New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 6	DOMAIN: Expressions and Equations (Inequalities)

CLUSTER: Apply and extend previous understandings of arithmetic to algebraic expressions.

Applying their prior knowledge from Grade 5, where whole number exponents were used to express powers of ten, students examine exponents and carry out the order of operations, now including exponents. Students further demonstrate the meaning and understanding of exponents as they write, generate equivalent, and evaluate numerical expressions. Numerical expressions may or may not contain exponents.

Students then extend their arithmetic work to include using letters to represent numbers. Students understand that letters are simply "stand-ins" for numbers and that arithmetic is carried out exactly as it is with numbers. Students explore operations in terms of verbal expressions and determine that arithmetic properties hold true with expressions because nothing has changed—they are still doing arithmetic with numbers. Students will generalize that there is a conventional order when evaluating expressions.

Grade Level Standard:

NY-6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Evaluate numerical expressions that contain whole-number exponents.
- Write numerical expressions (repeated multiplication) using whole-number exponents.
- Use informal and formal mathematical language to describe the repeated multiplication of a base number using whole number exponents.
- Use informal and formal mathematical language to explain the meaning of a numerical expression that uses whole-number exponents.

ASPECTS OF RIGOR

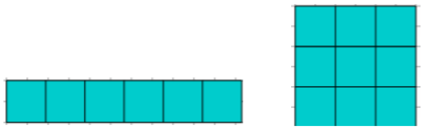
Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-5.NBT.2** Use whole-number exponents to denote powers of 10. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. **NY-3.MD.5b** Recognize a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **NY-5.MD.3b** Recognize that a solid figure which can be packed without gaps or overlaps unit *n* unit cubes is said to have a volume of *n* cubic units. **NY-6. G.5** Use area and volume models to explain perfect squares and perfect cubes.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 1: Introducing Exponential Notation for Whole Number Exponents using Visual (Area/Volume) Models

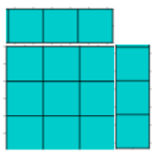
- Students brainstorm and write as many possible numerical expressions that can be used to find the total number of units.



6 square units plus 9 square units is 15 square units.

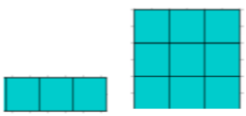
e.g., 6 +9 or 6 + 3 + 3 + 3 or 6 + 3 x 3 or 6 + 3^2 Discussions should include which method is the most efficient and why.

- What numerical expressions can be written to model the following diagram?



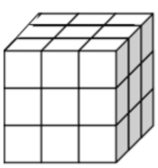
9 + 3 + 3
$3^2 + 6$
$4^2 - 1$

- What numerical expressions can be written to model the following diagram?



3 + (3+3+3)
4·3
3 + 9
$3 + 3^2$

- What numerical expression would represent the number of units in the following figure?



3 x 3 x 3
3^3

Example 2: Developing Mathematical Language
Students are introduced to exponential notation for whole number exponents and should see and understand the difference between repeated addition and repeated multiplication. See EngageNY Grade 6 Module 4, lesson 5 for possible tasks, like the following:

- Complete the table by filling in the blank cells. Use a calculator when needed.

Exponential Form	Expanded Form	Standard Form
3^5		
	$4 \times 4 \times 4$	
$(1.9)^2$		
$\left(\frac{1}{2}\right)^5$		

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- Why do whole numbers raised to an exponent get greater, while fractions raised to an exponent get smaller?

- The powers of 2 that are in the range 2 through 1,000 are 2, 4, 8, 16, 32, 64, 128, 256, and 512. Find all the powers of 3 that are in the range 3 through 1,000.

- Find all the powers of 4 in the range 4 through 1,000.

- Write an equivalent expression for $n \times a$ using only addition.

- Write an equivalent expression for w^b using only multiplication.
  Explain what w is in this new expression.
  Explain what b is in this new expression.

- What is the advantage of using exponential notation?

- What is the difference between $4x$ and x^4? Evaluate both expressions when $x = 2$.

Exponential Notation for Whole Number Exponents: Let m be a nonzero whole number. For any number a, the expression a^m is the product of m factors of a (i.e., $a^m = a \cdot a \cdot a \cdot a \cdots a$, m times). The number a is called the base, and m is called the exponent or power of a. When m is 1, "the product of one factor of *a*" just means *a* (i.e., $a^1 = a$). Raising any nonzero number, *a,* to the power of 0 is defined to be 1 (i.e., $a^0 = 1$ for all $a \neq 0$).

