New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

Course: **Algebra II**	**DOMAIN: Algebra (A) – Seeing Structure in Expressions (SSE)**

CLUSTER: Write expressions in equivalent forms to reveal their characteristics.

In order to reveal and explain their properties, a student will transform expressions into equivalent forms. The ability to manipulate expressions develops the skills necessary to work with linear, quadratic, and exponential equations.

Grade Level Standard:

AII-A.SSE.3c Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra I)
Note: Tasks include rewriting exponential expressions with rational coefficients in the exponent.

PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)

- Rewrite an expression employing the laws of rational exponents.
- Explain how different forms of equivalent exponential expressions reveal different information about the function they define, such as annual as opposed to monthly, quarterly, or daily rate of growth or decay.
- Apply exponent rules to change the base of an exponential expression to reflect growth rates of different time units, such as finding a yearly rate of decay for a given half-life of a substance or finding the effective annual rate of money that is compounded continuously, daily, monthly, quarterly, etc…

ASPECTS OF RIGOR

Procedural Conceptual Application

MATHEMATICAL PRACTICES	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
FOUNDATIONAL UNDERSTANDING	**AII-N.RN.1** Explore how the meaning of rational exponents follows from extending the properties of integer exponents. **AI-A.SSE.2** Recognize and use the structure of an expression to identify ways to rewrite it. **AI-A.SSE.3c** Use the properties of exponents to rewrite exponential expressions. Note: Exponential expressions will include those with integer exponents, as well as those whose exponents are linear expressions. Any linear term in those expressions will have an integer coefficient. Rational exponents are an expectation for Algebra II. **NY-8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. **NY-7.EE.2** Understand that rewriting an expression in different forms in real-world and mathematical problems can reveal and explain how the quantities are related. **NY-6.EE.4** Identify when two expressions are equivalent.

The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Students need to be comfortable with working with rational exponents, connecting work done with grade-level standard AII-N.RN.**1** (Explore how the meaning of rational exponents follows from extending the properties of integer exponents) and AII-N.RN.2 (Convert between radical expressions and expressions with rational exponents using the properties of exponents).
The following problems are from lessons 3 and 4 of EngageNY Algebra II Module 3.

- Write each expression in the form $b^{\frac{m}{n}}$. If a numeric expression is a rational number, then write your answer without exponents.

a) $b^{\frac{2}{3}} \cdot b^{\frac{1}{2}}$

b) $\left(b^{-\frac{1}{5}}\right)^{\frac{2}{3}}$

c) $64^{\frac{1}{3}} \cdot 64^{\frac{3}{2}}$

d) $\left(\frac{9^3}{4^2}\right)^{\frac{3}{2}}$

- Jefferson said that $8^{\frac{4}{3}} = 16$ because $8^{\frac{1}{3}} = 2$ and $2^4 = 16$. Use properties of exponents to explain why he is or is not correct.
- Rita said that $8^{\frac{2}{3}} = 128$ because $8^{\frac{2}{3}} = 8^2 \cdot 8^{\frac{1}{3}}$, so $8^{\frac{2}{3}} = 64 \cdot 2$, and then $8^{\frac{2}{3}} = 128$. Use properties of exponents to explain why she is or is not correct.
- Provide a written explanation for each question below.

a) Is it true that $\left(1000^{\frac{1}{3}}\right)^3 = (1000^3)^{\frac{1}{3}}$? Explain how you know.

b) Is it true that $\left(4^{\frac{1}{2}}\right)^3 = (4^3)^{\frac{1}{2}}$? Explain how you know.

c) Suppose that m and n are positive integers and b is a real number so that the principal nth root of b is a real number. In general, does $\left(b^{\frac{1}{n}}\right)^m = (b^m)^{\frac{1}{n}}$? Explain how you know.

Real-world tasks could include something like the following examples.

Example 1: A study of the annual population of gray squirrels in a suburban New York State park shows the population $S(t)$ can be represented by the function $S(t) = 110(1.07)^t$, where t represents the number of years since the study began. Write a function that could be used to determine the number of squirrels in this population t years since the study began, in terms of the **monthly** rate of growth.

This task is asking the students to generate an equivalent form of the given function, using a different time interval, months instead of years. It should be discussed that the growth of the population does not involve compounding monthly. Students can explore (e.g., table of values, graphically) the difference between $S(t) = 110(1.07)^t$ and $S(t) = 110\left(1 + \frac{0.07}{12}\right)^{12t}$. Students should see that the number of times a quantity compounds per year does have an effect on the future value.

