New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 6	DOMAIN: The Number System (NS)

CLUSTER: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Students will apply and extend previous understandings of multiplication and division, including their knowledge of multiplying a fraction or whole number by a fraction in order to develop strategies to divide fractions. Reasoning about the relationship between multiplication and division and utilizing the properties of operations will allow for the development of the standard algorithm $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$.

GRADE LEVEL STANDARD:

NY-6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions.

Note: Strategies may include but are not limited to the following: using visual fraction models, a standard algorithm, and equations to represent the problem.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Compute quotients of fractions.
- Justify the meaning of a dividing a fraction by a fraction visually using a model.
- Use their knowledge of the relationship between division and multiplication, as well as the properties of operations, to write a division problem as a multiplication equation with an unknown factor that can be used to solve a division problem involving two fractions.
- Recognize, solve and create word problems that involve the division of fractions by fractions.

ASPECTS OF RIGOR

Procedural Conceptual Application

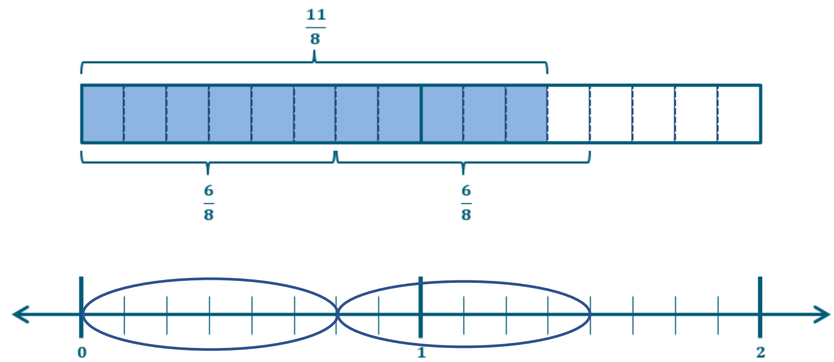
MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-3.OA.5** Apply properties of operations as strategies to multiply and divide. **NY-4.OA.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **NY-5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **NY-5.NF.4a.** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. **NY-5.NF.7** Apply and extend previous understanding of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 1: Division of a Fraction by a Fraction (Visual Representation)

This example, taken from [EngageNY Grade 6 Module 2](), Lesson 4, shows the measurement concept of division. Students are given the size of each group and being asked to determine the number of groups.

Xavier has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup of strawberries to make a batch of tarts. How many batches can he make?



This strategy has students utilizing knowledge of common denominators and least common multiples.

$\frac{11}{8} \div \frac{6}{8} = 11 \text{ eighths} \div 6 \text{ eighths} = \frac{11}{6} = 1\frac{5}{6}$

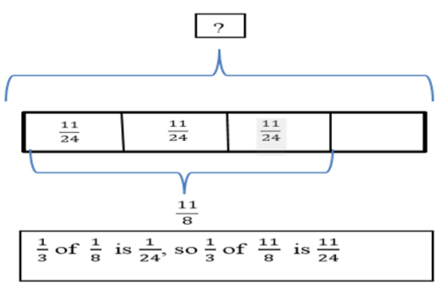
Xavier has enough to make 1 and $\frac{5}{6}$ batches.

This problem could be explained by utilizing the relationship between multiplication and division, as well as the commutative property:

$\frac{11}{8} \div \frac{3}{4} = ?$ How many groups of $\frac{3}{4}$ are there in $\frac{11}{8}$?

$? \times \frac{3}{4} = \frac{11}{8}$

$\frac{3}{4} \times ? = \frac{11}{8}$ By applying the commutive property, we can now ask $\frac{3}{4}$ of what number is $\frac{11}{8}$? ($\frac{3}{4}$ is $\frac{11}{8}$ group of what size?)



This strategy has students utilizing previous understandings of multiplication to multiply a fraction or whole number by a fraction. (NY-5.NF.4a)

The whole (?) would be equal to $4 \times \frac{11}{24}$ which is $\frac{44}{24} = \frac{11}{6}$.

This reasoning reinforces the standard algorithm $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$, where students can see that they first divided $\frac{11}{8}$ by 3, then multiplied the result by 4. This is the same as multiplying $\frac{11}{8}$ by the reciprocal (multiplicative inverse) of $\frac{3}{4}$.

$\frac{11}{8} \times \frac{4}{3} = \frac{44}{24} = \frac{11}{6}$

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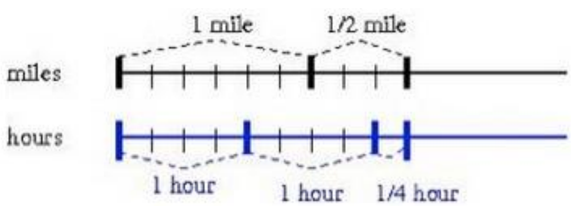
Example 2: Illustrative Math Task 6. NS. A. 1 Task: Traffic Jam (Content licensed under CC BY-NC-SA 4.0)

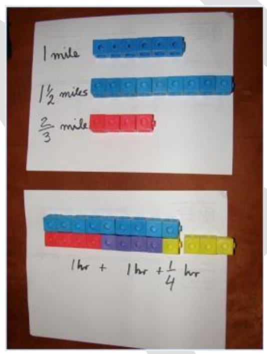
You are stuck in a big traffic jam on the freeway and you are wondering how long it will take to get to the next exit, which is $1\frac{1}{2}$ miles away. You are timing your progress and find that you can travel $\frac{2}{3}$ of a mile in one hour. If you continue to make progress at this rate, how long will it be until you reach the exit? Solve the problem with a diagram and explain your answer.

