

# New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

<b>GRADE: PK</b>	<b>DOMAIN: Counting and Cardinality</b>
<p><b>CLUSTER: Count to tell the number of objects.</b></p> <p>Students develop efficiency in saying the count sequence in standard order when counting objects and start making connections to number names and one-to-one correspondence with quantities to ten. Exposure to “how many” questions reinforces cardinality and the understanding that the last number name said tells the number of objects counted (See the <a href="#">Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking</a>. Common Core Standards Writing Team. <i>Progressions for the Common Core State Standards in Mathematics</i> (May 29, 2011 draft). Tucson, AZ: Institute for Mathematics and Education, University of Arizona).</p>	
<p><b>Grade Level Standard:</b></p> <p><b>NY-PK.CC.3</b> Understand the relationship between numbers and quantities to 10; connect counting to cardinality.</p> <p><b>NY-PK.CC.3a</b> When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (1:1 correspondence).</p> <p><b>NY-PK.CC.3b</b> Explore and develop the concept that the last number name said tells the number of objects counted (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.</p>	

<b>PERFORMANCE/KNOWLEDGE TARGETS (measurable and observable)</b>				
<ul style="list-style-type: none"> <li>• Count concrete sets of objects up to 10 accurately using one-to-one correspondence.</li> <li>• Identify the last number in a set as a quantity up to 10.</li> </ul>				
<b>ASPECTS OF RIGOR</b>				
<table style="width: 100%; border: none;"> <tr> <td style="width: 33%; border: none;">Procedural</td> <td style="width: 33%; border: none;">Conceptual</td> <td style="width: 33%; border: none;">Application</td> </tr> </table>		Procedural	Conceptual	Application
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<b>MATHEMATICAL PRACTICES</b>	<ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>			
<b>FOUNDATIONAL UNDERSTANDING</b>				

**The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.**

This standard should first be addressed using numbers 1-5 with teachers building to the numbers 1-10 later in the year. Students should encounter counting experiences throughout their day, not just in isolated directed math activities.

The progression of skills involves students saying number words in correspondence with the objects counted (one-to-one correspondence), connecting the last number word said when counting to tell how many objects have been counted (cardinality), to visually recognizing the number of items in a small set without counting (subitizing).

**Example 1:** Instructional Tasks taken from *A Learning Trajectory for Recognition of Number and Subitizing (Learning and Teaching Early Math: The Learning Trajectories Approach* by Douglas H. Clements, Julie Sarama, 2014 Taylor and Francis, pgs. 17-19).

**Maker of Small Collections**

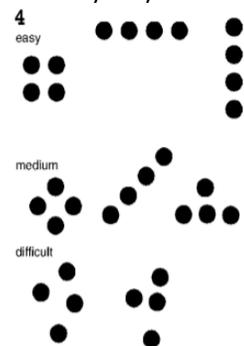
**Make Groups:** Lay out a small collection, say 3 blocks. Hide them. Ask students to make a group that has the same number of blocks as your group has. After they have finished, hide their version as well. Then uncover both groups, yelling “Ta-da!” Compare the two groups and ask students if they are the same. Name the number (e.g., “Both have three!”).

**Perceptual Subitizer to 4**

Instantly recognizes collections up to 4 briefly shown and verbally names the number of items.

When shown 4 objects briefly, student says “four”.

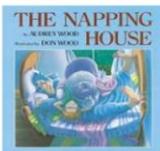
**Snapshots:** Play “Snapshots” with collections of 1 to 4 objects, arranged in a line or other simple arrangement, asking students to respond verbally with the number name. Start with the smaller numbers and easier arrangements, moving to those of moderate difficulty only as students are confident.



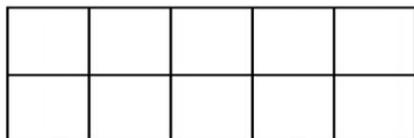
**Example 2: Illustrative Mathematics Task:** [The Napping House](#) (content licensed under [CC BY-NC-SA 4.0](#))

**Materials**

- The Napping House by Audrey Wood



- One ten-frame for each child (see PDF for black line master)



- 6-10 counters per child

The teacher reads *The Napping House* to the class, stopping each time a person or animal gets into the bed, so the students can add a counter to the ten-frame. After each page, stop to ask the students how many are sleeping in the bed after each counter is added to the ten-frame. For example, at the beginning there should be 1 counter for granny. When the child gets in the bed, there should be 2 counters. Have the students tell how many people there are in the bed now. Do this after each counter is added to the ten-frame. There should be 6 counters on the ten-frame once the wakeful flea is added. Once the flea bites the mouse, the students should begin taking the counters off the ten-frame to represent how many people/animals are still in the bed. For example, once the flea bites the mouse, there are only 4 people/animals left in the bed. By the end of the story, there should be no counters on the ten-frame.

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**Example 3: Number books**

The following is taken from [EngageNY Pre-K Module 1](#), Lesson 25.

Number Books: Allows students opportunities to use number words and connect to their everyday interactions.

