New York State Next Generation Mathematics Learning Standards Unpacking Document (DRAFT)

<table>
<tr><td>Course:
Algebra I</td><td>DOMAIN: Algebra (A) – Seeing Structure in Expressions (SSE)</td></tr>
<tr><td colspan="2">CLUSTER: Interpret the structure of expressions.
Students will develop a precise understanding of what it means for expressions to be algebraically equivalent. Students will see expressions as the sum of terms and the products of factors that can be utilized to explain meaning in a given context.</td></tr>
<tr><td colspan="2">Grade Level Standard:
AI-A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★
a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.
b. Interpret expressions by viewing one or more of their parts as a single entity.
<u>Note</u>: AI-A.SSE.1b is a fluency recommendation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.</td></tr>
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<tr><td colspan="2">PERFORMANCE/KNOWLEDGE TARGETS
(measurable and observable)</td></tr>
<tr><td colspan="2">• Rewrite a polynomial in its standard form;
• identify the parts of an expression by utilizing appropriate vocabulary: terms, factors, coefficients, leading coefficient, constant, degree, and standard form;
• describe the effect/role of the parts of an expression; and
• explain how a single part of an expression relates to a given context.</td></tr>
<tr><td colspan="2">ASPECTS OF RIGOR
Procedural Conceptual Application</td></tr>
<tr><td>MATHEMATICAL PRACTICES</td><td>1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>FOUNDATIONAL UNDERSTANDING</td><td>NY-8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
NY-7.EE.1 Add, subtract, factor, and expand linear expressions with rational coefficients by applying the properties of operations.
NY-7.EE.2 Understand that rewriting an expression in different forms in real-world and mathematical problems can reveal and explain how the quantities are related.
NY-6.EE.3 Apply the properties of operations to generate equivalent expressions.
NY-6.EE.4 Identify when two expressions are equivalent.</td></tr>
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The following pages contain EXAMPLES to support current instruction of the content standard and may be used at the discretion of the teacher and adapted to best serve the needs of the learners in the classroom.

Example 1: Polynomials in Standard Form

Rewrite the polynomial $8x - x^3 + 4 - 2x^2$ in standard form.
Answer: $-x^3 - 2x^2 + 8x + 4$

To reinforce mathematical language and vocabulary, students can be presented with scenarios like the following:

a. Write a third-degree polynomial in standard form with a constant value of 4, a leading coefficient of 2, a linear term with a coefficient of -5, and a quadratic term with a coefficient of -3.

   Answer: $2x^3 - 3x^2 - 5x + 4$

b. Write an expression for a second-degree polynomial that when written in standard form has a constant value of -70, a lead coefficient of 2, and a linear term with a coefficient of -4.

   Answer: $2x^2 - 4x - 70$

Example 2: Interpreting the Meaning of the Individual Parts of the Expression

a. Toni has been recording the height of a plant for her science lab. She notices that the plant grows at a constant rate and has written an equation h(n)=1.5n+2 to represent the height of the plant (cm.) on the n^{th} day. What does 1.5 represent in the context of the plant growth? What does 2 represent in the context of the plant growth?

b. The height of a ball t seconds after it is thrown into the air from the top of a building can be modeled by $h(t) = -16t^2 + 48t + 64$, where $h(t)$ is height in feet. What does 64 represent in the context of the problem?

   By factoring completely and expressing the function as $h(t) = -16(t-4)(t+1)$, how does this help to interpret the meaning of the function?

c. Lester's height in inches for a year can be predicted by the equation $h(m) = 60(1.005)^m$ where *m* is the number of months. What does 60 represent in the context of the problem? What does the 1.005 represent in the context of the problem?

