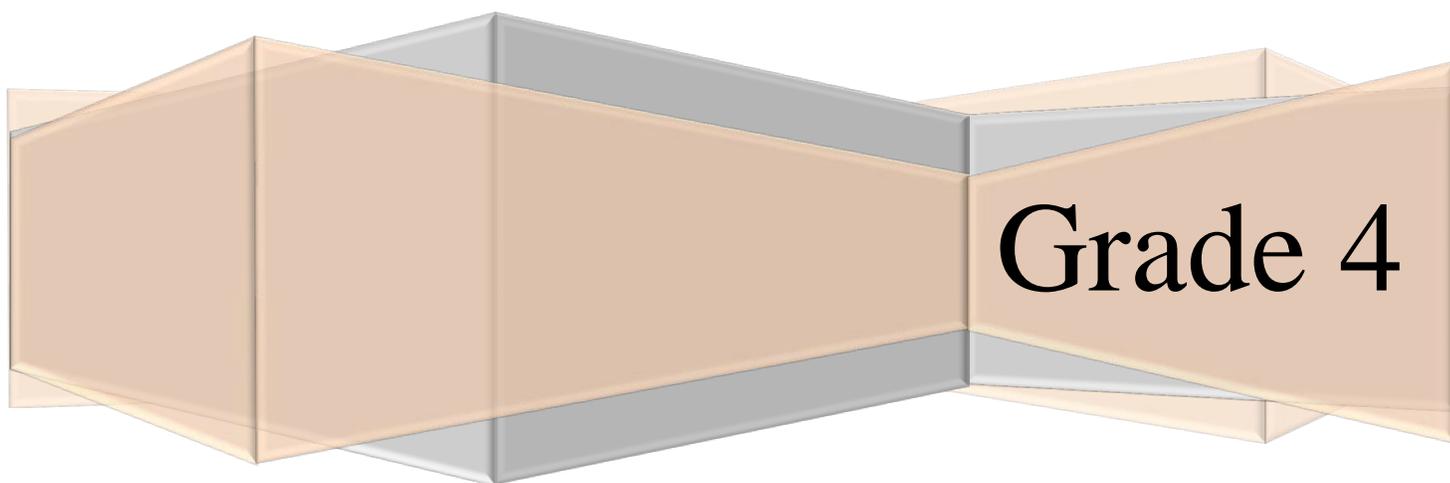


# **Scaffolding Instruction for All Students:**

**A Resource Guide for Mathematics**



The University of the State of New York  
State Education Department  
Office of Curriculum and Instruction  
and Office of Special Education  
Albany, NY 12234



# **Scaffolding Instruction for All Students: A Resource Guide for Mathematics Grade 4**

## **Acknowledgements**

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## Introduction

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The Next Generation English Language Arts (ELA) and Mathematics Learning Standards intend to foster the 21<sup>st</sup> century skills needed for college and career readiness and to prepare students to become lifelong learners and thinkers. Learning standards provide the “destination” or expectation of what students should know and be able to do while teachers provide the “map” for getting there through high-quality instruction. **Lessons need to be designed to ensure accessibility to a general education curriculum designed around rigorous learning standards for all students, including students who learn differently (e.g., students with disabilities, English Language Learners (ELLs)/Multilingual Learners(MLLs), and other students who are struggling with the content).** It is vital that teachers utilize a variety of research-based instructional and learning strategies while structuring a student-centered learning environment that addresses individual learning styles, interests, and abilities present among the students in the class. Classrooms should be supportive and nurturing, and factors such as the age, academic development, English and home language proficiency, culture and background knowledge, and disability, should be considered when designing instruction. The principles of Universal Design for Learning should be incorporated into curricula to provide students with learning experiences that allow for multiple means of representation, multiple means of expression, and multiple means of engagement. These learning experiences will reduce learning barriers and foster equal learning opportunities for all students.

The purpose of this guide is to provide teachers with examples of scaffolds and strategies to supplement their instruction of ELA and mathematics curricula. Scaffolds are instructional supports teachers intentionally build into their lesson planning to provide students support that is “just right” and “just in time.” Scaffolds do not differentiate lessons in such a way that students are working on or with different ELA texts or mathematical problems. Instead, scaffolds are put in place to allow all students access to grade-level content within a lesson. Scaffolds allow students to develop the knowledge, skills, and language needed to support their own performance in the future and are intended to be gradually removed as students independently master skills.

