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Our Students. Their Moment.

Regents Examination in Algebra II (Common Core)

Sample Questions Fall 2015

New York State Common Core Sample Questions: Regents Examination in Algebra II (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Algebra II (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Algebra II (Common Core) we have provided seventeen questions. These questions include multiple-choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Regents Examination in Algebra II measuring CCLS beginning in June 2016. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standards are assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.
- Notice the application of mathematical ways of thinking to real-world issues and challenges.

- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standards will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, distractors will be based on plausible missteps.

Constructed Response Questions

Math constructed response questions are similar to past questions, asking students to show their work in completing one or more tasks or solving more extensive problems. Constructed response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings follow each question to provide information for teacher use to help interpret the question, understand measurement with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice questions, the correct option.

Measures CCLS: This question measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.

Mathematical Practices: If applicable, this is a list of mathematical practices associated with the question.

Commentary: This is an explanation of how the question measures the knowledge, skills, and proficiencies characterized by the identified cluster(s).

Rationale: For multiple-choice questions, this section provides the correct option and demonstrates one method for arriving at that response. For constructed response questions, one or more possible approaches to solving the question are shown, followed by the scoring rubric that is specific to the question. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

1 What is the solution set of the equation $\dfrac{3x + 25}{x + 7} - 5 = \dfrac{3}{x}$?

(1) $\left\{\frac{3}{2}, 7\right\}$

(2) $\left\{\frac{7}{2}, -3\right\}$

(3) $\left\{-\frac{3}{2}, 7\right\}$

(4) $\left\{-\frac{7}{2}, -3\right\}$

Key: 4

Measures CCLS Cluster: A-REI.A

Mathematical Practice: 2, 7

Commentary: This question measures A-REI.A because students must solve a rational equation.

Rationale: Option 4 is correct.

$$x(x+7)\left[\frac{3x+25}{x+7}-5=\frac{3}{x}\right];\, x\neq -7,\, x\neq 0$$

$$x(3x+25)-5x(x+7)=3(x+7)$$

$$3x^2+25x-5x^2-35x=3x+21$$

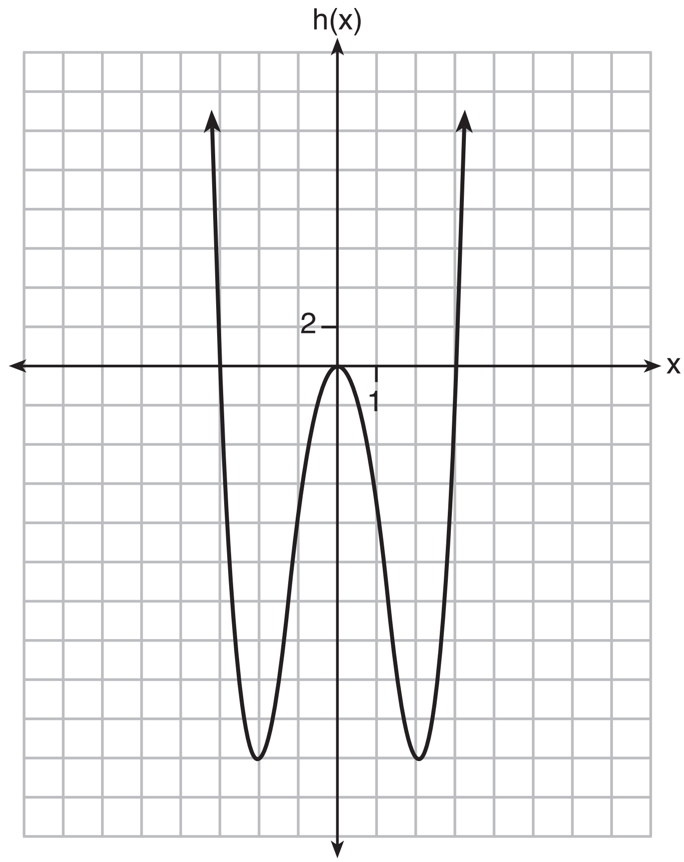
$$2x^2+13x+21=0$$

$$(2x+7)(x+3)=0$$

$2x+7=0$	$x+3=0$
$x=-\frac{7}{2}$	$x=-3$

2 Functions f, g, and h are given below.

$$f(x) = \sin(2x)$$
$$g(x) = f(x) + 1$$



Which statement is true about functions f, g, and h?

