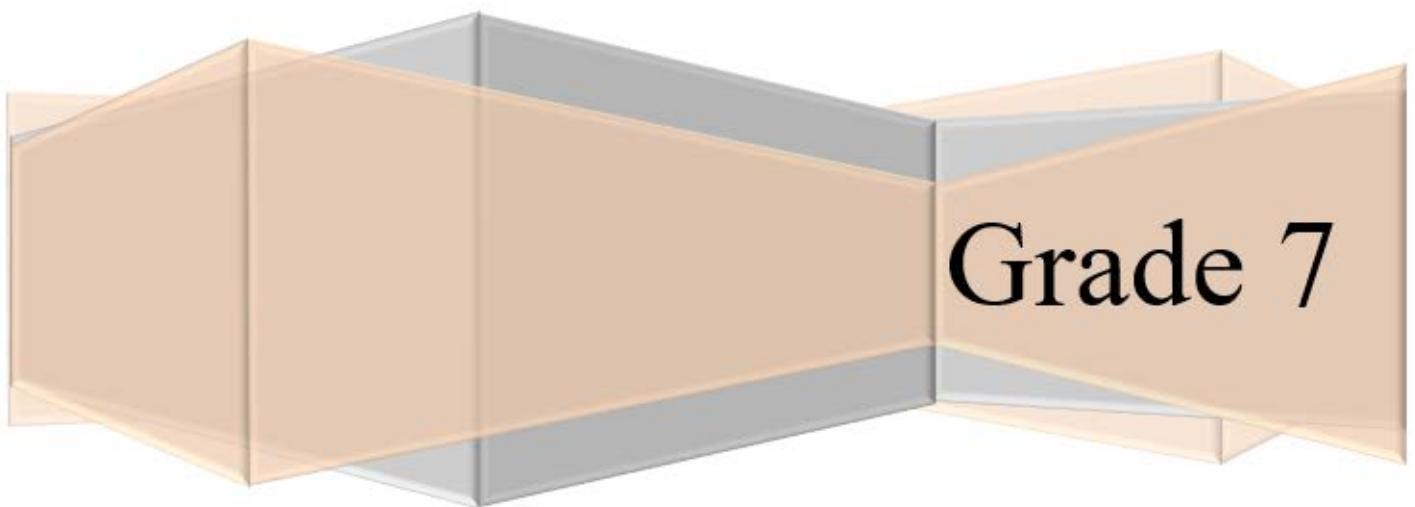


Scaffolding Instruction for All Students:

A Resource Guide for Mathematics



The University of the State of New York
State Education Department
Office of Curriculum and Instruction
and Office of Special Education
Albany, NY 12234



Scaffolding Instruction for All Students: A Resource Guide for Mathematics Grade 7

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Introduction

The Next Generation English Language Arts (ELA) and Mathematics Learning Standards intend to foster the 21st century skills needed for college and career readiness and to prepare students to become lifelong learners and thinkers. Learning standards provide the “destination” or expectation of what students should know and be able to do while teachers provide the “map” for getting there through high-quality instruction. **Lessons need to be designed to ensure accessibility to a general education curriculum designed around rigorous learning standards for all students, including students who learn differently (e.g., students with disabilities, English Language Learners (ELLs)/Multilingual Learners(MLLs), and other students who are struggling with the content).** It is vital that teachers utilize a variety of research-based instructional and learning strategies while structuring a student-centered learning environment that addresses individual learning styles, interests, and abilities present among the students in the class. Classrooms should be supportive and nurturing, and factors such as the age, academic development, English and home language proficiency, culture and background knowledge, and disability, should be considered when designing instruction. The principles of Universal Design for Learning should be incorporated into curricula to provide students with learning experiences that allow for multiple means of representation, multiple means of expression, and multiple means of engagement. These learning experiences will reduce learning barriers and foster equal learning opportunities for all students.

