New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 7	DOMAIN: Ratio and Proportional Reasoning

CLUSTER: Analyze proportional relationships and use them to solve real-world and mathematical problems.
Students build upon their reasoning about ratios, rates, and unit rates to formally define proportional relationships and the constant of proportionality. Reasoning is extended about ratios and proportional relationships by computing unit rates for ratios and rates specified by rational numbers. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions. Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation.

Grade Level Standard:

NY-7.RP.1 Compute unit rates associated with ratios of fractions.
Note: Problems may include ratios of lengths, areas, and other quantities measured in like or different units, including across measurement systems.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Divide two fractions.
- Compute unit rates that involve ratios of fractions in same or different units.
- Given a context, compute/identify and explain the unit rate.
- Solve unit rate problems that have fractional quantities.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison. **NY-5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers **NY-5.NF.7** Apply and extend previous understandings of division to divide fractions by whole numbers and whole numbers by unit fractions **NY-6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **NY-6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$ and use rate language in the context of a ratio relationship. **NY-6.RP.3b** Solve unit rate problems. **NY-6.NS.1** Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions.

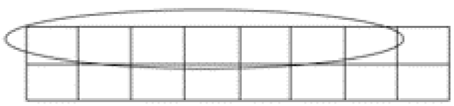
The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students will need to be comfortable with interpreting and computing the quotients of fractions. Their strategies may include the standard algorithm, as well as the use of visual fraction models which will support upcoming work with finding unit rates given a ratio that is comparing two fractional quantities. Examples that involve mixed fractions can be found in EngageNY Grade 7 Module 1, lesson 11, pg. 106-107 (scaffolding boxes).

$\frac{1}{2} \div \frac{1}{4} = 2$ (How many groups of $\frac{1}{4}$ are in $\frac{1}{2}$?)

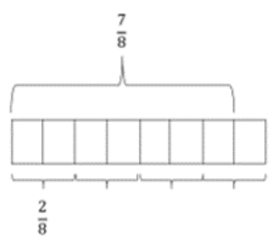
$\frac{2}{4} \div \frac{1}{4} = 2$

$\frac{7}{8} \div 2 = \frac{7}{16}$ ($\frac{1}{2}$ of $\frac{7}{8}$ is $\frac{7}{16}$)



$\frac{7}{8} \div 8 = \frac{7}{64}$ ($\frac{1}{8}$ of $\frac{7}{8}$ is $\frac{7}{64}$)

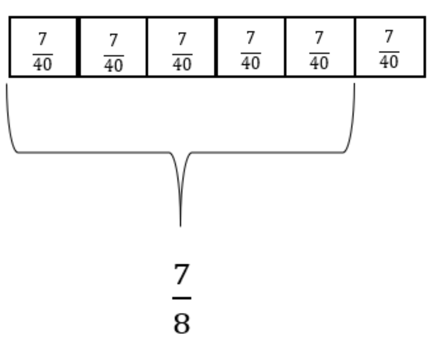
$\frac{7}{8} \div 50 = \frac{7}{400}$ ($\frac{1}{50}$ of $\frac{7}{8}$ is $\frac{7}{400}$)



$\frac{7}{8} \div \frac{1}{4} = \frac{7}{2}$ or $\frac{7}{8} \div \frac{2}{8} = 3\frac{1}{2}$

$\frac{7}{8} \div \frac{1}{6} = \frac{21}{4}$ or $\frac{21}{24} \div \frac{4}{24} = 21 \div 4 = \frac{21}{4} = 5\frac{1}{4}$

$\frac{7}{8} \div \frac{5}{6} = \frac{21}{20}$ or $\frac{21}{24} \div \frac{20}{24} = 21 \div 20 = \frac{21}{20} = 1\frac{1}{20}$ or $\frac{6}{5}$ of $\frac{7}{8}$ is 1 of $\frac{7}{8}$ plus $\frac{1}{5}$ of $\frac{7}{8}$ which is $\frac{7}{8} + \frac{7}{40} = \frac{42}{40}$ or



$$\frac{7}{8} \div 5 = \frac{7}{40}$$

$$\frac{7}{40} \times 6 = \frac{42}{40}$$

$\left(\frac{5}{6}\ of\ ? = \frac{7}{8}\right)$

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Students should see quotients of fractions written as complex fractions, numbers written in fraction form whose numerator and/or denominator is itself a fraction. Knowing that the fraction bar means "division", students should see that all complex fractions can be written as division problems.

$\frac{\frac{7}{8}}{2} = \frac{7}{8} \div 2$

When considering $7 \div 8 \div 2$, students should compare $(7 \div 8) \div 2$ and $7 \div (8 \div 2)$, and explain/show why these two expressions are not equivalent.

