New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 5	**DOMAIN: Numbers and Operations-Fractions**

CLUSTER: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students use their knowledge of fractions, of multiplication and division, and of the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. Students make connections between models (e.g., area models) and equations while reasoning about their results. Students interpret multiplication in Grade 3 as equal groups, and in Grade 4 students begin understanding multiplication as a comparison (times as much). Students will now extend their understanding of multiplication to include scaling, where they reason about the size of products when quantities are multiplied by numbers larger than 1 and smaller than 1.

Grade Level Standards:

NY-5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or fraction**.**

NY-5.NF.4a Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

NY-5.NF.4b Find the area of a rectangle with fractional side lengths by tiling it with rectangles of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Visually represent and explain the multiplication of a fraction and a whole number.
- Visually represent and explain the multiplication of a fraction by a fraction.
- Multiply fractions and whole numbers.
- Multiply fractions by fractions.
- Multiply fractional side lengths of a given rectangle to find the area of the rectangle.
- Using an area model, visually represent the product of two fractions.

ASPECTS OF RIGOR

Procedural Conceptual Application

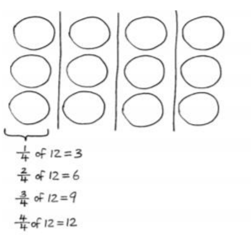
MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-3.OA. 5** Apply properties of operations as strategies to multiply and divide. **NY-3.MD.7** Relate area to the operations of multiplication and addition. **NY-4.NF.4** Apply and extend previous understandings of multiplication to multiply a whole number by a fraction. **NY-4.NF.4a** Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. **NY-4.NF.4b** Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a whole number by a fraction. **NY-5.NF.3** Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$) . Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students will use the commutative property to relate a fraction of a set to the Grade 4 repeated addition interpretation of multiplication by a fraction (NY-4.NF.4b). This offers opportunities for students to reason about various strategies for multiplying fractions and whole numbers. Students progress to multiplying a unit fraction by a unit fraction and multiplying two non-unit fractions. Using area models, rectangular arrays, and tape diagrams to model the multiplication, connections can be drawn that show/reinforce the parallels between the multiplication of two whole numbers and the multiplication of two fractions.

Example 1: Multiply a fraction by a whole number
The following is from EngageNY Grade 5 Module 4, Lesson 6.

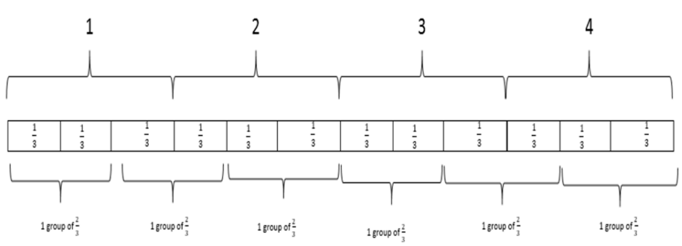
Use an array and manipulatives to solve $\frac{1}{4} \times 12$.
Make an array using 12 counters turned to the red side. Use your straws to divide the arrays into fourths.
How many counters did you place in each fourth?
Write the division equation.
$\frac{12}{4}=3$
What is 1 fourth of 12?



The language of multiplication:
2×3 can be translated as
2 groups of 3
2 of 3
2 times 3
2 taken three times
$\frac{1}{4} \times 12$ can be translated as
$\frac{1}{4}$ of 12

Example 2: Multiply a fraction by a whole number
The following is from EngageNY Grade 5 Module 4, Lesson 8.

Relate a fraction of a set to the repeated addition interpretation of fraction multiplication.
$\frac{2}{3} \times 6 =$
How do we read the expression $\frac{2}{3} \times 6$?
Interpretation could be $\frac{2}{3}$ taken 6 times, same as $6 \times \frac{2}{3}$.



$\frac{2}{3} \times 6 =$

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

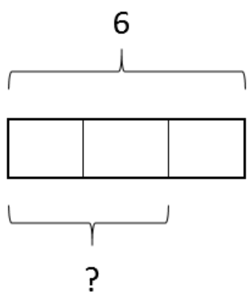
$= \frac{2+2+2+2+2+2}{3}$

$= \frac{2 \times 6}{3}$

$= \frac{12}{3}$

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

What if we read the expression $\frac{2}{3} \times 6$ as $\frac{2}{3}$ of 6? What would a visual look like for that interpretation?