The purpose of these guides is to provide teachers with examples of scaffolds and strategies to supplement their instruction of ELA and mathematics curricula. Scaffolds are instructional supports teachers intentionally build into their lesson planning to provide students support that is “just right” and “just in time.” Scaffolds do not differentiate lessons in such a way that students are working on or with different ELA texts or mathematical problems. Instead, scaffolds are put in place to allow all students access to grade-level content within a lesson. Scaffolds allow students to develop the knowledge, skills, and language needed to support their own performance in the future and are intended to be gradually removed as students independently master skills.

The scaffolds contained in these guides are grounded in the elements of explicit instruction as outlined by Archer and Hughes (2011). Explicit instruction is a structured, systematic approach to teaching which guides students through the learning process and toward independent mastery through the inclusion of clear statements regarding the purpose and rationale for learning the new skill/content; explanations and demonstrations of the instructional target; and supported practice with embedded, specific feedback.

The scaffolds in these guides can be adapted for use in any curricula and across content areas. While the exemplars were all drawn from the ELA and mathematics [EngageNY](#) modules, teachers are encouraged to customize the scaffolds in any lesson they deem appropriate. **All teachers (e.g., general, special education, English as a New Language, and Bilingual Education teachers) can use these scaffolds in any classroom setting to support student learning and to make the general education curriculum more accessible to all students without interfering with the rigor of the grade-level content.**

How to Use These Guides

The provision of scaffolds should be thoughtfully planned as to not isolate or identify any student or group of students as being “different” or requiring additional support. Therefore, in the spirit of inclusive and culturally responsive classrooms, the following is suggested:

- Make scaffolded worksheets or activities available to all students.
- Heterogeneously group students for group activities when appropriate.
- Provide ELLs/MLLs with opportunities to utilize their home language knowledge and skills in the context of the learning environment.
- Make individualized supports or adapted materials available without emphasizing the difference.
- Consistently and thoughtfully use technology to make materials more accessible to all students.

In the ELA guides, the *Table of Contents* is organized to allow teachers to access strategies based on the instructional focus (reading, writing, speaking and listening, and language) and includes a list of scaffolds that can be used to address those needs. In the mathematics guides, the *Table of Contents* is organized around the scaffolds themselves.

Each scaffold includes a description of what the scaffold is, who may benefit, and how it can be implemented in a lesson-specific model (see graphic below). Teachers are encouraged to make changes to presentation and language to best support the learning needs of their students. While lessons from the [EngageNY](#) modules are used to illustrate how each scaffold can be applied, **the main purpose of the exemplars is to show how teachers can incorporate these scaffolds into their lessons as appropriate.**

Title of Scaffold
Module: Unit: Lesson:
<u>Explanation of scaffold:</u> This section provides a deeper explanation of the scaffold itself including what it is and how it can and should be used. This section is helpful when implementing the scaffold in other lessons.
<u>Teacher actions/instructions:</u> This section provides specific instructions for the teacher regarding successful implementation of the scaffold.
<u>Student actions:</u> This section describes what the students are doing during the scaffolded portion of the lesson.
<u>Student handouts/materials:</u> This section indicates any student-facing materials that must be created to successfully use this scaffold.

Warm-up Review

Exemplar from:

[Module 1: Topic A: Lesson 5](#): Identifying Proportional and Non-Proportional Relationships in Graphs

Explanation of scaffold:

This scaffold provides students with the opportunity to review previously learned skills and concepts that are needed to build a strong foundation for new lesson material. Establishing a warm-up review routine at the beginning of class allows students to connect with prior knowledge and allows teachers to quickly assess student understanding of key concepts, build automaticity and fluency of important skills and concepts, and give targeted corrective feedback.

Teacher actions/instructions:

A warm-up review can be used at the beginning of class to engage students and activate prior knowledge, before introducing a new lesson, when reteaching skills and concepts to small groups of students, and as homework for struggling students. The procedures of the routine for completion of a warm-up review should be explicitly taught to students at the beginning of the school year.

