

New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 4	DOMAIN: Operations & Algebraic Thinking
<p>CLUSTER: Use the four operations with whole numbers to solve problems.</p> <p>Using their understanding of addition and subtraction properties, students connect them to multiplication and division. Students apply these skills as they solve multi-step problems. In problems that involve division, students must interpret and use remainders with respect to the context. Students put multiplicative comparison problems into context by using visual models to show the relationships between subjects being compared.</p>	
<p>Grade Level Standard:</p> <p>NY-4.OA.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <p>NY-4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison. Use drawings and equations with a symbol for the unknown number to represent the problem.</p>	

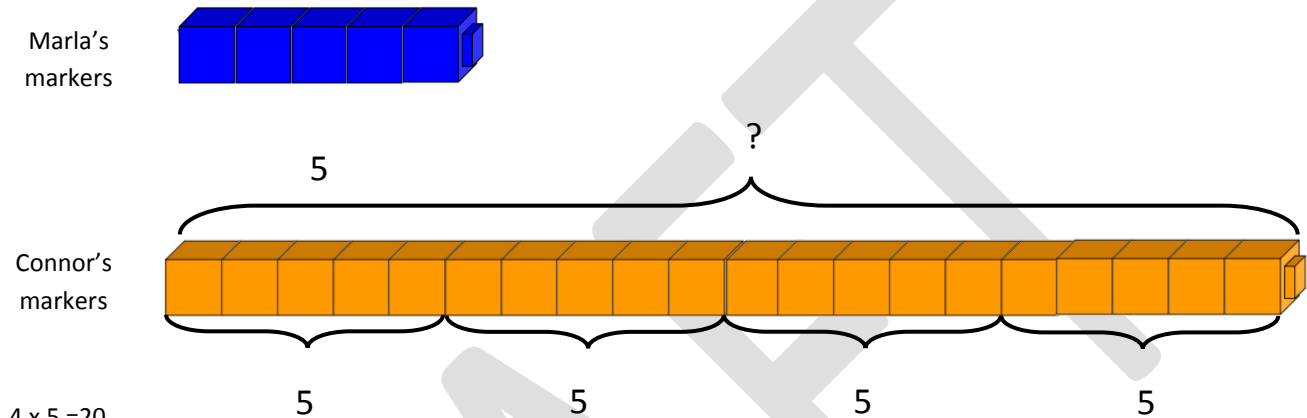
PERFORMANCE/KNOWLEDGE TARGETS (measurable and observable)				
<ul style="list-style-type: none"> • Demonstrate how repeated addition “equal groups of” relates to the multiplicative comparison of two quantities, showing that one quantity is a multiple of the other. • Explain a multiplication equation as a comparison of quantities. • Given a verbal multiplicative comparison context for two quantities, write a multiplication equation that models that comparison. 				
ASPECTS OF RIGOR				
<table style="width: 100%; border: none;"> <tr> <td style="width: 33%; border: none;">Procedural</td> <td style="width: 33%; border: none;">Conceptual</td> <td style="width: 33%; border: none;">Application</td> </tr> </table>		Procedural	Conceptual	Application
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MATHEMATICAL PRACTICES	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. 			
FOUNDATIONAL UNDERSTANDING	<p>NY-3.OA.1 Interpret products of whole numbers.</p> <p>NY-3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.</p>			

The following pages contain **EXAMPLES** to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students begin developing an understanding of multiplicative comparisons by building models (e.g., utilizing real objects such as rope, ribbon, string, and other manipulatives) and pictorial representations (e.g., tape diagrams) to represent multiplicative situations. Students should see the difference between multiplicative (times as much as) and additive comparisons (more than).

Example 1: Using Models and Pictorial Representations

- Connor has 4 times as many markers as Marla. If Marla has 5 markers, how many markers does Connor have?



$4 \times 5 = 20$
 4 groups of 5 is 20 markers.
 4 multiples of 5 is 20. $(5+5+5+5)$
 20 is 4 times as many as 5.

If the situation was the following, how would the model and the equation change?
 Connor has 4 more markers than Marla. If Marla has 5 markers, how many markers does Connor have?

- A bungee cord that is 15 inches long has a stretch length that is 5 times as long. Osman needs to stretch the cord to 62 inches. Will the bungee cord be long enough? Explain answer. Use a diagram or model in your explanation.