(1) $f(x)$ and $g(x)$ are odd, $h(x)$ is even.

(2) $f(x)$ and $g(x)$ are even, $h(x)$ is odd.

(3) $f(x)$ is odd, $g(x)$ is neither, $h(x)$ is even.

(4) $f(x)$ is even, $g(x)$ is neither, $h(x)$ is odd.

Key: 3

Measures CCLS Cluster: F-BF.B

Mathematical Practice: 2, 5

Commentary: This question measures F-BF.B because students must be able to recognize even and odd functions.

Rationale: Option 3 is correct.

$f(-x) = -f(x)$ or f is symmetric about the origin
$f(x) \rightarrow$ odd

$h(-x) = h(x)$ or h is symmetric about the y-axis
$h(x) \rightarrow$ even

For example, consider $x = 1$

$g(1) = 1.9093$
$g(-1) = .0907$
$g(-x) \neq g(x) \rightarrow$ not even
$g(-x) \neq -g(x) \rightarrow$ not odd
$g(x) \rightarrow$ neither

3 The expression $\dfrac{6x^3 + 17x^2 + 10x + 2}{2x + 3}$ equals

(1) $3x^2 + 4x - 1 + \dfrac{5}{2x + 3}$

(2) $6x^2 + 8x - 2 + \dfrac{5}{2x + 3}$

(3) $6x^2 - x + 13 - \dfrac{37}{2x + 3}$

(4) $3x^2 + 13x + \dfrac{49}{2} + \dfrac{151}{2x + 3}$

Key: 1

Measures CCLS Cluster: A-APR.D

Mathematical Practice: 8

Commentary: This question measures A-APR.D because students must rewrite a simple rational expression in quotient-remainder form.

Rationale: Option 1 is correct.

$$
\begin{array}{r}
3x^2+4x-1 \\
2x+3\overline{)6x^3+17x^2+10x+2} \\
\underline{6x^3+9x^2} \qquad\qquad \\
8x^2+10x \qquad \\
\underline{8x^2+12x} \qquad \\
-2x+2 \\
\underline{-2x-3} \\
5
\end{array}
$$

$$3x^2+4x-1+\frac{5}{2x+3}$$

4 The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

(1) $-6 \pm 2i$

(2) $-6 \pm 2\sqrt{19}$

(3) $6 \pm 2i$

(4) $6 \pm 2\sqrt{19}$

Key: 3

Measures CCLS Cluster: A-REI.B

Mathematical Practice: 7

Commentary: This question measures A-REI.B because students must solve a quadratic equation with complex solutions.

Rationale: Option 3 is correct.

Method 1:

$$-\frac{1}{2}x^2 = -6x + 20$$

$$-\frac{1}{2}x^2 + 6x - 20 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 4\left(-\frac{1}{2}\right)(-20)}}{2\left(-\frac{1}{2}\right)}$$

$$x = \frac{-6 \pm \sqrt{-4}}{-1}$$

$$x = 6 \pm 2i$$

Method 2:

$$-2\left(-\frac{1}{2}x^2 = -6x + 20\right)$$

$$x^2 = 12x - 40$$

$$x^2 - 12x + 40 = 0$$

$$x^2 - 12x + 36 = -40 + 36$$

$$(x-6)^2 = -4$$

$$x - 6 = \pm 2i$$

$$x = 6 \pm 2i$$

5 What is the completely factored form of $k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$?

(1) $(k-2)(k-2)(k+3)(k+4)$

(2) $(k-2)(k-2)(k+6)(k+2)$

(3) $(k+2)(k-2)(k+3)(k+4)$

(4) $(k+2)(k-2)(k+6)(k+2)$

Key: 4

Measures CCLS Cluster: A-SSE.A

Mathematical Practice: 5, 7

Commentary: This question measures A-SSE.A because students use the structure of an expression to identify ways to rewrite it.