The following is a model of how a warm-up review could be used to focus on the prerequisite skills needed for this lesson:

1. Display a large version of the *Warm-up Review* sheet on chart paper or use a document camera to project your work. Hand out student copies.
2. Give students five minutes to complete warm-up.
 - Walk about the classroom and monitor student work.
 - Give corrective feedback to individual students as needed.
 - Give struggling students the option to work with a partner.
 - Remind students of information from previous lessons:
 - Two quantities are proportional to each other when there exists a constant (number) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
 - If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.
3. Review answers as a class.
 - Have students explain steps;
 - Review steps, but involve students by eliciting unison responses; or
 - Have students use thumbs up/thumbs down to indicate agreement/disagreement with answers. Have them explain why.

Answers to Problem 1:

- a. The relationship is proportional because when you multiply the value of the numerator and denominator of the first fraction by 7, the products equal the values in the second fraction.
- b. The relationship is non-proportional because you cannot multiply the value of 3 and 8 by the same number to get 21 and 41 ($3 \times 7 = 21$, but $8 \times 7 \neq 41$).

Answers to Problem 2:

- a. The table does not show a proportional relationship because when you divide the total price of the comic books by the number of comic books, the answer is not the same for each row of the table.
- b. The table does show a proportional relationship because when you divide the earnings by the number of cups sold, the answer (4) is the same for each row of the table.

Student actions:

Students complete the *Warm-up Review* sheet and participate in the warm-up review routine as directed.

Student handouts/materials:

Warm-up Review sheet (found on the next page)

Warm-up Review
Module 1, Topic A, Lesson 5: Identifying Proportional and Non-proportional Relationships in Graphs

Name _____ Date _____

1. Is the relationship between the two quantities proportional or non-proportional? Explain your reasoning.

a. $\frac{5}{4}$ and $\frac{35}{28}$ The relationship is **proportional/non-proportional** because _____

b. 3 : 8 and 21 : 41 The relationship is **proportional/non-proportional** because _____

2. Do the tables below show a proportional relationship? Why or why not?

a.

Number of Comic Books	Total Price (Dollars)
2	6
4	12
6	16
8	24

The table **does/does not** show a proportional relationship because _____

b.

Number of Cups Sold	Earnings (Dollars)
3	12
5	20
7	28
9	36

The table **shows/does not show** a proportional relationship because _____

Guided Notes with Partially Completed Problems

Exemplar from:

[Module 1: Topic A: Lesson 3](#): Identifying Proportional and Non-Proportional Relationships in Tables

Explanation of scaffold:

This scaffold supports students who require new information to be presented in smaller steps and increased opportunities to respond. It provides a structure in which difficult tasks are broken down and student practice is guided. When completed, guided notes with partially completed problems serve as a useful reference tool.

Teacher actions/instructions:

Guided notes with partially completed problems can be used with individuals, small groups, or the whole class when introducing a new skill or concept. It is best to use following a review of prerequisite skills or an opening activity and should be used in combination with teacher materials. Monitor student responses, adjust instruction as needed, and fade to independent practice of the skill being taught.

The following is a model of how guided notes with partially completed problems could be used to provide structure and guidance for introducing the identification of proportional and non-proportional relationships:

1. Hand out student copies of the *Guided Notes*.
2. Direct students to complete the second column of the table in the *Example* as indicated in the lesson. Monitor for accuracy and provide modeling or guidance as needed.
3. Guide students in completing the notes and partially completed problems from the classwork and discussion questions.
4. Monitor for accuracy and adjust instruction as needed.

Student actions:

Students participate in class discussion and complete the *Guided Notes* as directed.

Student handouts/materials:

Guided Notes (found on the following pages)

Guided Notes
Module 1, Topic A, Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

Name _____ Date _____

Classwork

Example

You have been hired by your neighbors to babysit their children on Friday night. You are paid \$8 per hour. Complete the table relating your pay to the number of hours you worked.

Hours Worked	Pay
1	
2	
3	
4	
$4\frac{1}{2}$	
5	
6	
6.5	

1. What did you need to do to complete the table? I _____ **the number of hours worked by the amount of dollars paid per hour, which is _____.**
2. How did you determine the pay for $4\frac{1}{2}$ hours? Show your work.

