**New York State Next Generation Mathematics Learning Standards**

**Unpacking Document (DRAFT)**

**GRADE: 6**

**DOMAIN: Expressions and Equations (Inequalities)**

**CLUSTER: Apply and extend previous understandings of arithmetic to algebraic expressions.**

Applying their prior knowledge from Grade 5, where whole number exponents were used to express powers of ten, students examine exponents and carry out the order of operations, including exponents. Students continue to demonstrate the meaning of exponents to write, generate equivalent, and evaluate numerical expressions. These numerical expressions may include whole number exponents.

Students then extend their arithmetic work to include using letters to represent numbers. Students understand that letters are simply “stand-ins” for numbers and that arithmetic is carried out exactly as it is with numbers. Students explore operations in terms of verbal expressions and determine that arithmetic properties hold true with expressions because nothing has changed—they are still doing arithmetic with numbers. Students will generalize that there is a conventional order when evaluating expressions.

**Grade Level Standard:**

- **NY-6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers.
- **NY-6.EE.2a** Write expressions that record operations with numbers and with letters standing for numbers.
- **NY-6.EE.2b** Identify parts of an expression using mathematical terms (term, coefficient, sum difference, product, factor, and quotient); view one or more parts of an expression as a single entity.
- **NY-6.EE.2c** Evaluate expressions given specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order (Order of Operations).

*Note: Expressions may or may not include parentheses. Nested grouping symbols are not included.*

**PERFORMANCE/KNOWLEDGE TARGETS**

*(measurable and observable)*

- Translate verbal expressions (word phrases) to algebraic expressions with letters standing for numbers.
- Identify parts of an expression using mathematical vocabulary.
- Evaluate expressions by substituting a numerical value for a variable.
- Simplify expressions using order of operations.
- Solve real-world problems when given a formula.

**ASPECTS OF RIGOR**

<table>
<thead>
<tr>
<th>Procedural</th>
<th>Conceptual</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**MATHEMATICAL PRACTICES**

**FOUNDATIONAL UNDERSTANDING**

- **NY-5.OA.1** Apply the order of operations to evaluate numerical expressions.
- **NY-5.OA.2** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them.
- **NY-6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.
Students learn to express operations in algebraic form. They read and write expressions in which letters stand for and represent numbers. They also learn to use the correct terminology for operation symbols when reading expressions. For example, the expression \( \frac{3}{2x-4} \) is read as “the quotient of three and the difference of twice a number and four.” Similarly, students write algebraic expressions that record operations with numbers and letters that stand for numbers. Students determine that \( 3a + b \) can represent the story: “Martina tripled her money and added it to her sister’s money”. While evaluating algebraic expressions, they build on their understanding of the order of operations by including work with exponents. They also understand that the placement of parentheses can alter the final solution when evaluating expressions. Students also write and evaluate expressions and formulas. Students continue to evaluate given formulas such as the volume of a cube, \( V = s^3 \), given the side length, or the volume of a rectangular prism, \( V = l \cdot w \cdot h \), given those dimensions.

Examples similar to the following can be found in EngageNY Grade 6 Module 4.

**Example 1: Writing of Expressions (Implied Multiplication is introduced in Grade 6)**

- Students write numeric and algebraic expressions, given the word expressions.
  - Four more than twice a number, \( x \) is written as \( 4 + 2x \)
  - Three times four squared is written as \( 3 \times 4^2 \) or \( 3 \cdot 4^2 \) or \( 3(4)^2 \) (What are the factors for this expression?)
- Give students a number, such as 24, and ask them to generate as many numerical expressions that would evaluate to 24. Students should read out-loud their expressions and justify.
  - \( 4^2 + 8 \) (i.e., the sum of 16 and 8)
  - \( 5^2 - 3^2 + 2^2 \)
  - \( 5(5-3)^2 + 2^3 \)
  - \( \frac{2(6)^2}{3} \)
- Rewrite the expression in standard form (use the fewest number of symbols and characters possible).
  - \( 5g \cdot 7h \)
  - \( 3 \cdot 4 \cdot 5 \cdot m \cdot n \)
- Name the parts of the expression. Then, write it in expanded form.
  - \( 14b \)
  - \( 30jk \)
- Write the division expression in words and as a fraction.
  - \( \frac{g + 12}{h} \)
- Write the following division expression using the division symbol and as a fraction.
  - \( f \) divided by the quantity \( h \) minus 3.

