the center of the regular pentagon, the minimum degrees necessary to carry a regular polygon onto itself are equal to the measure of an exterior angle of the regular polygon.



- 3 The equation of line *h* is 2x + y = 1. Line *m* is the image of line *h* after a dilation of scale factor 4 with respect to the origin. What is the equation of the line *m*?
 - (1) y = -2x + 1(2) y = -2x + 4(3) y = 2x + 4(4) y = 2x + 1

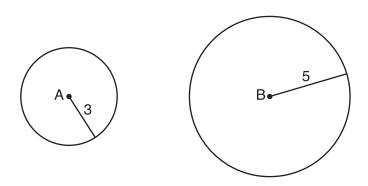
Key: 2

Meaures CCLS: G-SRT.A

Mathematical Practice: 2

- **Commentary:** This question measures G-SRT.A because a line that is dilated and does not pass through the center of dilation results in a parallel line.
- **Rationale:** Option 2 is correct. The given line h, 2x + y = 1, does not pass through the center of dilation, the origin, because the *y*-intercept is at (0,1). The slope of the dilated line, *m*, will remain the same as the slope of line h, -2. All points on line h, such as (0,1), the *y*-intercept, are dilated by a scale factor of 4; therefore, the *y*-intercept of the dilated line is (0,4) because the center of dilation is the origin, resulting in the dilated line represented by the equation y = -2x + 4.

4 As shown in the diagram below, circle *A* as a radius of 3 and circle *B* has a radius of 5.



Use transformations to explain why circles *A* and *B* are similar.

Key: See explanation in the rationale below. A correct explanation must include a written verbal statement.

Measures CCLS: G-C.A

Mathematical Practice: 3, 6

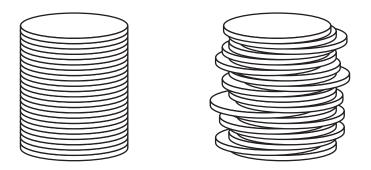
- **Commentary:** This question measures G-C.A because the student must explain why two given circles are similar.
- **Rationale:** Circle *A* can be mapped onto circle *B* by first translating circle *A* along vector \overline{AB} such that *A* maps onto *B*, and then dilating circle *A*, centered at *A*, by a scale factor of $\frac{5}{3}$. Since there exists a sequence of transformations that maps circle *A* onto circle *B*, circle *A* is similar to circle *B*.

Rubric:

- [2] A complete and correct explanation is written.
- [1] An appropriate explanation is written, but one computational error is made.

- [1] An appropriate explanation is written, but one conceptual error is made.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

5 Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder.



Use Cavelieri's principle to explain why the volumes of these two stacks of quarters are equal.

Key: See explanation in rationale below.

Measures CCLS: G-GMD.A

Mathematical Practice: 3, 6

- **Commentary:** This question measures G-GMD.A because the student is required to explain the relationship of the volumes of two objects using Cavelieri's principle.
- **Rationale:** Each quarter in both stacks has the same base area. Therefore, each corresponding cross-section of the stacks will have the same area. Since the two stacks of quarters have the same height of 23 quarters, the two volumes must be the same.

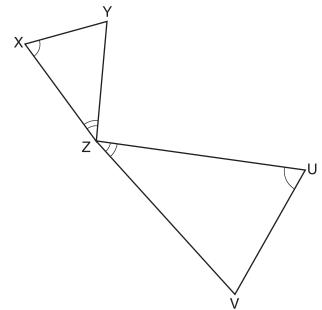
Rubric:

- [2] A complete and correct explanation is written.
- [1] An appropriate explanation is written, but one conceptual error is made.

or

- [1] An incomplete or partially correct explanation is written.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

6 In the diagram below, triangles *XYZ* and *UVZ* are drawn such that $\angle X \cong \angle U$ and $\angle XZY \cong \angle UZV$.



Describe a sequence of similarity transformations that shows $\triangle XYZ$ is similar to $\triangle UVZ$.

Key: See the description in the rationale below.

Measures CCLS: G-SRT.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.A because students must describe a sequence of similarity transformations to show two triangles are similar when they have two pairs of corresponding angles congruent.