Example 3: Crafting and Interpreting Expressions
Although the standard focuses on the interpretation of the parts of an expression, a possible task might be to have students create an expression for a given context, allowing for the interpretation and the mathematical questions behind the expression to be self-generated. Crafting the expression allows students to define the roles of the individual parts of the expression, further developing conceptual understanding. The opportunity/ability to communicate the roles of the individual parts of the expression they have defined allows the development of mathematical language in the process, as well as seeing how different expressions can be utilized to represent the same context when they share their expressions with others. This work also supports Algebra I standards NY-A.SSE.3, NY-A.CED.1, NY-F.IF.8, NY-F.BF.1a and NY-F.LE.2. An example of a possible task follows, taken from the Poster Problem "How Do You Generate Equations from Patterns, SERP Strategic Education Research Partnership, math.serpmedia.org. (Content licensed under CC BY-NC-SA 4.0)

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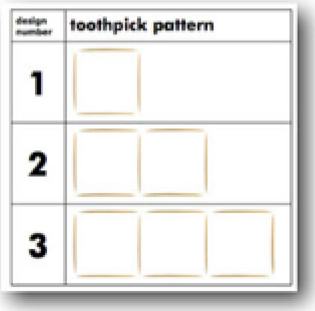
In the Poster Problem "Toothpick Patterns", students write symbolic expressions for the numbers of toothpicks in a sequence of designs. One strategy is to look for a pattern in the numbers, separate from the situation. Suppose the pattern is



{ 4, 7, 10, 13, ... }.

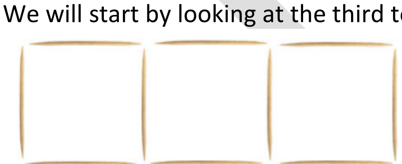
If you put it on a number line, it looks like this:

Students might notice a difference of three between any two adjacent numbers. This difference suggests that the solution will have something to do with multiplying by three. Encourage students to make a table rather than try to figure it out in their heads.

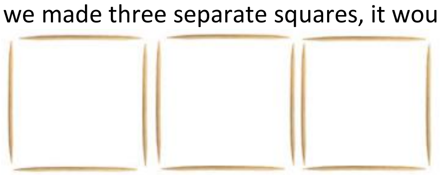
pattern (n)	toothpicks (T)	$3 \times n$
1	4	3
2	7	6
3	10	9
4	13	12
5		
n	?	$3n$

By making the table, students will see that the relationship jumps right out: the number of toothpicks (middle column) is one more than the right-hand, 3n column. So, the formula is T=3n+1. We developed that expression just by looking at the numbers. But doing so still requires the "leap" of deciding to multiply by three and gives no insight into what three has to do with the toothpicks. Why three? Which brings us to the other strategy: looking more deeply at the pattern in the toothpicks. There are many approaches here, all good. This is the point of the problem: that there are several good formulas, all of which lead to the same correct result. Encourage students to come up with as many as possible.

We will start by looking at the third toothpick pattern:



It looks like three attached squares.
If we made three separate squares, it would look like this:



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The three squares would require 12 toothpicks. In fact, if we made separate squares in every pattern, the formula would be easy: T=4n, because there are n squares and four toothpicks in each square. But the squares are not separate. How do we fix the formula? In our picture, you can see that there are two places with "double" toothpicks. To turn the second diagram into the first one, we must remove two toothpicks-one from each double. That gives us 10 toothpicks, which is correct.

How many toothpicks do we have to remove if we are looking at toothpick pattern with a different number of squares? One less than the number of squares. That is, the number of toothpicks is what we need for n squares, minus (n-1). That gives us the formula T=4n-(n-1).

Here is the third toothpick pattern again:

Let's look at it as two horizontal lines, with connectors like this:

Each horizontal line has n toothpicks, so the two horizontals are 2n (six in this case). How many vertical connectors? One more than the number in the horizontals or (n+1). This yields: T= 2n + (n+1).

Both of our formulas give the same result no matter what value you use for n. They are equivalent. And if you simplify them, you get T=3n+1 in each case.

Students can generate any equivalent expression for their formula but should be able to explain the parts of the expression and why it works for the pattern.