New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GRADE: 3	DOMAIN: Operations & Algebraic Thinking

CLUSTER: Solve problems involving the four operations and identify and extend patterns in arithmetic.

Students apply the tools, representations and conceptual understandings of the four operations to solve multi-step word problems and develop their algebraic language by using a letter for the unknown quantity in expressions or equations. Students will estimate during the problem-solving process, and then revisit their estimate, checking for reasonableness. Students will utilize structure and repeated reasoning when identifying and extending numerical patterns that are related to operations.

Grade Level Standard:

NY-3.OA.9 Identify and extend arithmetic patterns (including patterns in the addition table or multiplication table).

PERFORMANCE/KNOWLEDGE TARGETS (measurable and observable)

- Create and explain numeric patterns using addition & multiplication.
- Explain a given numeric pattern shown in a list of terms, table or chart.
- Conclude a rule from a given list, table or chart.
- Solve for a missing number (term) in a given arithmetic pattern.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-2.OA.3a** Determine whether a group of objects (up to 20) has an odd or even number of members. **NY-2.OA.3b** Write an equation to express an even number as a sum of two equal addends. **NY-2.NBT.2** Count within 1000; skip-count by 5's, 10's and 100's. **NY-3.OA.5** Apply properties of operations as strategies to multiply and divide.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students are utilizing strategies for addition and multiplication (i.e., decomposing, properties of operations, partial products) to explain addition and multiplication patterns. Students should engage in tasks that involve verbal explanations of patterns in order to reinforce mathematical language such as difference, pairs, doubles, even, odd, factors and multiples. The use of pictures and visuals should be part of their experience in order to help with the transition of concrete, pictorial to the abstract.

Example 1: Explaining patterns in addition

- Given students the pattern 5, 9, 13, 17, 21, 25, 29. Ask them to explain what rule could have been used to make the pattern, such as start with 5 and add 4 to get the next number, adding 4 six times to get to the last term 29. Students can be asked what the next two terms in the pattern would be and how they determined those terms?

- The following task is taken from Illustrative Mathematics, Addition Patterns , (content licensed by CC BY-NC-SA 4.0).

  Below is a table showing addition of numbers from 1 through 5.

+	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

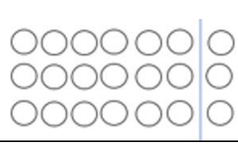
  a. In each column and each row of the table, even and odd numbers alternate. Explain why.
  b. Explain why the diagonal, from top left to bottom right, contains the even numbers 2, 4, 6, 8, and 10.
  c. Explain why all numbers in the other diagonal, from bottom left to top right, are 6's.

  This activity can be expanded upon by giving students a larger addition table and asking them to identify a pattern that they see and defend why that pattern is there (i.e., patterns that run along the diagonals, the sum of the diagonals of any square drawn on the table is the same, the sums of each row (column) increase by the same amount).

- Students can be asked to justify a claim like the following: The sum of two odd numbers is always even.
  With manipulatives, students can see that pairs can be removed from an odd number, with one always remaining. From both odd numbers, the two remaining ones will make the sum even.

Example 2: Explaining patterns in multiplication

- Students can be asked to justify a claim like the following: The product of two odd numbers is always odd.
  Students can apply the distributive property, connecting grade-level work with standards NY-3.OA.5 and NY-3.MD.7c as follows:
  $3 \times 7 = 21$
  3 groups of 6 will be even, why?
  3 groups of 1 will be odd, why?
  So, the total product (sum of 3 groups of 6 and 3 groups of 1) will be odd, why?



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- The following task is taken from [EngageNY Grade 3 Module 3](), lesson 13.

  Skip-count by nine.

  9, ______, ______, 36, ______, ______, ______, 72, ______, ______

  Look at the *tens* place in the count-by. What is the pattern?
  Look at the *ones* place in the count-by. What is the pattern?

  Complete to make true statements.

10 more than 54 is ____.	10 more than 63 is _____.	10 more than 72 is _____.	10 more than 81 is _____.
1 less is _____.	1 less is ______.	1 less is ______.	1 less is _____.
7 x 9 = _____.	8 x 9 = ______.	9 x 9 = _____.	10 x 9 = _____.

  Analyze the equations above. What is the pattern?
  Extension: Use the pattern to find the next 4 facts. Show your work.

  11 × 9 = 12 × 9 = 13 × 9 = 14 × 9 =

  Kent notices a different pattern. His work is shown below. He sees the following:
  The tens digit in the product is 1 less than the number of groups.
  The ones digit in the product is 10 minus the number of groups.

		Tens digit	Ones digit
2 × 9 = 18	→	1 = 2 – 1	8 = 10 – 2
3 × 9 = 27	→	2 = 3 – 1	7 = 10 – 3
4 × 9 = 36	→	3 = 4 – 1	6 = 10 – 4
5 × 9 = 45	→	4 = 5 – 1	5 = 10 – 5

  Use Kent's strategy to solve 6 × 9 and 7 × 9.
  Extension: Show an example of when Kent's pattern does not work.

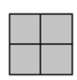
The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

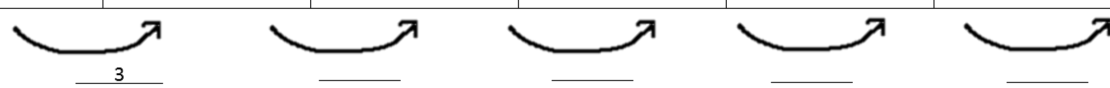
- The following task is taken from EngageNY Grade 3 Module 3, lesson 17.

In the multiplication table below, only the products on the diagonal are shown. Ask each student to label each product on the diagonal.

1 × 1					
	2 × 2				
		3 × 3			
			4 × 4		
				5 × 5	
					6 × 6

Have each student draw an array to match each expression in the table below. Have them also label the number of squares added to make each new array.

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6
					



Ask students the following:
What pattern do you notice in the number of squares that are added to each new array?
Using that pattern, what would 7 × 7 and 8 × 8 be?
Using the pattern, prove that 9 × 9 is the sum of the first 9 odd numbers.