Rationale: Triangle X'Y'Z' is the image of $\triangle XYZ$ after a rotation about point Z such that \overline{ZX} coincides with \overline{ZU} . Since rotations preserve angle measure, \overline{ZY} coincides with \overline{ZV} , and corresponding angles X and Y, after the rotation, remain congruent, so $\overline{XY} \parallel \overline{UV}$.

Then, dilate $\triangle X'Y'Z'$ by a scale factor of $\frac{ZU}{ZX}$ with its center at point *Z*. Since dilations preserve parallelism, \overline{XY} maps onto \overline{UV} . Therefore, $\triangle XYZ \sim \triangle UVZ$.

Rubric:

- [2] A correct description of sequence of similarity transformations is written.
- [1] One conceptual error is made, but an appropriate sequence of similarity transformations is written.

or

- [1] An incomplete or partially correct sequence of similarity transformations is written.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

7 Explain why cos(x) = sin(90 - x) for x such that 0 < x < 90.

Key: See explanation in the rationale below. A correct explanation must include a written verbal statement.

Measures CCLS: G-SRT.C

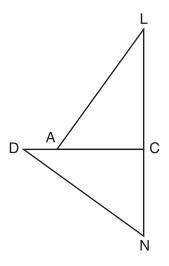
Mathematical Practice: 3, 6

- **Commentary:** This question measures G-SRT.C because the student is required to explain why the sine and cosine of complementary angles are equal.
- Rationale: The acute angles in a right triangle are always complementary. The sine of any acute angle is equal to the cosine of its complement.

Rubric:

- A correct and complete explanation is written. [2]
- [1] One conceptual error is made, but an appropriate explanation is written.
- [1] An incomplete or partially correct explanation is written.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

8 In the diagram of $\triangle LAC$ and $\triangle DNC$ below, $\overline{LA} \cong \overline{DN}$, $\overline{CA} \cong \overline{CN}$, and $\overline{DAC} \perp \overline{LCN}$.



a) Prove that $\triangle LAC \cong \triangle DNC$.

b) Describe a sequence of rigid motions that will map $\triangle LAC$ onto $\triangle DNC$.

Key: See rationale below. A sequence of one transformation is acceptable.

Measures CCLS: G-SRT.B, G-CO.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.B and G-CO.A because students are required to prove two triangles are congruent and demonstrate congruence using rigid motion.

Rationale:

Statement	Reasons
1. $\overline{LA} \cong \overline{DN}, \overline{CA} \cong \overline{CN}, \text{ and } \overline{DC} \perp \overline{LN}$	1. Given
2. $\angle LCA$ and $\angle DCN$ are right angles	2. Definition of perpendicular lines
3. $\triangle LAC$ and $\triangle DNC$ are right triangles	3. Definition of a right triangle
4. $\triangle LAC \cong \triangle DNC$	4. H.L. Theorem

Triangle *LAC* will map onto $\triangle DNC$ after rotating $\triangle LAC$ counterclockwise 90 degrees about point *C* such that point *L* maps onto point *D*.

Rubric:

Part a)

- [2] A complete and correct proof that includes a conclusion is written.
- [1] Only one correct statement and reason are written.

or

- [1] One conceptual error is made.
- [0] The "given" and/or the "prove" statements are written, but no further correct relevant statements are written.

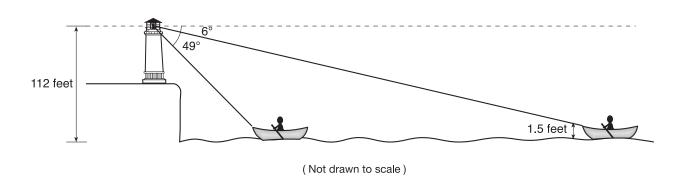
or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Part b)

- [2] A complete and correct description mapping $\triangle LAC$ onto $\triangle DNC$ is written.
- [1] An appropriate description is written, but one conceptual error is made.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

9 As shown below, a canoe is approaching a lighthouse on the coastline of a lake. The front of the canoe is 1.5 feet above the water and an observer in the lighthouse is 112 feet above the water.



At 5:00, the observer in the lighthouse measured the angle of depression to the front of the canoe to be 6°. Five minutes later, the observer measured and saw the angle of depression to the front of the canoe had increased by 49° . Determine and state, to the *nearest foot per minute*, the average speed at which the canoe traveled toward the lighthouse.