3. How could you use the information to determine the pay for a week in which you worked 20 hours?

_____ × _____ = \$ _____

4. Using the completed table, what is another way the answer can be determined?

Take the amount of money made from working 4 hours and multiply it by _____ or take the amount money made from working _____ hours and multiply it by 4.

5. If the quantities in the table were graphed, would the point (0, 0) be on that graph? What would it mean in the context of the problem?

Yes / No because if you multiply _____ by any c _____ number, you would get _____.

For this problem, the point (0, 0) represents working _____ hours and earning \$ _____.

6. Describe the relationship between the amount of money earned and the number of hours worked.

The two quantities are in a p _____ r _____. This relationship exists because when every measure of time is multiplied by the same number, the corresponding measures of pay are obtained (\$ _____ for every _____ hour worked).

7. How can multiplication and division be used to show the earnings are proportional to the number of hours worked?

Every measure of time (hours) can be _____ by the constant _____ to determine each measure of pay. Division can be used by _____ each measure of y (pay) by 8 to get the corresponding x (hours) measure.

8. Based on the table above, is the pay proportional to the hours worked? How do you know?

Yes / No because every ratio of the amount of pay to the number of hours worked is the _____. The ratio is _____ : _____, and every measure of hours worked _____ by 8 will result in the corresponding measure of pay.

$\frac{8}{1} = 8$ $\frac{16}{2} = \underline{\hspace{1cm}}$ $\frac{\square}{3} = 8$ $\frac{32}{\square} = 8$ $\frac{36}{\square} = 8$ $\frac{\square}{5} = \underline{\hspace{1cm}}$

$\frac{\square}{\square} = 8$ $\frac{\square}{\square} = \underline{\hspace{1cm}}$

Cooperative Learning

Exemplar from:

[Module 1: Topic C: Lesson 11](#): Ratios of Fractions and Their Unit Rates

Explanation of scaffold:

Cooperative learning includes those strategies where small groups of students contribute equally toward shared learning goals. This scaffold provides students of different ability levels an opportunity to engage with, assist, and learn from their peers. It motivates students to take responsibility for their own learning and can be used in any lesson to support students while they improve their understanding of a concept or skill without changing the rigor of the content.

Teacher actions/instructions:

Cooperative learning can be used at almost any point of instruction, but in mathematics, it is most beneficial after material has been presented by the teacher. This means it is best used when students are reviewing and practicing concepts or skills to reinforce their learning. It can also be used to assess student learning in the form of group or team projects and tests.

Although instructions will vary depending on the cooperative learning strategy being used, specific directions and explicit expectations should always be provided to minimize off-topic conversations and other distracting behaviors. Student groups or teams should be thoughtfully assigned and mixed heterogeneously by ability.

The following is a model of how cooperative learning could be used in this lesson:

1. Read and display a large version of *Example 1: Who is Faster?* (see *Cooperative Learning/Group Work* sheet on page 11) or use a document camera to project your work, and hand out student copies.
2. Fill in the cells of the table as a class, using information from the problem. Lead a class discussion to answer the questions presented on pages 106-107 of the *Teacher Version* of the lesson, and model how to solve the problem using the formula $d = rt$ as indicated on page 107.
3. Tell the class there are a variety of other ways to represent the problem, including bar models, double number lines, and clock diagrams. Review these representations to be sure students remember how to use them. Tell the class they will be working in groups to use one of these approaches to represent and solve the problem (examples for teacher reference are on pages 107-109 of the *Teacher Version* of the lesson).
4. Split the class into mixed-ability groups and assign each group to one of the three types of representations. Provide graph paper and rulers for students in the bar model and number line groups, and provide rulers and circles drawn on graph paper (clock diagram template) for the clock diagram groups.
5. Give groups about 15 minutes to work together to figure out how to represent and solve the problem using their assigned type of representation. Circulate around the classroom to answer students' questions and provide guidance as needed.

6. Give each group about three minutes to present their problem-solving strategy and solution to the class.
7. When presentations are finished, ask the class: “After seeing various representations, which do you think was the most useful or efficient? Why? If you were given a similar problem, which strategy would you use to solve it?” Elicit student responses as needed.