Students see that the product of $\frac{2}{3} \times 6$ can be interpreted as 2 parts when 6 is partitioned into 3 parts.

3 units =6

1 unit $= \frac{6}{3}$

2 units $= 2 \times \frac{6}{3}$

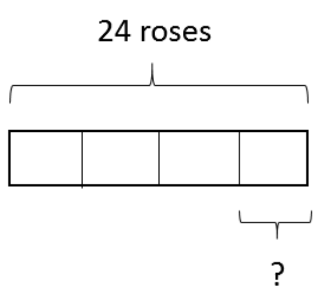
$= \frac{6}{3} + \frac{6}{3}$

$= \frac{2 \times 6}{3}$

$= \frac{12}{3}$

Students can be given Compare and Connect opportunities (Understanding Language/SCALE, Principles for the Design of Mathematics Curricula: Promoting Language and Content Development, content licensed by CC BY 4.0) where students understand one another's strategies by relating and connecting other students' approaches to their own approach. For example, when evaluating $\frac{2}{3} \times 6$ vs. $6 \times \frac{2}{3}$, students can explore what is similar, what is different, are they the same?

A similar example from EngageNY Grade 5 Module 4, involving the use of a tape diagram follows:
Aurelia buys 2 dozen roses. Of these roses, $\frac{3}{4}$ are red, and the rest are white. How many white roses did she buy?



Students see that the $\frac{3}{4} \times 24$ can be interpreted as 3 parts when 24 is partitioned into 4 parts.

4 units=24

1 unit $= \frac{24}{4}$

=6

Aurelia bought 6 white roses.

$\frac{1}{4}$ of 24 = 6

$\frac{3}{4}$ of 24 = 18 red roses

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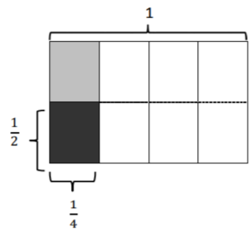
Students now progress to multiplying fractions by fractions, being able to generalize that for the product of two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, where the product shows *a* parts when $\frac{c}{d}$ is broken into *b* parts.

Example 3: Multiply unit fractions by unit fractions

The following is taken from [EngageNY Grade 5 Module 4](), Lesson 13

Half of $\frac{1}{4}$ pan of brownies = $\underline{\frac{1}{8}}$ pan of brownies.

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$



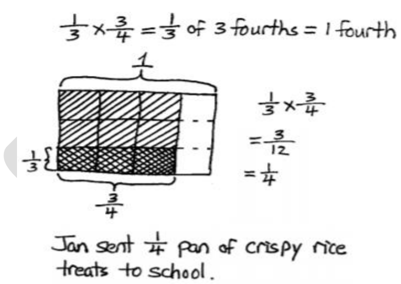
Students see that $\frac{1}{2} \times \frac{1}{4}$ can be interpreted as 1 part when $\frac{1}{4}$ is partitioned into 2 parts.
$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4}$

Students will use the pictorial level of representation, but some may still need concrete models such as paper folding to fully comprehend the concept. For an example of a paper-folding task, see Illustrative Mathematics, [Folding Strips of Paper]() (Content licensed under [CC BY-NC-SA 4.0]()).

Example 4: Multiply unit fractions by non-unit fractions

The following is taken from [EngageNY Grade 5 Module 4](), Lesson 14.

Jan had $\frac{3}{4}$ pan of crispy rice treats. She sent $\frac{1}{3}$ of the treats to school. What fraction of the whole pan did she send to school?