Students can be presented with scenarios that would involve utilizing a complex fraction, as well as tasks that involve them creating their own scenarios/word problems, such as the following (additional examples can be found in lessons 5 and 6 of [EngageNY Grade 6 Module 2]):

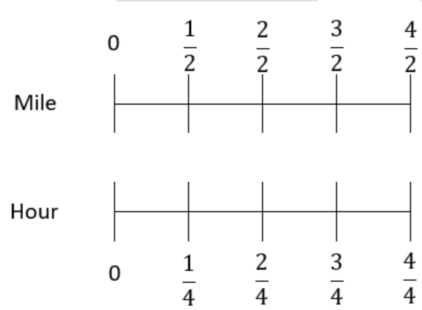
Example 1: Using the same dividend and divisor (two fractions), work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as cups, yards, or miles if you prefer.

- Tiffany uses $\frac{1}{2}$ cup of glycerin each time she makes a batch of soap bubble mixture. How many batches can she make if she has $\frac{3}{4}$ cup left in her glycerin bottle?
- Each jug holds $\frac{3}{4}$ gallon. Each bottle holds $\frac{1}{2}$ gallon. One jug has the same capcity as how many bottles?

A ratio relationship between two types of quantities, such as 5 miles per 2 hours, can be described as a rate. The unit for the rate is miles/hour, read miles per hour. When the rate describes how many units of the first type of quantity corresponds to one unit of the second type of quantity (i.e., the quantity 2.5 miles/1 hour), we can determine the unit rate. The numerical part of the rate is called the unit rate and is simply the value of the ratio, in this case 2.5. This means that in 1 hour the car travels 2.5 miles.

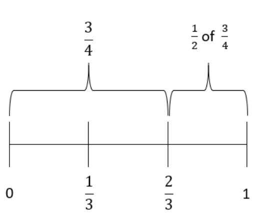
Examples involving unit rate scenarios follow:

Example 2: A person jogs $\frac{1}{2}$ mile in $\frac{1}{4}$ hour. If this person continues to jog at the same speed, what is this person's unit rate and what does this unit rate mean?



The unit rate is 2, meaning that this person jogs 2 miles in 1 hour.

Example 3: Erik can read $\frac{3}{4}$ of a page in $\frac{2}{3}$ minute. At this same rate, how many pages can Erik read in a minute?



$\frac{3}{4} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$

$\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{9}{8} = 1\frac{1}{8}$ page

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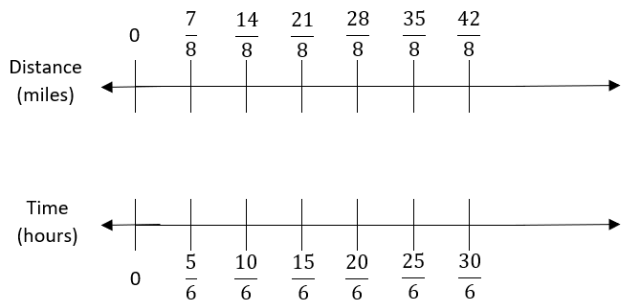
Example 4: A turtle walks $\frac{7}{8}$ of a mile in 50 minutes. What is the unit rate when the turtle's speed is expressed in miles per hour? Possible methods of solution might include:

Method 1: $\frac{7}{8} \div 50 = \frac{7}{400}$ mile for every 1 minute. Multiply by 60 to get the unit rate in miles per hour. $\frac{7}{400} \times 60 = \frac{420}{400}$, which is $\frac{42}{40}$ or $1\frac{1}{20}$.

Method 2: 50 minutes is $\frac{5}{6}$ of an hour. Use a double number line to find the distance traveled in miles in 5 hours.



$\frac{\frac{42}{8}}{5} = \frac{42}{40} = 1\frac{1}{20}$

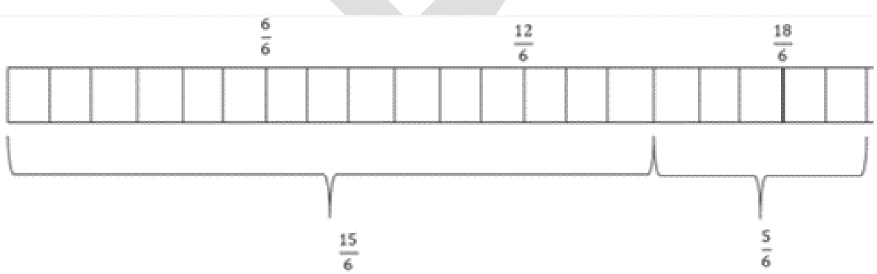
Method 3: Using the rate $\frac{\frac{7}{8}\text{ mile}}{\frac{5}{6}\text{ hour}}$ and simplifying the complex fraction $\frac{\frac{7}{8}}{\frac{5}{6}}$ by rewriting as a division problem $\frac{7}{8} \div \frac{5}{6} = \frac{42}{40} = 1\frac{1}{20}$.