Rationale: Option 4 is correct.

$$k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$$

$$\left(k^4 - 4k^2\right) + \left(8k^3 - 32k\right) + \left(12k^2 - 48\right)$$

$$k^2\left(k^2 - 4\right) + 8k\left(k^2 - 4\right) + 12\left(k^2 - 4\right)$$

$$\left(k^2 - 4\right)\left(k^2 + 8k + 12\right)$$

$$(k+2)(k-2)(k+6)(k+2)$$

6 Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

(1) The period is 6.

(2) The amplitude is 3.

(3) The range is [4,10].

(4) The midline is $y = -4$.

Key: 4

Measures CCLS Cluster: F.IF.C

Mathematical Practice: 5, 7

Commentary: This question measures F-IF.C because students must determine key features of the graph of a given trigonometric function.

Rationale: Option 4 states an incorrect midline.

The midline is $y = 7$ since 7 is the average of the endpoints of the range.

7 Algebraically determine the values of x that satisfy the system of equations below.

$$y = -2x + 1$$
$$y = -2x^2 + 3x + 1$$

Key: $0, \frac{5}{2}$

Measures CCLS Cluster: A-REI.C

Mathematical Practice: 2, 7

Commentary: This question measures A-REI.C because students must be able to solve a linear-quadratic system in two variables.

Rationale: $-2x+1=-2x^2+3x+1$

$$2x^2-5x=0$$

$$x(2x-5)=0$$

$$x=0 \quad \Big| \quad 2x-5=0$$

$$x=\frac{5}{2}$$

Rubric:

[2] $0, \frac{5}{2}$ and correct algebraic work is shown.

[1] Appropriate work is shown, but one computational or factoring error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $0, \frac{5}{2}$ but a method other than algebraic is used.

or

[1] $0, \frac{5}{2}$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

8 The results of a poll of 200 students are shown in the table below:

	Preferred Music Style		
	Techno	**Rap**	**Country**
Female	54	25	27
Male	36	40	18

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

Key: See rationale below

Measures CCLS Cluster: S-CP.A

Mathematical Practice: 1, 2, 3

Commentary: This question measures S-CP.A because students must demonstrate an understanding of conditional probability and interpret independence of events.

Rationale: Based on these data, the two events do not appear to be independent. The probability that a student is female given that she prefers techno music is $\frac{54}{90} = 0.6$ while the probability that a student is female is $\frac{106}{200} = 0.53$. These probabilities are not the same. This suggests that the events are not independent.

Other music styles can be used such as

$$P(\text{Female}|\text{Rap}) = \frac{25}{65} = 0.385; P(\text{Female}|\text{Country}) = \frac{27}{45} = 0.6$$

Rubric:

[2] The events are not independent and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

[0] Not independent, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

9 For the function $f(x) = (x-3)^3 + 1$, find $f^{-1}(x)$.

Key: $(x-1)^{\frac{1}{3}}+3$

Measures CCLS Cluster: F-BF.B

Mathematical Practice: 7

Commentary: This question measures F-BF.B because students must write the inverse of a given function.