Students see that $\frac{1}{3} \times \frac{3}{4}$ can be interpreted as 1 part when $\frac{3}{4}$ is partitioned into 3 parts.
$\frac{1}{3} \times \frac{3}{4} = \frac{1 \times 3}{3 \times 4}$

What are we finding $\frac{1}{3}$ of? $\frac{1}{3}$ *of 3 fourths.*
We are taking 1 third of 3 units. The units are fourths.
Work with a neighbor to solve one-third of 3 fourths. One of you can draw the rectangular fraction model, while the other writes a matching number sentence.
In your area model when you partitioned each of the fourths into 3 equal parts, what new unit did you create? *Twelfths*
How many twelfths represent 1 third of 3 fourths? *3 twelfths.*
Say 3 twelfths in its simplest form. *1 fourth.*
So, $\frac{1}{3}$ of 3 fourths is equal to what? *1 fourth.*

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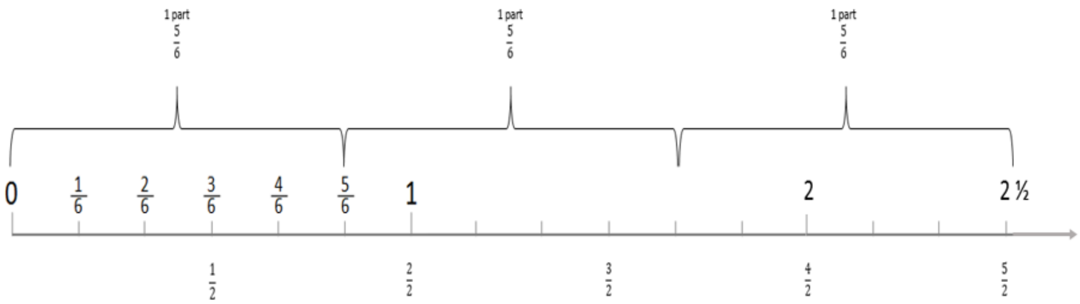
Example 5: Multiplying a fraction by a fraction

See Progressions Documents for the Common Core State Standards in Mathematics (draft). Grades 3–5, Number and Operations – Fractions, pg. 17. Common Core Standards Writing Team. (August 10, 2018). *Progressions for the Common Core State Standards in Mathematics*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Content licensed under CC BY 4.0.

$\frac{2}{3} \times \frac{5}{2}$ is seen 2 parts when $\frac{5}{2}$ is broken into 3 parts.



$$\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$$

$$\frac{2}{3} \times \frac{5}{2} = \frac{10}{6}$$

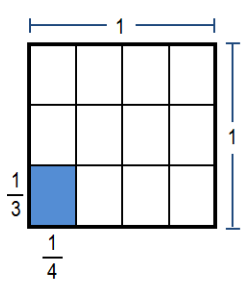
Example 6: Connecting area to finding the product of two fractions

See Progressions Documents for the Common Core State Standards in Mathematics (draft). Grades 3–5, Number and Operations – Fractions, pg. 18. Common Core Standards Writing Team. (August 10, 2018). *Progressions for the Common Core State Standards in Mathematics*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Content licensed under CC BY 4.0.

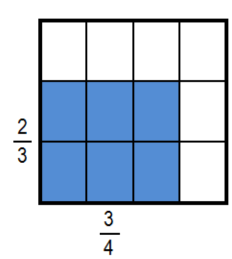
$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

$\frac{6}{12}$ is 2 parts when $\frac{3}{4}$ is broken into 3 parts.

The shaded portion shows the rectangle with the appropriate unit fraction side lengths.



The area of a $\frac{2}{3} \times \frac{3}{4}$ rectangle is $\frac{6}{12}$ because the whole is partitioned into 12 parts with 6 of them shaded.



