New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

GEOMETRY	**DOMAIN:** Congruence

CLUSTER: Experiment with transformations in the plane.

Students are reintroduced to rigid transformations, specifically rotations, reflections, and translations. Students first saw the topic in Grade 8 (NY-8. G.1–4) and developed an intuitive understanding of the transformations, observing their properties by experimentation. Students now examine each transformation more closely building on their hands-on work and will develop precise definitions that will serve as a logical basis for all theorems that students prove in geometry. Students will differentiate rigid motions from non-rigid motions. Rotations and reflections will be used to verify symmetries within polygons. Using their construction skills, in conjunction with their understanding of transformations, as well as properties of parallel and perpendicular lines, students will perform transformations and will use these transformations to examine correspondence and its place for further upcoming discussion of congruency.

Grade Level Standard:

GEO-G.CO.4 Develop definitions of rotations, reflections, and translations in terms of points, angles, circles, perpendicular lines, parallel lines, and line segments.

- **Include point reflections.**
- **A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector.**
- **A rotation requires knowing the center/point and the measure/direction of the angle of rotation.**
- **A line reflection requires a line and the knowledge of perpendicular bisectors.**

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Identify what specific information/parameters are needed to perform a given rigid motion.
- Represent and analyze the results of rotations, reflections and translations using appropriate tools when necessary.

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**NY-8. G.1** Verify experimentally the properties of rotations, reflections, and translations. a. Verify experimentally lines are mapped to lines, and line segments to line segments of the same length. b. Verify experimentally angles are mapped to angles of the same measure. c. Verify experimentally parallel lines are mapped to parallel lines. **NY-8. F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

With the help of manipulatives (technology, tracing paper, and construction tools), students observe how reflections, translations, and rotations behave individually and in concert, supporting work with course-level standards GEO-G.CO.2 and 6. Students will identify the resulting properties that stay the same and those that change. Discussion of fixed points should be included here.

The three basic properties of rigid motions are:

- map lines to lines, rays to rays, and segments to segments;
- preserve segment length;
- preserve angle measure.

Students will be formalizing the definitions and outcomes of each transformation using precise language. This language includes the set of physical properties determined by the nature of the transformation. Students should be able to explain that

- when a figure is reflected over a line, the line of reflection is the perpendicular bisector of the segment that connects a point on the figure with its reflected image.
- when figures are rotated around a center point, the points of the figure travel in a circular path over a specified angle of rotation and direction.
- when polygonal figures are translated, the segments of the preimage are parallel to the corresponding segments of the image.
  Note: Vectors can be discussed prior to describing and defining a translation. Refer to Grade 8 Module 2 Lesson 2 for supplementary materials on the use of vectors, as well as on translations in general.

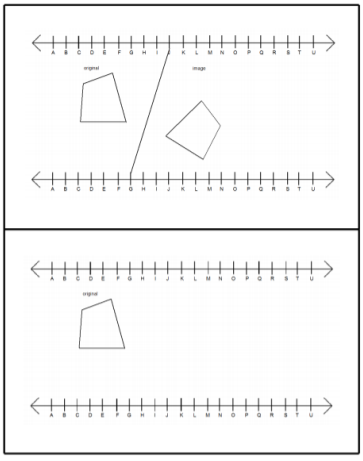
Example 1: In Search of Language

Students will be developing working definitions of rigid motions using precise language that produces accurate results if that rigid motion is performed. Terms such as line segment, perpendicular lines, perpendicular bisector, right angles, midpoint should be part of their definitions. Situations that involve fixed points should be included.

Students will be answering the following:

- For a rotation, we need to know …
- For a reflection (both line and point), we need to know …
- For a translation, we need to know …

Mathematical Modeling Exercise (taken from Lesson 12, , EngageNY Geometry Module 1) : Students discover what they need to know about rotations, reflections, and translations through a partner exercise. The cards mentioned are in the Lesson 12 supplement; there are three pairs of cards. Photocopy and split the pair of images from each transformation card. For the first round of the activity, assign Partner A with pre-image/image cards, and assign Partner B with the pre-image cards. Switch card assignments after the first round.



You will work with a partner on this exercise and are allowed a protractor, compass, and straightedge.

- Partner A: Use the card your teacher gives you. Without showing the card to your partner, describe to your partner how to draw the transformation indicated on the card. When you have finished, compare your partner's drawing with the transformed image on your card. Did you describe the motion correctly?
- Partner B: Your partner is going to describe a transformation to be performed on the figure on your card. Follow your partner's instructions and then compare the image of your transformation to the image on your partner's card.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 2: Formalizing Definitions

The following sample tasks involve formalizing definitions and are taken from Illustrative Mathematics Tasks: Defining Rotations, Defining Reflections, Identifying Translations and Identifying Rotations (content licensed under CC BY-NC-SA 4.0).

Defining Rotations

Consider the following definitions for rotation of the plane by an angle of measure *a* about the point *P:*

a. If *Q* is a point in the plane, then we send *Q* to the point *R* so that the measure of $<RPQ$ is *a*.

b. If *Q* is a point in the plane, then we send *Q* to *R*, where $QP=RP$ and the measure of $<RPQ$ is *a*.

c. If *Q* is a point in the plane, then we send *Q* to *R*, where $QP=RP$ and the measure of $<QPR$ is *a*.

d. If *Q* is a point in the plane and *C* is the circle with center *P* containing *Q,* then we send *Q* to the point R on *C* so that the measure of $<QPR$ is *a*.

Which, if any, of these definitions are valid (that is, do they make sense and have the desired effect) for all points *P* and *Q*?

Defining Reflections

Carlos finds the following definition of a reflection in a math book:

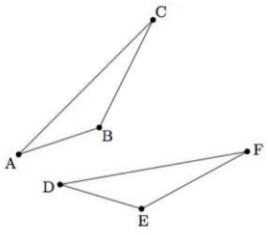
The reflection *r* over a line ℓ takes each point *P* on ℓ to itself and takes each point *Q* not on ℓ to the point $r_\ell(Q)$ such that ℓ is the perpendicular bisector of line segment $Q\ r_\ell(Q)$.

Carlos does not find this definition very helpful. He says ''the reflection over line ℓ sends each point to its mirror image on the other side of ℓ.''

- In what ways is Carlos's definition of reflection more helpful than one from the math book?
- In what ways is the math book definition of reflection more helpful than Carlos's definition?

Identifying Rotations

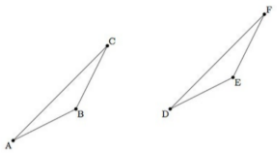
Below is triangle *ABC* and a rotated image, triangle *DEF*.



- Explain how to identify the center of the rotation.
- Once you have found the center, how do you identify the angle of rotation for point *C*.

Identifying Translations

Below is triangle *ABC* and a translated image, triangle *DEF*.



- Explain why *ABED, ACFD*, and *BCFE* are parallelograms.
- Explain why $AD=BE=CF$.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 3: Understanding Definitions through Constructions

Students show understanding of the definitions (properties) of translations, reflections and rotations by performing various constructions utilizing tools such as mirrors, tracing paper, compass and straightedge.

Construction tasks could include, but not limited to the following:

- Construct the line of reflection of a figure and its reflected image.
- Construct the image of a figure when provided the line of reflection.
- Find the center of a rotation for a figure and its image.
- Construct the image of a figure when provided a point of reflection.
- Perform a translation given a vector.