Example 2: Open-ended Questions

- 18 is ____ times as many as _____. Support answer with a diagram/model and an equation.
- Given the equation $4 \times 7 = 28$, create a story/scenario that models the equation and uses a multiplicative comparison phrase (i.e., “times as much” or “times as many”). Draw a diagram for your story/scenario. The following chart from the NYS Next Generation Mathematics Learning Standards Document (NY-4.OA.2, pg. 56) provides some examples.

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$
Compare	A blue hat costs \$ b . A red hat costs a times as much as the blue hat. How much does the red hat cost?	A red hat costs \$ p and that is a times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$ p and a blue hat costs \$ b . How many times as much does the red hat cost as the blue hat?
	<i>Measurement example:</i> A rubber band is b cm long. How long will the rubber band be when it is stretched to be a times as long?	<i>Measurement example:</i> A rubber band is stretched to be p cm long and that is a times as long as it was at first. How long was the rubber band at first?	<i>Measurement example:</i> A rubber band was b cm long at first. Now it is stretched to be p cm long. How many times as long is the rubber band now as it was at first?

Multiplicative Compare problems appear first in Grade 4, with whole-number values for a , b , and p , and with the “times as much” language in the table. In Grade 5, unit fractions language such as “one third as much” may be used.

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- Illustrative Mathematics, [Comparing Growth Variation 2](#), licensed under [\(CC BY-NC-SA 4.0\)](#).

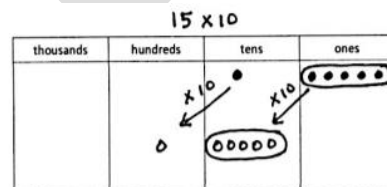
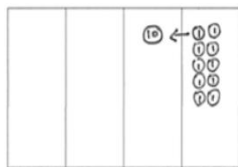
There are two snakes at the zoo, Jewel and Clyde. Jewel was six feet and Clyde was eight feet. A year later Jewel was eight feet and Clyde was 10 feet. When asked which one grew more, students gave varying answers.

Mia said, "Since the two snakes both grew two feet ($8-6=2$ and $10-8=2$) then I would say that they grew the same amount." Raul said, "They both grew 2 feet, but Jewel was only 6 feet to start with, while Clyde was 8 feet to start with. That means Jewel grew more compared to her original length (2 is a larger part of six than it is of eight)."

Compare the two arguments. Describe the difference in the way the two students are thinking about the problem. Suppose a one-foot snake grew two feet and a 20-foot snake grew two feet. Could the two students still make the same type of argument?

The idea of "times as much (many)" in multiplication can be connected to the work that students do with place value (NY-4.NBT.1), units of measure in feet, inches, kilometers, meters, and centimeters (NY-4.MD.1-2), and with the multiplication of a fraction by a whole number (NY-4.NF.4).

Example 3: Connecting to Place Value



1 ten is 10 times as many as 1 one.
 $10 \times 1 = 10$

1 hundred is 10 times as many as 1 ten, and 1 ten is 10 times as many as 1 one.
 $10 \times 10 = 100$
 $5 \times 10 = 50$
 $15 \times 10 = 150$

How do the values represented by the digit 7 compare in the following two numbers? Explain your answer. Represent the comparison using a multiplication equation.

12,752 17,268

7000 is 10 times as much as 700. The multiplication equation would be $700 \times 10 = 7000$.

Example 4: Connecting to Converting Units

- It takes Mallory 45 cm of ribbon to tie a gift box. If she has 2 meters of ribbon, how many boxes can Mallory wrap?
- Renting a bike costs \$1.25 for every 30 minutes. After using the bike for $1\frac{1}{2}$ hours, how much will it have cost?

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Example 5: Multiplicative Comparisons Utilizing Fractions

Doubling, tripling, quadrupling a recipe.

Using this recipe, determine how much of each ingredient you will need to make 4 batches of oatmeal cookies.

$\frac{3}{4}$ cup packed dark brown sugar.

1 cup (2 sticks) salted butter, softened.

2 teaspoons vanilla extract.

2 eggs.

$2\frac{1}{3}$ cups all-purpose flour.

1 teaspoon salt.

$\frac{1}{2}$ teaspoon baking soda.

3 cups old-fashioned oats.

If this recipe makes 12 cookies, how many cookies will 4 batches make?

Additional problems involving multiplicative comparisons can be found in [EngageNY Grade 4 Module 3](#), lesson 2 (area and perimeter) and lesson 12 (two-step word problems). These problems connect with grade-level standard NY-4.OA.2.