Key: 195

Measures CCLS: G-SRT.C

Mathematical Practice: 1, 4

Commentary: This question measures G-SRT.C because students need to use modeling and trigonometric ratios to find the average speed.

Rationale: *x* represents the distance between the lighthouse and the canoe at 5:00. *y* represents the distance between the lighthouse and the canoe at 5:05.

$\tan 6 = \frac{112 - 1.5}{x}$	$\tan(6+49) = \frac{112 - 1.5}{y}$	
$x = \frac{110.5}{\tan 6}$	$y = \frac{110.5}{\tan 55}$	
<i>x</i> = 1051.337272	<i>y</i> = 77.37293297	
Average speed = $\frac{973.964339 \text{ ft}}{5 \text{ min}} = 194.7928678$		

Rubric:

- [4] 195, and correct work is shown.
- [3] Appropriate work is shown, but one computational or rounding error is made.

or

- [3] Appropriate work is shown to find the distance traveled, but no further correct work is shown.
- [2] Appropriate work is shown, but two computational or rounding errors are made.
- [2] Appropriate work is shown, but one conceptual error is made.

or

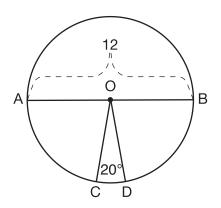
- [2] Appropriate work is shown to find the distance from the lighthouse at 5:00 and at 5:05, but no further correct work is shown.
- [1] Appropriate work is shown, but one computational or rounding error and one conceptual error are made.

[1] Appropriate work is shown to find the distance from the lighthouse at either 5:00 or at 5:05, but no further correct work is shown.

or

- [1] 195, but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

10 In the diagram below of circle *O*, diameter \overline{AB} and radii \overline{OC} and \overline{OD} are drawn. The length of \overline{AB} is 12 and the measure of $\angle COD$ is 20 degrees.



If $\widehat{AC} \cong \widehat{BD}$, find the area of sector *BOD* in terms of π .

Key: 8π

Measures CCLS: G-C.B

Mathematical Practice: 2

Commentary: This question measures G-C.B because students are required to find the area of a sector.

Rationale: A_o represents the area of circle O and A_s represents the area of sector BOD.

$$A_{o} = \pi r^{2} \qquad m \angle BOD = \frac{180 - 20}{2} \qquad \frac{A_{s}}{36\pi} = \frac{80}{360}$$
$$A_{o} = \pi (6)^{2} \qquad m \angle BOD = 80 \qquad 360(A_{s}) = 2880\pi$$
$$A_{o} = 36\pi \qquad A_{s} = 8\pi$$

Rubric:

- [4] 8π , and correct work is shown.
- [3] Appropriate work is shown, but one computational error is made.

or

- [3] Appropriate work is shown, but the area of the sector is written as an appropriate decimal.
- [2] Appropriate work is shown, but two computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or [2] Appropriate work is shown to find 36π, the area of the circle, and 80, the measure of angle *BOD*, but no further correct work is shown.

[1] Appropriate work is shown, but one computational error and one conceptual error are made.

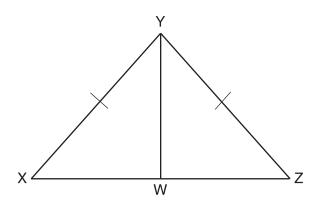
or

or

- [1] Appropriate work is shown to find either 36π , the area of the circle, or 80, the measure of angle *BOD*, but no further correct work is shown.
- [1] 8π , but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

11 Given: $\triangle XYZ$, $\overline{XY} \cong \overline{ZY}$, and \overline{YW} bisects $\angle XYZ$

Prove that $\angle YWZ$ is a right angle.