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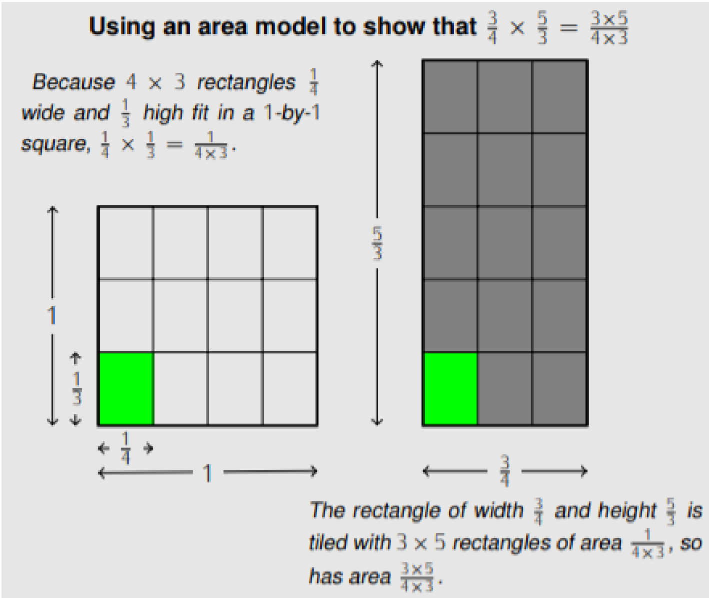

Example 7: Using an area model and decomposition to find the product of two mixed numbers

$3\frac{2}{5} \times 2\frac{1}{4}$

	2	$\frac{1}{4}$
3	6	$\frac{3}{4}$
$\frac{2}{5}$	$\frac{4}{5}$	$\frac{2}{20}$

$6 + \frac{3}{4} + \frac{4}{5} + \frac{2}{20} = 7\frac{13}{20}$

$\frac{17}{5} \times \frac{9}{4} = \frac{153}{20}$

Example 8: Solve and create word problems
Sample word problem scenarios like the following can be found throughout EngageNY Grade 5 Module 4, lessons 10-16.

- Lillian and Darlene plan to get their homework finished within one hour. Darlene completes her math homework in $\frac{3}{5}$ hour. Lillian completes her math homework with $\frac{5}{6}$ hour remaining. Who completes her homework faster, and by how many minutes? Bonus: Give the answer as a fraction of an hour.

  Method 1:
  Darlene: $\frac{3}{5}$ of an hour is $\frac{3}{5} \times 60$ = 36 minutes ($\frac{3\times60}{5} = \frac{180}{5}$).
  Lillian: $\frac{5}{6}$ of an hour remaining means that it took $\frac{1}{6}$ of an hour to complete homework, so $\frac{1}{6} \times 60$ = 10 minutes ($\frac{1\times60}{6} = \frac{60}{6}$).
  36 minutes – 10 minutes = 26 minutes
  Lillian completed the homework faster. She did it 26 minutes faster.
  Bonus: 26 minutes as a fraction of an hour is $\frac{26}{60}$ or $\frac{13}{30}$.

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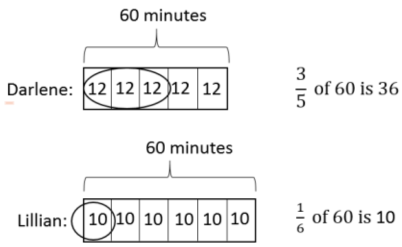
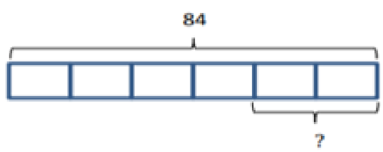
Method 2:



36 minutes – 10 minutes = 26 minutes
Lillian completed the homework faster. She did it 26 minutes faster.

- Create a story problem about a fish tank for the tape diagram below. Your story must include a fraction.



In this problem, students are shown a tape diagram marking 84 as the whole and partitioned into 6 equal units (or sixths). The question mark should signal students to find $\frac{2}{6}$ of the whole. Students should be encouraged to use their creativity while generating a word problem but remain mathematically sound. Examples of stories:

There are 84 fish in the fish tank. $\frac{4}{6}$ of the fish are gold fish, and the rest are guppies. How many guppies are in the tank?

There are 84 organisms in a freshwater aquarium. One-half of the organisms are top-feeding fish. One-sixth are crustaceans. The remaining organisms are snails. How many snails are in the tank?

- Three-quarters of the boats in a marina are white, $\frac{4}{7}$ of the remaining boats are blue, and the rest are red. If there are 9 red boats, how many boats are in the marina?
- A newspaper's cover page is $\frac{3}{8}$ text and photographs fill the rest. If $\frac{2}{5}$ of the text is an article about endangered species, what fraction of the cover page is the article about endangered species?