Part 1: Concept Introduction

Materials: (T) 3 trays, easel (if possible, for demonstration), piece of chart paper, sample number book, manipulatives (e.g., straws, puffballs), pre-cut magazine pictures and shapes (or cutouts 1–5, Template), 1-inch strips of construction paper (to create 5-groups), various art supplies (e.g., scissors, glue, glue sticks, dot painters, crayons, stickers)

Gather materials on the carpet. On the first tray, place various manipulatives. On the second tray, place empty 5-group strips and pre-cut magazine pictures, shapes, or cutouts 1–5 template. On the third tray, place various art supplies, as noted in the materials.

1. Tell students, “We are going to make a number book to celebrate how much we know about 1, 2, 3, 4, and 5.” Show students a sample book, and briefly explain materials.
2. Turn to the 1 page, displaying it on the easel. Describe the page using self-talk, e.g., “Look, I see the number 1 at the top. I drew 1 red circle to match the number. Then, I glued a picture of 1 bumblebee, (counting) 1.”
3. Ask students, “What can I add to my page to show 1?” Add suggestions, such as gluing 1 puffball.
4. Tell students, “Now, let’s make a giant 2 page together.” (Use chart paper.) Encourage students to think of different ways to show the number 2.
5. Invite students to come forward to share ideas, adding them to the class page (e.g., “I can trace my two fingers.”)



**NOTES ON  
MULTIPLE MEANS  
OF REPRESENTATION:**

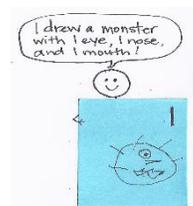
Some students may benefit from continuing to work at the concrete level. Provide children with teddy bear counters, linking cubes, etc., that they can arrange and count before drawing them on their number page. Then comment, e.g., “Pia, I see you drew two teddy bears to match your counters.”

Part 2: Practice

Materials: (S) Per student: 5 sheets of 8.5" × 11" colored construction paper with numbers 1–5 written in the upper right-hand corner; per table: caddy with art supplies

Send students to tables to make their own books. Have supplies ready on each table. Start all students with the number 1.

1. As students work, circulate and describe what they are doing using parallel talk, e.g., “Andrew is drawing 1 car on his paper to match the car he plays with in our classroom,” or “Ian is showing 1 on his paper strip with the green dot painters.”
2. Circulate and ask how the number is shown. Record the dictation on the page. For example, write, “I drew a monster with 1 nose, 1 mouth, and 1 eye.”
3. Children will work at different rates. As each child finishes one page, write his or her name on the back of the page, and set it aside. Then provide the next numeral page for the child to continue working.
4. Call students to the carpet with the page they are currently working on, in preparation for the Student Debrief.



The utilization of questions such as *what are some ways that you can show or how did you show the number* \_\_\_\_\_, may be used to help students express ideas, make connections, and use new vocabulary.

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**Example 4: Teaching Strategies for Specific Counting Errors**

The following is taken from *A Learning Trajectory for Recognition of Number and Subitizing (Learning and Teaching Early Math: The Learning Trajectories Approach* by Douglas H. Clements, Julie Sarama, 2014 Taylor and Francis, pg. 32).

Box 3.1 Teaching Strategies for Specific Counting Errors.

- *One-to-one errors (includes keeping-track-of-what's-been counted errors)*: Emphasize the importance of accuracy and encourage the students to count slowly and carefully to “count each item exactly once.” When relevant, explain a keeping-track strategy. If moving objects is possible and desirable in the activity, suggest the strategy of moving items to a different pile or location. Otherwise, explain making a verbal plan, such as “Go from top to bottom. Start from the top and count every one.” – then carry out the plan together.  
If the students return and re-count objects (e.g., in a circular arrangement): Stop and tell them they counted that item already. Suggest that they start on one they can remember (e.g., one at the “top” or “the corner” or “the blue one” – whatever makes sense in the activity; if there is not identifier, highlight an item in some way).
- *Cardinality (the “how many?” rule) errors*: Ask the students to re-count. Demonstrate the cardinality rule on the collection. That is, count the collection, pointing to each item in turn, then gesture at them all, saying, “Five in all!” Demonstrate the cardinality rule on a smaller (subitizable) collection in an easily recognizable arrangement (see the Snapshots activity).
- *Cardinality errors (production tasks – knowing when to stop)*: Remind the students of the goal number and ask to re-count. Count the collection, say that is not the requested number, and ask the students to try again. If there were too few, count the existing collection quickly and ask the students to put on another object, saying when that has been done, “And that makes -.” Allow the students to add more than one as long as this does not exceed a total. If there were too many, ask the students to remove one or more items, and then re-count. So, count the existing collection quickly and say, “There are too many. Take some away so that we have -.” Demonstrate.
- *Guided counting sequence (when the above are not sufficient)*: Ask the students to count out loud as they point to each object. Suggest a keeping-track strategy if necessary. If there are still errors after this remediation, say, “Count with me,” and name the keeping-track strategy you will model. Have the students point to each item and say the correct counting word, thus walking the students through the counting. Demonstrate the cardinality rule- repeat the last counting number, gesture in a circular motion to all the items, and say, ‘That’s how many there are in all.’ For “Counter to” activities, emphasize the goal number, saying, “[Five!] That’s what we wanted!”