Rationale:

$$x=(y-3)^3+1$$
$$x-1=(y-3)^3$$
$$(x-1)^{\frac{1}{3}}=\left[(y-3)^3\right]^{\frac{1}{3}}$$
$$(x-1)^{\frac{1}{3}}=y-3$$
$$(x-1)^{\frac{1}{3}}+3=y$$
$$f^{-1}(x)=(x-1)^{\frac{1}{3}}+3$$

Rubric:

[2] $(x-1)^{\frac{1}{3}}+3$ or an equivalent expression and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $x=(y-3)^3+1$ is written, but no further correct work is shown.

or

[1] $(x-1)^{\frac{1}{3}}+3$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

10 Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$$k(x) = -|0.7x| + 5$$

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

Key: –5.17, –1.13, and 1.75.

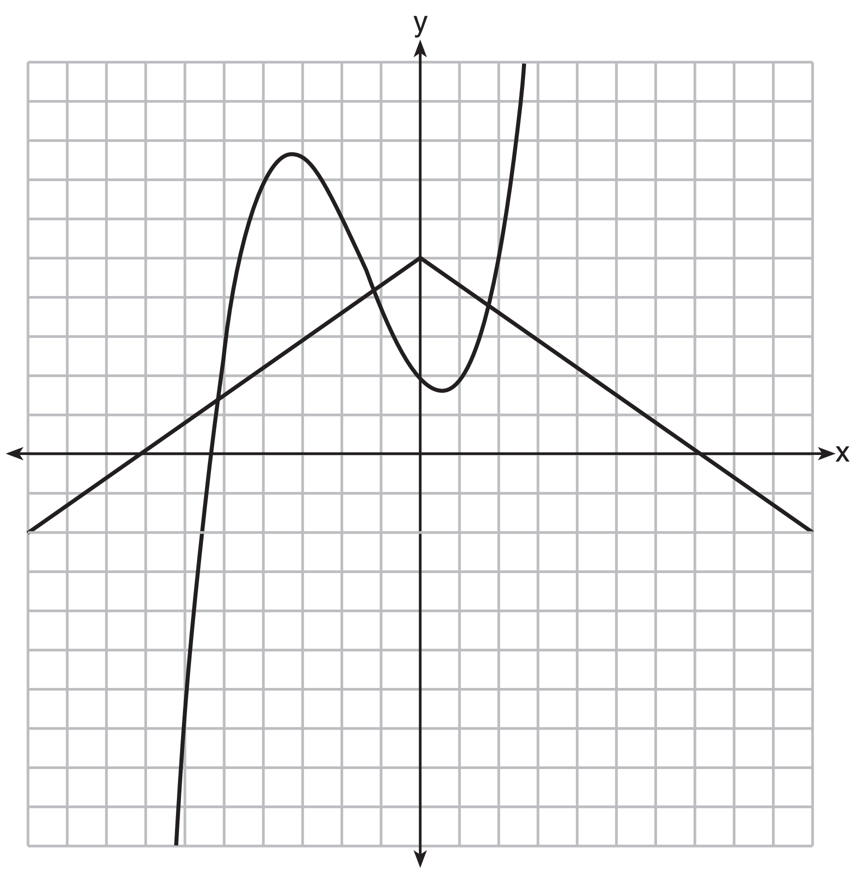
Measures CCLS Cluster: A-REI.D

Mathematical Practice: 5, 6

Commentary: This question measures A-REI.D because students are required to find the approximate solutions to $h(x) = k(x)$.

Rationale: Using technology and $y_1 = h(x)$ and $y_2 = k(x)$, the intersect function is used to determine all values of x for which $y_1 = y_2$.

On their calculator screens, students should see an image similar to the one below.



Rubric:

[2] -5.17, -1.13, and 1.75.

[1] One computational or rounding error is made.

or

[1] One conceptual error is made.

or

[1] Only two correct values are found.

or

[1] $(-5.17, 1.38)$, $(-1.13, 4.21)$, and $(1.75, 3.77)$ are written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

11 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

Key: See rationale below.

Measures CCLS Cluster: A-APR.C

Mathematical Practice: 1, 8

Commentary: This question measures A-APR.C because students must prove a polynomial identity.