Student actions:

Students work with their classmates to solve the problem using the assigned strategy and participate in class discussions.

Student handouts/materials:

Cooperative Learning/Group Work sheet (found on the next page)

Graph paper

Rulers

Clock Diagram Templates (found on page 12)

Cooperative Learning/Group Work
Module 1, Topic C, Lesson 11: Ratios of Fractions and Their Unit Rates

Name _____ Date _____

Example 1: Who is faster?

During their last workout, Izzy ran $2\frac{1}{4}$ miles in 15 minutes, and her friend Julia ran $3\frac{3}{4}$ miles in 25 minutes. Each girl thought she was the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

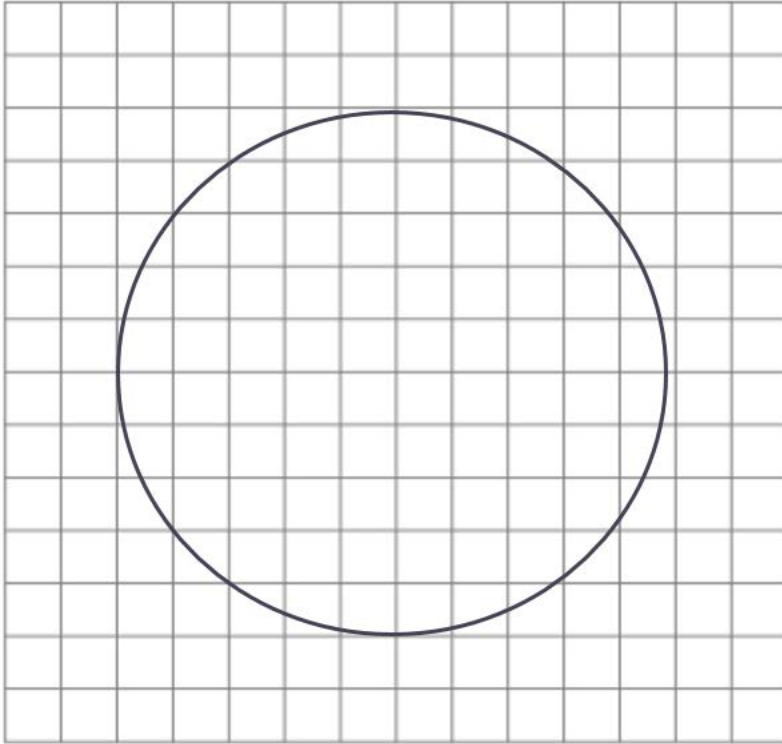
Tables

IZZY		
Time (minutes)	Time (hours)	Distance (miles)

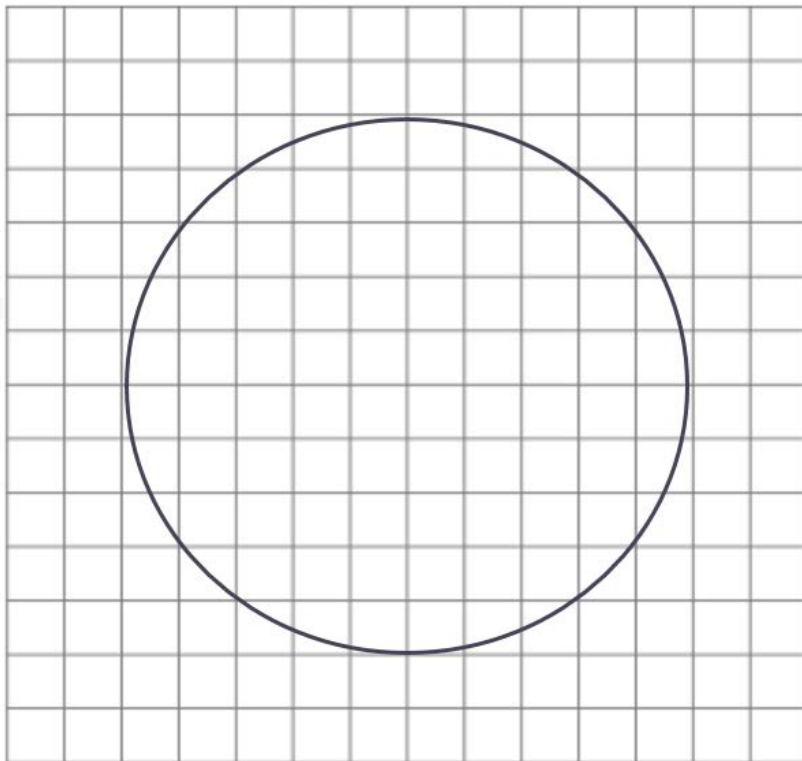
JULIA		
Time (minutes)	Time (hours)	Distance (miles)