Write an equivalent numerical expression for 5 x 5 x 5 x 5 x 5 x 5 that uses exponents. Have students read aloud their expression(s). Encourage exponential language such as 5 raised to the 6th power, the factor 5 being multiplied 6 times, the product of 6 factors of 5, the 6th power of 5, the base 5 raised to the 6th power (exponent), the base 5 raised to the power of 6.
Some responses could include ones like the following: $5^2 \cdot 5^4$ or $5^5 \cdot 5$.

Write an equivalent numerical expression without using the given operation for both of the following: 2+2+2+2 and $2 \cdot 2 \cdot 2 \cdot 2$.
Have students read aloud their expression(s). Discussions should center on the difference between repeated addition vs. repeated multiplication.

Example 3: Meaning of Exponents, taken from Open-Up Resources (licensed by CC BY 4.0).

- Magic Coins
  You are walking along, and you find a brass bottle that looks really old. There appears to be some writing on the bottle! You try to clean off some dirt to read it better. A genie appears! He is so happy to be free. He wants to repay you. He offers two ways to repay and you must choose one:

  He will give you $50,000; or
  He will give you one magical $1 coin. The magic coin will turn into 2 coins on the first day. The 2 coins will turn into 4 coins on the second day. On the third day, the 4 coins will magically turn into 8 coins. The genie explains that the magic coins will continue this pattern for 28 days.

  1. The number of coins on the third day will be $2 \cdot 2 \cdot 2$. Can you write another expression using exponents for the number of coins there will be on the third day?
  2. What do 2^5 and 2^6 represent in this situation? Evaluated 2^5 and 2^6 without a calculator.
  3. How many days would it take or the number of magical coins to exceed $50,000?
  4. Will the value of the magical coins exceed a million dollars within the 28 days? Explain or show your reasoning.

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- Here are some expressions. All but one of them equals 16. Find the one that is not equal to 16 and explain how you know.

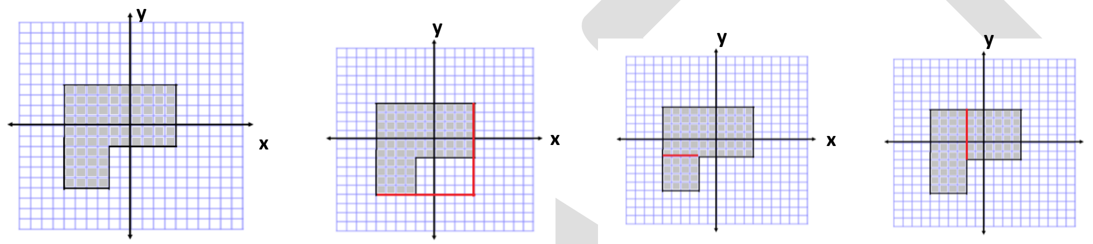
  $2^3 \cdot 2$ 4^2 $\frac{2^5}{2}$ 8^2

- Write three expressions containing exponents so that each expression equals 81.

Example 4: Geometry Connections

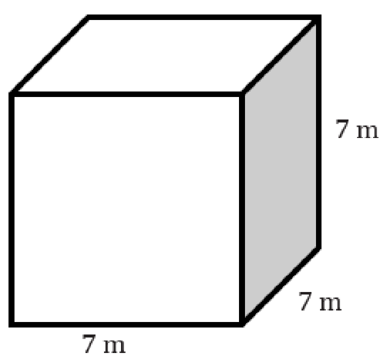
- Write an expression, in exponential form, to represent the area of a square with a side length of 50.

- Write an expression that uses exponents to determine the area of the following figure. Use that expression to calculate the area. Compare your expression to an expression written by a classmate. Are they different? Using exponents, write another expression that could be used to determine the area of the figure (link to grade-level standard NY-6. G.3, NY-6.EE.1 and 2c).



- Sketch and label a net of a cubic gift box that measures 7 cm. on each side. Write a numerical expression for the total area of the net. Tell what each of the terms in your expression means. Taken from EngageNY Grade 6 Module 5, Lesson 18.

  Examine the figure below.



  a. What is the most specific name of the three-dimensional shape?

  b. Write two different expressions for the surface area.

  c. Explain how these two expressions are equivalent.

- All the edges of a cube have the same length. A classmate claims that the formula SA = $6s^2$, where *s* is the length of each side of the cube, can be used to calculate the surface area of a cube. Is your classmate correct and if so, explain why? A cube is a rectangular prism. Will this formula work for calculating the surface area for all rectangular prisms (link to grade-level standards NY-6.EE.2a, 2b, 2c and NY-6. G.4)?