Rationale: Let $x =$ the first integer
$x + 1 =$ the next integer

The difference of their squares is

$(x+1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$
$2x$ is an even integer, therefore $2x + 1$ is an odd integer.

or

$x^2 - (x+1)^2 = x^2 - (x^2 + 2x + 1) = -2x - 1$
$-2x$ is an even integer, therefore $-2x - 1$ is an odd integer.

Rubric:

[2] A correct algebraic proof is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Appropriate work is shown to find $2x + 1$ or $-2x - 1,$ but no concluding statement is written.

or

[1] An incomplete algebraic proof is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

12 Rewrite the expression $\left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6$ as a product of four linear factors.

Key: See rationale below.

Measures CCLS Cluster: A-SSE.A

Mathematical Practice: 1, 2, 7

Commentary: This question measures A-SSE.A because students produce an equivalent form of an expression.

Rationale: The problem is of the form $y^2 - 5y - 6,$ which factors to $(y-6)(y+1)$. Therefore:

$$\left(4x^2+5x\right)^2 - 5\left(4x^2+5x\right) - 6$$
$$\left(4x^2+5x-6\right)\left(4x^2+5x+1\right)$$
$$(4x-3)(x+2)(4x+1)(x+1)$$

Rubric:

[2] $(4x-3)(x+2)(4x+1)(x+1)$ and correct work is shown.

[1] Appropriate work is shown, but one computational or factoring error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $\left(4x^2+5x-6\right)\left(4x^2+5x+1\right)$ is written, but no further correct work is shown.

or

[1] $(4x-3)(x+2)(4x+1)(x+1)$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

13 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_o - T_a)e^{-kt}$$

$T_a =$ the temperature surrounding the object
$T_o =$ the initial temperature of the object
$t =$ the time in hours
$T =$ the temperature of the object after t hours
$k =$ decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after t hours.

Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

Key: $k = 0.066, T = 325 - 257e^{-0.066t}, 163$

Measures CCLS Cluster: A-CED.A

Mathematical Practice: 1, 4

Commentary: This question measures A-CED.A because students must create an exponential equation and use it to solve problems.

Rationale: $100 = 325 + (68 - 325)e^{-2k}$

$$-225 = -257e^{-2k}$$

$$k = \frac{ln\left(\frac{-225}{-257}\right)}{-2}$$

$$k \approx 0.066$$

$$T = 325 - 257e^{-0.066t}$$

At 3 pm, $t = 7$.

$$T = 325 - 257e^{-0.066(7)}$$

$$T \approx 163$$

Rubric:

[4] $k = 0.066, T = 325 - 257e^{-0.066t}$, 163, and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] Appropriate work is shown, $T = 325 - 257e^{-0.066t}$ is written, but no further correct work is shown.

or

[3] Appropriate work is shown, but the equation is written without *T* or *t*.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find $k = 0.066$, but no further correct work is shown.

or

[2] The expression $325 - 257e^{-0.066t}$ is written, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual and one computational or rounding error is made.