Clock Diagram Templates

IZZY



JULIA



Instruction with Computer Technology

Exemplar from:

[Module 1: Topic D: Lesson 19](#): Computing Actual Areas from a Scale Drawing

Explanation of scaffold:

Instruction with computer technology involves using computer programs and websites to increase academic engagement and reinforce understanding of concepts. This scaffold provides visual and conceptual support for students who need additional models and practice opportunities to learn new information. Videos and game applications are an engaging way for students to interact with new information, practice skills, and receive immediate feedback. Guided notes or checkout activities can be used to assist students in attending and allow teachers to check for understanding. These notes or activities can also serve as reference tools for students.

Teacher actions/instructions:

Instruction with computer technology is beneficial when introducing or reteaching a concept or skill. It can be used as a homework assignment, during whole class instruction with a smart board, or during small group or individual instruction using computers. Ensure students have the prerequisite skills for operating computers and navigating the internet. Ensure websites are accessible to all students and assistive technology needs are satisfied.

The following is a model of how instruction with computer technology could be used to complement this lesson by reinforcing students' ability to identify a scale factor and interpret a scale drawing:

1. Hand out student copies of the *Guided Notes*.
2. Access the Khan Academy video, [Solving a Scale Drawing Word Problem](#).
3. Pause the video as needed to allow students to fill in their guided notes.
4. Carefully monitor students' activities to confirm on-task behaviors.

Student actions:

Students view the video and complete the *Guided Notes* as directed.

Student handouts/materials:

Computer access

Guided Notes (found on the following pages)

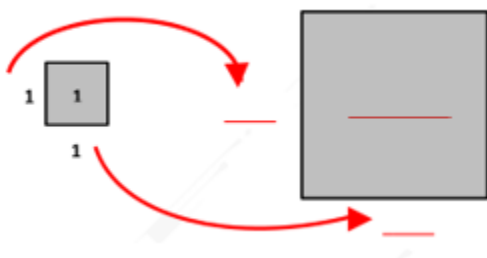
Guided Notes
Solving a Scale Drawing Word Problem

Name _____ Date _____

Underline the important information and circle the question.

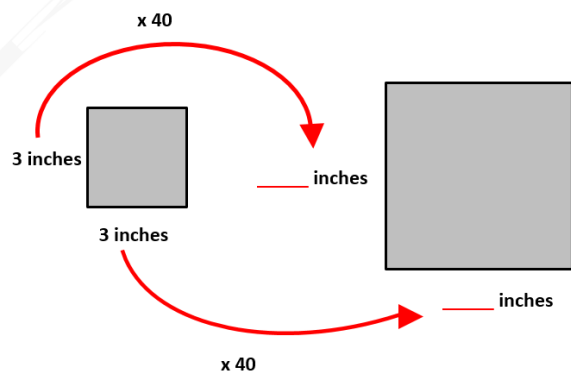
The area of the actual dining room is 1600 times larger than the area of the dining room on the blueprint. The length of the dining room on the blueprint is 3 inches. What is the length of the actual dining room in feet?

- Complete the scale factor and area of the actual dining room:



- The scale factor is _____

- Use the scale factor to find the length of the actual dining room in inches:



- Convert the length of the dining room in inches to feet:

$$\frac{120}{\boxed{}} = \underline{\hspace{2cm}} \text{ feet}$$

- The length of the actual dining room in feet is _____ feet.