The monthly growth rate can be determined by substituting $t=\frac{1}{12}$: $S\left(\frac{1}{12}\right)=110(1.07)^{\frac{1}{12}}$ {one month is $\frac{1}{12}$ of a year}
$(1.07)^{\frac{1}{12}} \approx 1.0057$, so the monthly growth rate is approximately 0.57%. To re-write an equivalent function, the annual growth factor is replaced with the monthly growth factor and the exponent is replaced with $12t$, since t is in years and the growth factor is in terms of months. (For each year, 12 monthly rates need to be applied.)
The final manipulated, but equivalent function is: $S(t) = 110(1.0057)^{12t}$
Students can compare $S(t) = 110(1.0057)^{12t}$ to the original $S(t) = 110(1.07)^t$ (e.g., table of values, graphically), noticing that both functions produce approximately the same result. Why is $(1.07)^t = (1.07^{\frac{1}{12}})^{12t}$?

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Compounding periods and percent rates of change that are based upon different units are explained in lesson 26 of EngageNY Algebra II Module 3.

Example 2: Given the half-life of C_{14} is 5760 years, the exponential model would be A=P$(0.5)^{\frac{t}{5760}}$, where t represents the time in years. Find the annual decay rate.

Using the properties of exponents, A=P$(0.5)^{\frac{t}{5760}}$=P $(0.5^{\frac{1}{5760}})^t$ ≈P $(0.99988)^t$. So, the decay rate is 1-0.99988=0.00012 or 0.012% per year.

Example 3: Gas prices are decreasing at a constant rate of 3.5% per week. If this decline continues, determine what the percent of decrease will be over the next year to the nearest tenth of a percent.

Using the formula for growth and decay, $A = P(1 \pm r)^t$, where t represents the time in terms of weeks, we can create the equations $A = P(1 - 0.035)^t$. As we are interested in the yearly decay rate, we would substitute $t = 52$ and evaluate $(1 - 0.035)^{52} \approx 0.1568$. Therefore, the percent of decrease can be found by solving the equation $1 - r = 0.1568$; $r = 0.8431 \approx 84.31\%$

Example 4: Taken from lesson 27 of EngageNY Algebra II Module 3.

The table below gives the average annual cost (e.g., tuition, room, and board) for four-year public colleges and universities.

1. Explain why a linear model might not be appropriate for this situation.

Year	Average Annual Cost
1981	$2,550
1991	$5,243
2001	$8,653
2011	$15,918

2. Write an exponential function to model this situation.

   If you calculate the growth factor every 10 years, you get the following values

$$1981 - 1991: \frac{5243}{2550} = 2.056$$
$$1991 - 2001: \frac{8653}{5243} = 1.650$$
$$2001 - 2011: \frac{15918}{8653} = 1.840$$

   The average of these growth factors is 1.85, so the average annual cost in dollars t years after 1981 is $C(t) = 2550(1.85)^{\frac{t}{10}}$.

3. Use the properties of exponents to rewrite the function from Problem 2 to determine an annual growth rate.

   We know that $2550(1.85)^{\frac{t}{10}} = 2250\left(1.85^{\frac{1}{10}}\right)^t$ and $1.85^{\frac{1}{10}} \approx 1.063$, thus the annual growth rate is 6.3%.

4. If this trend continues, when will the average annual cost exceed $35,000?

   We need to solve the equation $C(t) = 35000$ for t.

$$\mathbf{2550(1.85)^{\frac{t}{10}} = 35000}$$
$$\mathbf{(1.85)^{\frac{t}{10}} = 13.725}$$
$$\mathbf{\log\left((1.85)^{\frac{t}{10}}\right) = \log(13.725)}$$

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$\frac{\boldsymbol{t}}{\mathbf{10}} = \frac{\mathbf{log(13.725)}}{\mathbf{log(1.85)}}$ $\boldsymbol{t} = \mathbf{10}\left(\frac{\mathbf{log(13.725)}}{\mathbf{log(1.85)}}\right)$ $\boldsymbol{t} \approx \mathbf{42.6}$ The cost will exceed $\$\mathbf{35,000}$ after $\mathbf{43}$ years, in the year 2024.