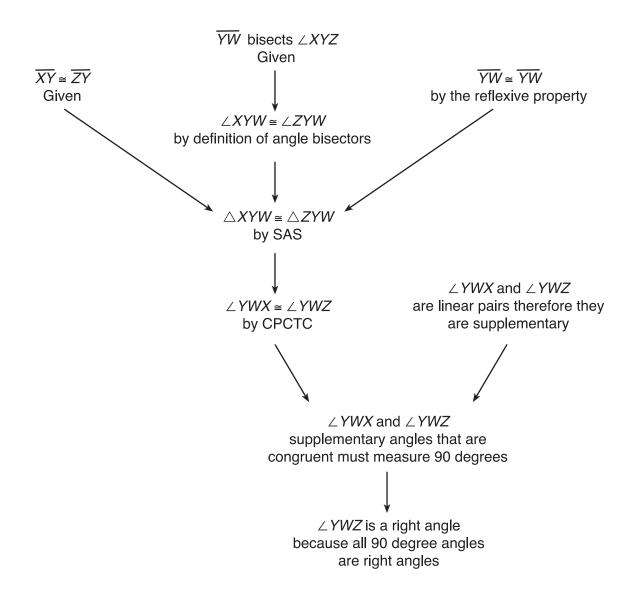
Key: See proof in the rationale below.

Measures CCLS: G-CO.C

Mathematical Practice: 3, 6

Commentary: This question measures G-CO.C because students are required to prove that the altitude of an isosceles triangle forms right angles.

Rationale: Multiple methods of proof are acceptable.



Statements	Reasons
1. $\triangle XYZ, \overline{XY} \cong \overline{ZY}, \overline{YW}$ bisects $\angle XYZ$	1. Given
2. $\triangle XYZ$ is isosceles	2. Definition of isosceles triangle
3. \overline{YW} is an altitude of $\triangle XYZ$	3. The angle bisector of the vertex of an isosceles triangle is also the altitude of that triangle.
4. $\overline{YW} \perp \overline{XZ}$	4. Definition of altitude
5. $\angle YWZ$ is a right angle	5. Definition of perpendicular lines

Rubric:

- [4] A complete and correct proof that includes a concluding statement is written.
- [3] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect, or the concluding statement is missing.
- [2] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.

or

- [2] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
- [1] Only one correct relevant statement and reason are written.
- [0] The "given" and/or the "prove" statements are written, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

12 Trees that are cut down and stripped of their branches for timber are approximately cylindrical. A timber company specializes in a certain type of tree that has a typical diameter of 50 cm and a typical height of about 10 meters. The density of the wood is 380 kilograms per cubic meter, and the wood can be sold by mass at a rate of \$4.75 per kilogram. Determine and state the minimum number of whole trees that must be sold to raise at least \$50,000.

Key: 15

Measures CCLS: G-MG.A

Mathematical Practice: 1, 4

Commentary: This question measures G-MG.A because a cylinder is used to model a tree trunk to solve the problem. This problem requires students to navigate multiple steps and develop an appropriate model.

Rationale:

Volume of one tree	<u>Weight of one tree</u> (x)	Whole trees needed (n)
$V = \pi r^2 h$	$\frac{380 \text{ K}}{2} = \frac{x}{100000000000000000000000000000000000$	n = 50,000
	$\frac{1}{1} \text{ m}^3 = \frac{1}{0.625 \pi}$	$n = \frac{1}{(4.75)(746.1282552)}$
$V = \pi (0.25)^2 (10)$	x = 746.1282552 K	<i>n</i> = 14.10791739
$V = 0.625\pi$		

15 whole trees

Rubric:

- [4] 15, and correct work is shown.
- [3] Appropriate work is shown, but one computational error is made.

or

- [3] Appropriate work is shown, but 15 is not identified as the answer.
- [3] Appropriate work is shown to find the volume and weight of one tree and amount of money for the sale of one tree. No further correct work is shown.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

- or
- [2] Appropriate work is shown, but one conceptual error is made.

or

- [2] Appropriate work is shown to find the volume and weight of one tree, but no further correct work is shown.
- [1] Appropriate work is shown, but one conceptual and one computational or rounding error are made.

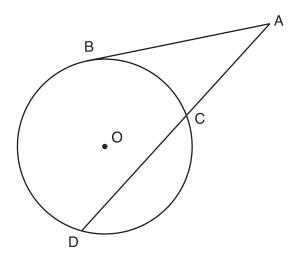
or

[1] Appropriate work is shown to find the volume of one tree, but no further correct work is shown.

or

- [1] 15, but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

13 In the diagram below, secant \overline{ACD} and tangent \overline{AB} are drawn from external point A to circle O.



Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. $(AC \cdot AD = AB^2)$

Key: See proof in the rationale below.