or

[1] 0.066, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

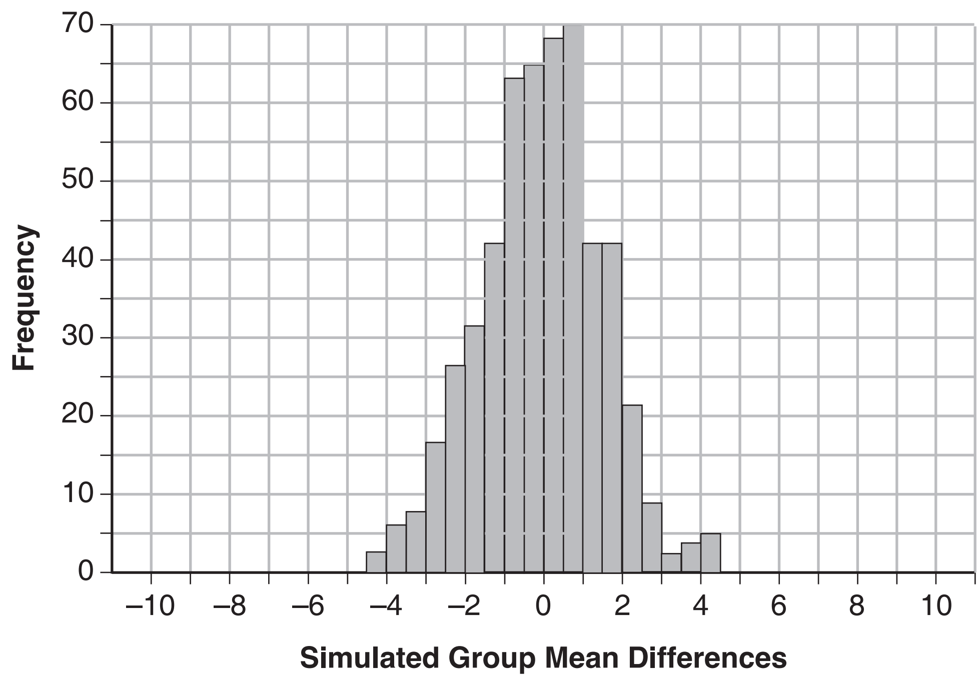
14 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year.

A summary of the two groups' final grades is shown below:

	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem.

A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

Key: See rationale below.

Measures CCLS Cluster: S-IC.B

Mathematical Practice: 1, 3, 6

Commentary: This question measures S-IC.B because students use a simulation to determine if a difference in sample means in an experiment is significant.

Rationale: The mean difference between the students' final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription.

One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students' final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

The rerandomization process always involves the following steps:

1. Shuffle all observations.
2. Divide the observations into 2 equal groups.
3. Find the mean difference between the groups.
4. Repeat steps 1 through 3 many times.

Rubric:

[4] −3.64 and a correct explanation, and yes and a correct explanation is given.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Appropriate work is shown, but the mean difference is calculated incorrectly.

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] −3.64 and a correct explanation, but no further correct work is shown.

[1] −3.64, but no work is shown.

[0] Yes, but no explanation is given.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

15 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

Key: $\frac{3}{2}, \frac{1}{3}$ and -5

Measures CCLS Cluster: A-APR.B

Mathematical Practice: 1, 7

Commentary: This question measures A-APR.B because students must apply the Remainder Theorem and then identify the zeros of a polynomial when a suitable factorization is available.

Rationale: Find b:

$$35 = 6(2)^3 + b(2)^2 - 52(2) + 15$$
$$35 = 48 + 4b - 104 + 15$$
$$35 = -41 + 46$$
$$76 = 4b$$
$$19 = b$$

or

$$0 = 6(-5)^3 + b(-5)^2 - 5$$
$$0 = -475 + 25b$$
$$475 = 25b$$
$$19 = b$$

$$z(x) = 6x^3 + 19x^2 - 52x + 15$$

$z(-5) = 0$, by the Remainder Theorem;

$$\begin{array}{r|rrrr} -5 & 6 & 19 & -52 & 15 \\ & & -30 & 55 & -15 \\ \hline & 6 & 11 & 3 & 0 \end{array}$$

$$6x^2 - 11x + 3 = 0$$

$$6x^2 - 9x - 2x + 3 = 0$$
$$3x(2x-3) - 1(2x-3) = 0$$
$$(2x-3)(3x-1) = 0 \qquad z(-5) = 0$$

$2x - 3 = 0$	$3x - 1 = 0$	$z(-5) = 0$
$x = \frac{3}{2}$	$x = \frac{1}{3}$	$x = -5$

Rubric:

[4] $\frac{3}{2}, \frac{1}{3}$, and -5, and correct algebraic work is shown.