It is much easier to visualize division of fraction problems with contexts where the quantities involved are continuous. It makes sense to talk about a fraction of an hour. The context suggests a linear diagram, so this is a good opportunity for students to draw a number line or a double number line to solve the problem. Linker cubes are also an appropriate tool to solve this problem. The linker cube solution suggests an algorithm for dividing fractions using a common denominator. The context of this problem would also work in the case where the dividend is smaller than the divisor, e.g. $\frac{1}{4} \div \frac{2}{3}$.

Solution: Number Line (and Linker cubes)

Using a double number line where one line is measured in miles and the other one is measure in hours, we get the following diagram.

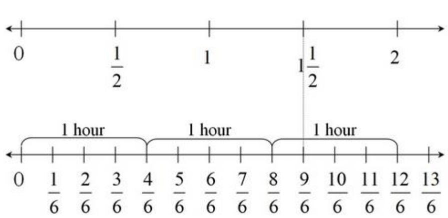




In order to measure both $\frac{1}{2}$ miles and $\frac{1}{3}$ miles, we divide the 1 mile into $\frac{1}{6}$ mile pieces. This way we can find1 $\frac{1}{2}$ miles and $\frac{2}{3}$ miles. Driving two $\frac{2}{3}$ mile stretches takes two hours. That leaves $\frac{1}{6}$ mile, which will take $\frac{1}{4}$ hour to drive. Therefore, it takes $2\frac{1}{4}$ hours to drive $1\frac{1}{2}$ miles. Since we are asking "How many $\frac{2}{3}$ are there in $1\frac{1}{2}$?" this is a "How many groups?" division problem:
$1\frac{1}{2} \div \frac{2}{3} = ?$ We have found that the answer to this division problem is $2\frac{1}{4}$.

Note: This problem could lead into a discovery of a "common denominator" procedure for dividing fractions: Find a common denominator of both fractions, then just divide the numerators:
$1\frac{1}{2} \div \frac{2}{3} = \frac{9}{6} \div \frac{4}{6} = 9\left(\frac{1}{6}\right) \div 4\left(\frac{1}{6}\right) = 9 \div 4 = \frac{9}{4} = 2\frac{1}{4}$.

Solution: Number line solution Since $1\frac{1}{2} = \frac{9}{6}$ and it takes an hour to travel $\frac{2}{3} = \frac{4}{6}$ miles, we can look at the number lines below and see that it will take $2\frac{1}{4}$ hours to travel the distance to the exit. Since we are asking "How many $\frac{2}{3}$ are there in $1\frac{1}{2}$?" this is a "How many groups?" division problem: $1\frac{1}{2} \div \frac{2}{3} = ?$ We have found that the answer to this division problem is $2\frac{1}{4}$.



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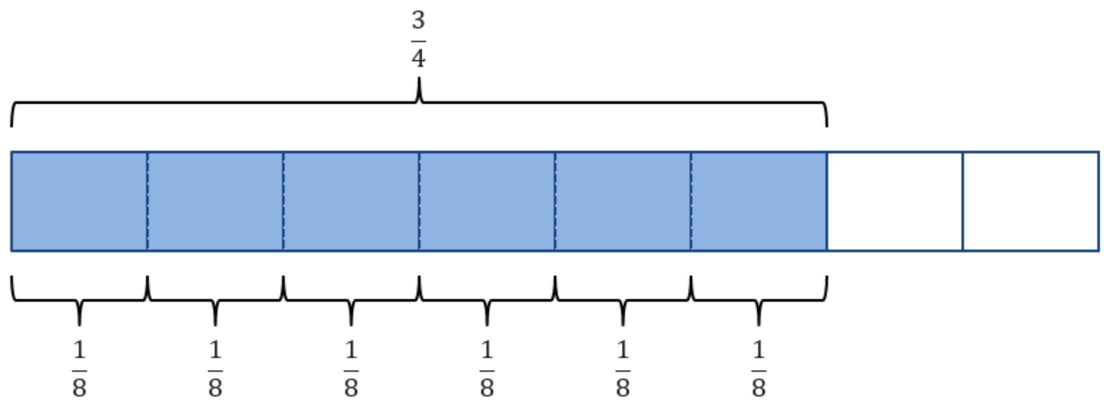
Example 3: Create a Division Problem
The following problem is taken from EngageNY Grade 6 Module 2, Lesson 5.

Students create a story problem for $\frac{3}{4} \div \frac{1}{8} = 6$, using the measurement (or partitive) interpretation of division.

Arthur divided $\frac{3}{4}$ of his kingdom into parcels of land, each being $\frac{1}{8}$ of the entire kingdom. How many parcels did he make?



A student could also provide a visual/context from the perspective $\frac{3}{4}$ is $\frac{1}{8}$ group of what size, connecting it to the invert and multiply algorithm (multiplying by the multiplicative inverse).