Measures CCLS: G-SRT.B, G-C.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.B because the student is required to use similarity criteria to prove relationships in a geometric figure. It also aligns to G-C.A because the student would use angles formed by chords to prove the triangle are similar.

Rationale:

Statements	Reasons
1. Circle O, Secant \overline{ACD} , Tangent \overline{AB}	1. Given
2. Chords \overline{BC} and \overline{BD} are drawn	2. Auxiliary lines
3. $\angle A \cong \angle A, \widehat{BC} \cong \widehat{BC}$	3. Reflexive property
4. m $\angle BDC = \frac{1}{2} m \widehat{BC}$	4. The measure of an inscribed angle is half the measure of the intercepted arc.
5. m $\angle CBA = \frac{1}{2} m \widehat{BC}$	5. The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.
6. $\angle BDC \cong \angle CBA$	6. Angles equal to half of the same arc are congruent.
7. $\triangle ABC \sim \triangle ADB$	7. AA
8. $\frac{AB}{AC} = \frac{AD}{AB}$	 Corresponding sides of similar triangles are proportional.
9. $AC \bullet AD = AB^2$	9. In a proportion, the product of the means equals the product of the extremes.

Rubric:

- [6] A complete and correct proof that includes a concluding statement is written.
- [5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect.

[4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.

or

- [4] $\triangle ABC \sim \triangle ADB$, but no further correct work is shown.
- [3] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
- [2] A proof is written that demonstrates a method of proof, but one conceptual error is made, and one statement and/or reason is missing or is incorrect.

or

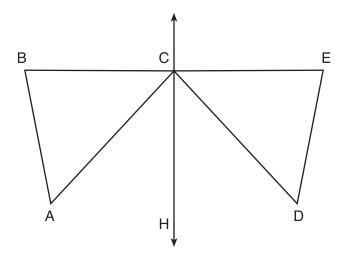
- [2] Some correct relevant statements about the proof are made, but three or four statements or reasons are missing or are incorrect.
- [1] Only one correct relevant statement and reason are written.
- [0] The "given" and/or the "prove" statements are rewritten in the style of a formal proof, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

14 Given: *D* is the image of *A* after a reflection over \overrightarrow{CH} . \overrightarrow{CH} is the perpendicular bisector of \overrightarrow{BCE} $\triangle ABC$ and $\triangle DEC$ are drawn

Prove: $\triangle ABC \cong \triangle DEC$



Key: See proof in the rationale below.

Measures CCLS: G-CO.B

Mathematical Practice: 3, 6

Commentary: This question measures G-CO.B because the student is required to prove that two triangles are congruent using the definition of congruence in terms of rigid motion.

Rationale:

It is given that point *D* is the image of point *A* after a reflection in line *CH*.

It is given that \overrightarrow{CH} is the perpendicular bisector of \overrightarrow{BE} at point C. Since a bisector divides a segment into two congruent segments at its midpoint, $\overrightarrow{BC} \cong \overrightarrow{EC}$. Point E is the image of point B after a reflection over the line CH, since points B and E are equidistant from point C and it is given that \overrightarrow{CH} is perpendicular to \overrightarrow{BE} .

Point C is on \overrightarrow{CH} therefore, point C maps to itself after the reflection over \overrightarrow{CH} .

Since all three vertices of triangle *ABC* map to all three vertices of triangle *DEC* under the same line reflection, then $\triangle ABC \cong \triangle DEC$ because a line reflection is a rigid motion and triangles are congruent when one can be mapped onto the other using a sequence of rigid motions.

Rubric:

- [6] A complete and correct proof that includes a concluding statement is written.
- [5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one supporting statement and/or reason is missing or is incorrect.
- [4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two supporting statements and/or reasons are missing or are incorrect.
- [3] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
- [2] A proof is written that demonstrates a method of proof, but one conceptual error is made, and one supporting statement and/or reason is missing or is incorrect.
- [1] Only one correct relevant statement and reason are written.

[0] The "given" and/or the "prove" statements are rewritten, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.