Check for Understanding

Sally creates another blueprint where the area of the actual dining room is 900 times larger than the area of the dining room on the blueprint. The length of the dining room on the blueprint is 5 inches. What is the length of the actual dining room in feet? Show your work.

The length of the actual dining room is _____ feet.

Frayer Model

Exemplar from:

[Module 1: Topic A: Lesson 2](#): Proportional Relationships

Explanation of scaffold:

The Frayer model is a graphic organizer that can be used in any lesson to help students understand unfamiliar vocabulary, including mathematical terms. This four-square model includes a student-friendly definition, a description of important characteristics, examples, and nonexamples. It provides a format to organize information and visual representations of the mathematical term being defined. Developing vocabulary skills is essential for students as they learn to “speak mathematically” and develop their abstract reasoning and problem-solving skills. The term *proportional* is used to demonstrate how to apply this strategy when working with students.

Teacher actions/instructions:

Select key mathematical terms. These terms should be limited in number and essential to developing a deeper understanding of the mathematical concepts or skills in the lesson.

Instruct students to complete Frayer models as follows:

1. Write the mathematical term in the middle circle.
2. Define the term, using student-friendly language, in the **Definition** box. Use your own words.
3. Write words to describe the term in the **Characteristics** box. Again, use your own words.
4. List examples of the definition in the **Examples** box. Draw a picture and/or write an equation to help you understand the term if needed.
5. List nonexamples of the definition in the **Nonexamples** box. Again, draw a picture and/or write an equation if needed.
6. Test yourself.
 - The study step is critical to student success in using vocabulary strategies such as the Frayer model. Students need to study the terms to internalize them for later use.
 - Students can quiz each other during “down times,” or the models/cards can be used as part of a center activity.

Student actions:

Students work either individually or in pairs to make and study Frayer models.

Student handouts/materials:

Frayer Model template (found on page 18)