[3] Appropriate work is shown to find $\frac{3}{2}$ and $\frac{1}{3}$**,** only.

or

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual and one computational error are made.

or

[1] Appropriate work is shown to find $b = 19$, but no further correct work is shown.

or

[1] $\frac{3}{2}, \frac{1}{3}$, and -5 but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

16 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed.

Joanne took the April version and scored in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval?

Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

Key: See rationale below.

Measures CCLS Cluster: S-ID.A

Mathematical Practice: 1, 3, 5

Commentary: This question measures S-ID.A because students must be able to use their calculators to estimate the area under the curve.

Rationale: The probability of a score being between 510 and 540 on the April exam can be found using the normal probability cumulative density function,
normcdf(510, 540, 480, 24) = 0.0994.

Use z-scores to compare the two sets of data. Joanne's scores correspond to
$z = \frac{510-480}{24} = 1.25$ to $z = \frac{540-480}{24} = 2.5$.

Calculating equivalent scores,

$1.25 = \frac{x-510}{20}$	$2.5 = \frac{x-510}{20}$
$x = 535$	$x = 560$

Maria must score in the interval 535–560.

Rubric:

[4] 0.0994 and [535,560] and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find 0.0994, but no further correct work is shown.

or

[2] Appropriate work is shown to find [535,560], but no further correct work is shown.

or

[2] 0.0994 and [535,560], but no work is shown.

[1] Appropriate work is shown, but one conceptual and one computational or rounding error are made.

or

[1] 0.0994 or [535,560], but no work is shown.

[0] A zero response if completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

17 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables.

Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the *nearest tenth*.

Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

Key: See rationale below.

Measures CCLS Cluster: F-BF.A

Mathematical Practice: 2, 4

Commentary: This question measures F-BF.A because students must write a function that describes a relationship between two quantities. This question also measures F-IF.B because students must calculate and interpret the average rate of change of a function.

Rationale: Method 1:

$A(t) = 100(0.5)^{\frac{t}{63}}$, where t = time in years and

$A(t) =$ amount of titanium-44 remaining after t years.

$$\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868$$

At $t = 40$, the estimated mass is

$100 - 40(1.041868)$

$58.32528 \approx 58.3$ g

The actual mass is

$A(40) = 100(0.5)^{\frac{40}{63}} = 64.3976$

The estimation is less than the actual.

Method 2:

$$y = 100e^{-kt}$$

$$\frac{1}{2}(100) = 100e^{-63k}$$

$$\frac{1}{2} = e^{-63k}$$

$$\ln\frac{1}{2} = -63k$$

$0.011002 = k$

$y = 100e^{-0.011002t}$, where $y =$ amount of titanium-44 remaining after t years and t = time in years.

$$\frac{y(10) - y(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868$$

At $t = 40$, the estimated mass is

$100 - 40(1.041868) \approx 58.3$ grams

The actual mass is

$$y = 100e^{-0.011002(40)} = 64.3976$$

The estimation is less than the actual.

Rubric:

[6] A correct function with defined variables is written, 58.3, actual, and a correct justification is given.

[5] Appropriate work is shown, but one computational error is made.

or

[5] Appropriate work is shown, but the function's variables are not defined.

[4] Appropriate work is shown, but two computational errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

or

[4] Appropriate work is shown to find 58.3, actual, and a correct explanation are stated, but no further correct work is shown.

[3] Appropriate work is shown, but three or more computational errors are made.

or

[3] Appropriate work is shown, but one conceptual and one computational error is made.

or

[3] A correct function and 58.3 are stated, but no further correct work is shown.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Appropriate work is shown, but one conceptual and two or more computational errors are made.

or

[2] A correct function is written with defined variables, but no further correct work is shown.

[1] Appropriate work is shown, but two conceptual and one computational errors are made.

or

[1] 58.3, but no work is shown.

[0] Actual, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.