Example 5: Taken from EngageNY Grade 7 Module 1, lesson 11.

For Anthony's birthday, his mother is making cupcakes for his 12 friends at his daycare. The recipe calls for $3\frac{1}{3}$ cups of flour. This recipe makes $2\frac{1}{2}$ dozen cupcakes. Anthony's mother has only 1 cup of flour. Is there enough flour for each of his friends to get a cupcake? Explain and show your work.

$\frac{\text{cups}}{\text{dozen}}$ $\qquad \frac{3\frac{1}{3}}{2\frac{1}{2}} = \frac{\frac{10}{3} \times \frac{2}{5}}{\frac{5}{2} \times \frac{2}{5}} = \frac{\frac{20}{15}}{1} = 1\frac{5}{15} = 1\frac{1}{3}$

No, since Anthony has 12 friends, he would need 1 dozen cupcakes. This means you need to find the unit rate. Finding the unit rate tells us how much flour his mother needs for 1 dozen cupcakes. Upon finding the unit rate, Anthony's mother would need $1\frac{1}{3}$ cups of flour; therefore, she does not have enough flour to make cupcakes for all of his friends.



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Example 6: Taken from EngageNY Grade 7 Module 1, lesson 12.

Which car can travel farther on 1 gallon of gas?
Blue Car: travels $18\frac{2}{5}$ miles using 0.8 gallons of gas
Red Car: travels $17\frac{2}{5}$ miles using 0.75 gallons of gas

Finding the Unit Rate:

Blue Car:

$$\frac{18\frac{2}{5}}{\frac{4}{5}} = \frac{\frac{92}{5}}{\frac{4}{5}} = 23$$

Red Car:

$$\frac{17\frac{2}{5}}{\frac{3}{4}} = \frac{\frac{87}{5}}{\frac{3}{4}} = 23\frac{1}{5}$$

Rate:

23 *miles/gallon*

$\mathbf{23\frac{1}{5}}$ ***miles/gallon***

The red car traveled $\frac{1}{5}$ mile farther on one gallon of gas.

Example 7: Taken from Illustrative Mathematics, Track Practice (content licensed by CC BY-NC-SA 4.0)

Angel and Jayden were at track practice. The track is $\frac{2}{5}$ kilometers around.

- Angel ran 1 lap in 2 minutes.
- Jayden ran 3 laps in 5 minutes.

How many minutes does it take Angel to run one kilometer? What about Jayden?
How far does Angel run in one minute? What about Jayden?
Who is running faster? Explain your reasoning.

Number of laps	Number of km
1	$\frac{2}{5}$
2	$\frac{4}{5}$
3	$\frac{6}{5}$

We can see from the table that 1 km is exactly half way between 2 and 3 laps. So it will take 2.5 laps to run 1 km.

Since it takes Angel 2 minutes to run 1 lap, she will take

$$\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{2 \text{ minutes}}{1 \text{ lap}} = \frac{5 \text{ minutes}}{1 \text{ km}}.$$

So it takes Angel 5 minutes to run 1 km.

Since it takes Jayden 5 minutes to runs 3 laps, she runs 1 lap in $\frac{5}{3}$ minutes. Thus, it takes Jayden

$$\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{5 \text{ minutes}}{3 \text{ laps}} = \frac{5}{2} \cdot \frac{5}{3} \text{ minutes/km} = \frac{25}{6} \text{ minutes/km} = 4\frac{1}{6} \text{ minutes/km}.$$

So it takes Jayden $4\frac{1}{6}$ minutes to run 1 km.

Angel runs 1 lap in 2 minutes, so she runs $\frac{1}{2}$ lap in 1 minute. Since 1 lap is $\frac{2}{5}$ km, $\frac{1}{2}$ lap is $\frac{1}{5}$km, so she also runs $\frac{1}{5}$km in one minute. Since Jayden runs 1 lap in $\frac{5}{3}$ minutes, she will run $\frac{3}{5}$laps in 1 minute. Since Jayden runs 1 km in $\frac{25}{6}$minutes, she will run $\frac{6}{25}$km in 1 minute. Jayden runs the same distance in less time than Angel (alternatively, Jayden runs farther in the same time than Angel), so Jayden is running faster than Angel.