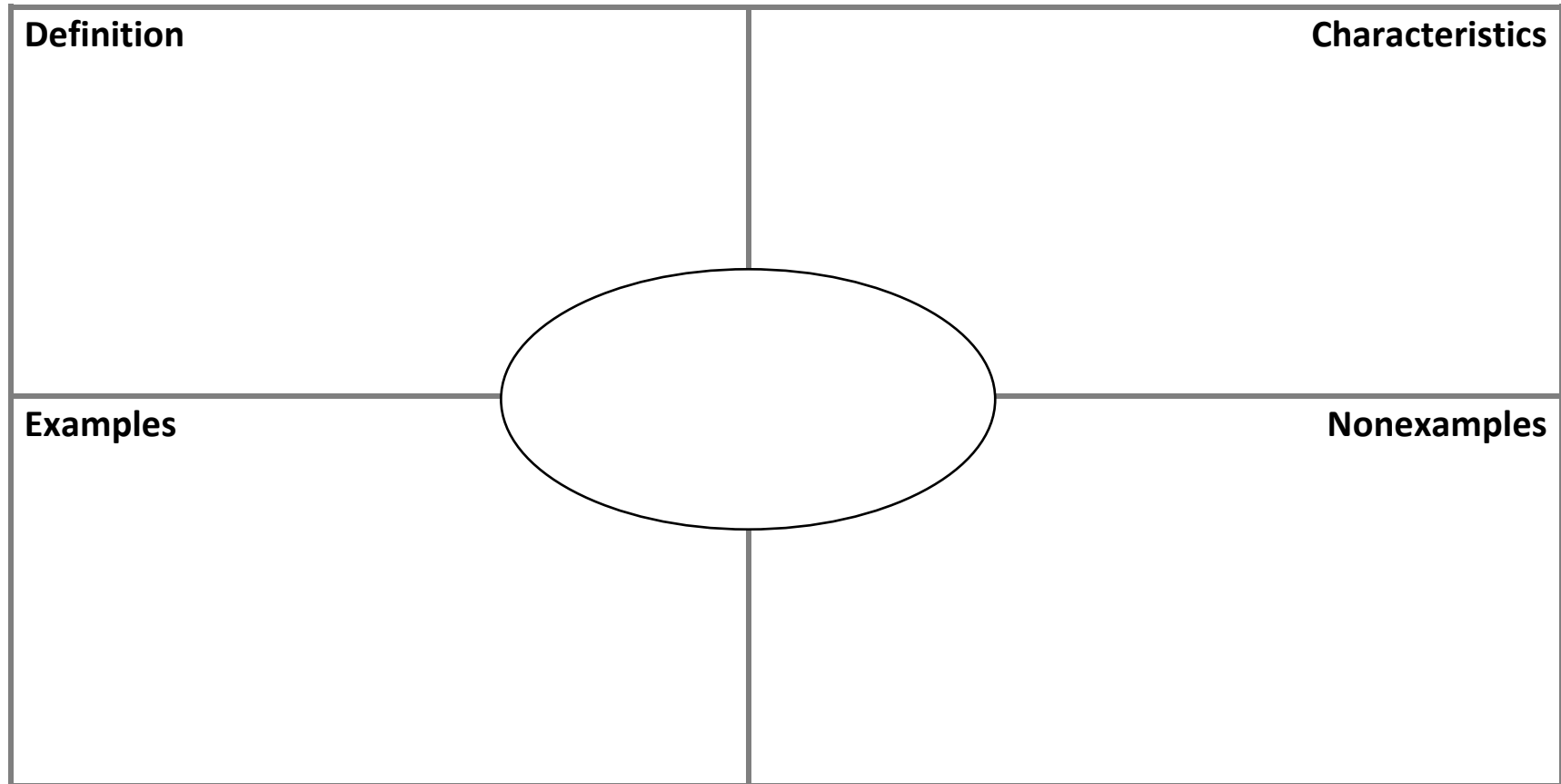
NAME: _____

Frayer Model (example)

<p>Definition</p> <p>Two quantities are proportional to each other if there is one constant number that is multiplied by each measure in the first quantity to give the corresponding measure in the second quantity.</p> <p>If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.</p>	<p style="text-align: right;">Characteristics</p> <p>Each measure of x multiplied by the constant (k) gives the corresponding y-value</p> <p>k is a positive constant</p> <p>$y = kx$</p>																								
<div style="border: 1px solid black; border-radius: 50%; width: 150px; height: 100px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> <h2 style="margin: 0;">Proportional</h2> </div>																									
<p>Examples</p> <p>*Miles driven is proportional to gallons of gasoline consumed.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">Gallons Consumed</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">9</td> <td style="padding: 5px;">11</td> </tr> <tr> <td style="padding: 5px;">Miles Driven</td> <td style="padding: 5px;">44</td> <td style="padding: 5px;">110</td> <td style="padding: 5px;">176</td> <td style="padding: 5px;">198</td> <td style="padding: 5px;">242</td> </tr> </table> <p style="text-align: center; margin-top: 10px;"> $\frac{44}{2} = 22$ $\frac{110}{5} = 22$ $\frac{176}{8} = 22$ $\frac{198}{9} = 22$ $\frac{242}{11} = 22$ </p>	Gallons Consumed	2	5	8	9	11	Miles Driven	44	110	176	198	242	<p>Nonexamples</p> <p>*Miles driven is <u>not</u> proportional to gallons consumed.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">Gallons Consumed</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">9</td> <td style="padding: 5px;">11</td> </tr> <tr> <td style="padding: 5px;">Miles Driven</td> <td style="padding: 5px;">44</td> <td style="padding: 5px;">120</td> <td style="padding: 5px;">208</td> <td style="padding: 5px;">189</td> <td style="padding: 5px;">275</td> </tr> </table> <p style="text-align: center; margin-top: 10px;"> $\frac{44}{2} = 22$ $\frac{120}{5} = 24$ $\frac{208}{8} = 26$ $\frac{189}{9} = 21$ $\frac{275}{11} = 25$ </p>	Gallons Consumed	2	5	8	9	11	Miles Driven	44	120	208	189	275
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NAME: _____

Fruyer Model



Evidence of Effectiveness

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