**Example 2: Expressions with Parentheses**

- Consider a family of 4 that goes to a soccer game. Tickets are $5.00 each. The mom also buys a soft drink for $2.00. How would you write a numerical expression for this scenario? How much does this outing cost?

  Consider a different scenario: The same family goes to the game as before, but each of the family members wants a drink. Would your expression change? How much will this outing cost? Students can consider the expressions \( 4 \cdot 5 + 4 \cdot 2 \) and \( 4(5+2) \). How does the model change?
The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

- Evaluate the following expressions. Why do these similar numeric expressions have different values? Justify answer.
  - 12 \times (3 + 2^2) + 2 - 10
  - 12 \times (3 + 2)^2 + 2 - 10
  - 12 \times 3 + 2^2 + 2 - 10

Example 3: Writing Equivalent Expressions

The following is taken from Illustrative Mathematics, Distance to School (Content licensed under CC-BY-NC-SA 4.0).

- Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains. Let \(d\) be the distance in miles from a student’s home to the school. Write two different expressions that represent how far a student travels by bike in a four-week period if there is one rainy day each week.

  Possible extensions: How would your expressions change if the time period was 6 weeks? (Students should identify what parts of the expression changed using mathematical language, such as coefficient, product, factor)

  Suppose twice a week, a student needed to stop at the post office, adding 3 miles on to the trip. How would this change your expressions?

Example 4: Real World Applications

The following is taken from lesson 21 of EngageNY Grade 6 Module 4.

- Pizza Queen has a special offer on lunch pizzas: $4.00 each. They charge $2.00 to deliver, regardless of how many pizzas are ordered. Determine the cost for various numbers of pizzas and determine the expression that describes the cost of having \(P\) pizzas delivered.

<table>
<thead>
<tr>
<th>Number of Pizzas Delivered</th>
<th>Total Cost in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>(P)</td>
<td></td>
</tr>
</tbody>
</table>

If the booster club had $400 to spend on pizza, what is the greatest number of pizzas they could order?

If the pizza price was raised to $5.00 and the delivery price was raised to $3.00, create a table that shows the total cost (pizza plus delivery) of 1, 2, 3, 4 and 5 pizzas. Include the expression that describes the new cost of ordering \(P\) pizzas.

Additional examples include:

- Tom wants to order tickets online so that he and three of his friends can go together to a water park. The cost of the tickets is $16.00 per person. There is also a $2.50 one-time service fee for ordering tickets online. Write an expression in terms of \(n\) that represents the cost for ordering \(n\) tickets online. Use your expression to find the total cost for ordering 4 tickets online.
The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

- In reverse, students can be given expressions and asked to develop a context. For example, given 20 - 0.75t students might say that at an amusement park, each ride needs a ticket that costs $0.75. What is the change given back if you go on t rides? Students should explain the role of each term in the expression.

- The formula $F=1.8C+32$ is used to convert a temperature in degrees Celsius, $C$, to a temperature in degrees Fahrenheit, $F$. The high temperature in a mountain city was 15°C. What was the high temperature in degrees Fahrenheit?

- The following example shows a grade-level Geometry connection NY-6. G.4 (Using nets to find the surface area of three-dimensional figures).

  The surface area, $S$, of a right rectangular prism with length $l$, width $w$ and height $h$ can be found using the formula below. $S=2(lw+wh+hl)$. What is the surface area, in square inches, of a prism with a length of 12 inches, a width of 9 inches, and a height of 2 inches?