The scaffolds contained in this guide are grounded in the elements of explicit instruction as outlined by Archer and Hughes (2011). Explicit instruction is a structured, systematic approach to teaching which guides students through the learning process and toward independent mastery through the inclusion of clear statements regarding the purpose and rationale for learning the new skill/content; explanations and demonstrations of the instructional target; and supported practice with embedded, specific feedback.

The scaffolds in this guide can be adapted for use in any curricula and across content areas. While the exemplars were all drawn from the ELA and mathematics [EngageNY](#) modules, teachers are encouraged to customize the scaffolds in any lesson they deem appropriate. **All teachers (e.g., general, special education, English as a New Language, and Bilingual Education teachers) can use these scaffolds in any classroom setting to support student learning and to make the general education curriculum more accessible to all students without interfering with the rigor of the grade-level content.**

## How to Use This Guide

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The provision of scaffolds should be thoughtfully planned as to not isolate or identify any student or group of students as being “different” or requiring additional support. Therefore, in the spirit of inclusive and culturally responsive classrooms, the following is suggested:

- Make scaffolded worksheets or activities available to all students.
- Heterogeneously group students for group activities when appropriate.
- Provide ELLs/MLLs with opportunities to utilize their home language knowledge and skills in the context of the learning environment.
- Make individualized supports or adapted materials available without emphasizing the difference.
- Consistently and thoughtfully use technology to make materials more accessible to all students.

In the ELA guides, the *Table of Contents* is organized to allow teachers to access strategies based on the instructional focus (reading, writing, speaking and listening, and language) and includes a list of scaffolds that can be used to address those needs. In the mathematics guides, the *Table of Contents* is organized around the scaffolds themselves.

Each scaffold includes a description of what the scaffold is, who may benefit, and how it can be implemented in a lesson-specific model (see graphic below). **The scripts provided are only for demonstrating what a scaffold might look like in action.** Teachers are encouraged to make changes to presentation and language to best support the learning needs of their students. While lessons from the [EngageNY](#) modules are used to illustrate how each scaffold can be applied, the main purpose of the exemplars is to show how teachers can incorporate these scaffolds into their lessons as appropriate.

<b>Title of Scaffold</b> <b>Module: Unit: Lesson:</b>
<b>Explanation of scaffold:</b> This section provides a deeper explanation of the scaffold itself, including what it is and how it can and should be used. This section is helpful when implementing the scaffold in other lessons.
<b>Teacher actions/instructions:</b> This section provides specific instructions for the teacher regarding successful implementation of the scaffold.
<b>Student actions:</b> This section describes what the students are doing during the scaffolded portion of the lesson.
<b>Student handouts/materials:</b> This section indicates any student-facing materials that must be created to successfully use this scaffold.

## Graphic Organizer (*RDW (Read, Draw, Write) Template*)

### Exemplar from:

[Module 1: Topic A: Lesson 1](#): Application Problem

### Explanation of scaffold:

The *RDW Template* is a graphic organizer that can be used to support students who have difficulty organizing information and recalling multistep, problem-solving strategies. This template is intended to help students keep track of the steps involved in the RDW process that can be used to solve real world/application word problems. Some students may need additional scaffolding and explicit instruction to use this tool to structure their work to solve a mathematical problem. The following example shows one way to instruct students who need modeling and guided practice to use this tool and learn this new problem-solving strategy. Although the application problem in this lesson is used as an exemplar, graphic organizers such as the *RDW Template* can be used in any lesson to support students while learning a multistep problem-solving strategy without changing the rigor of the content.

### Teacher actions/instructions:

Instruct students in the use of the RDW process and completion of the RDW template to solve a mathematical problem as follows:

1. Read the problem.
2. Draw and label. Use a tape diagram, number bond, or array to make your drawing. Ask yourself, “What do I know? What do I need to find? How can I draw what I’m looking for?” Label your drawing.
3. Read the problem again.
4. Write an equation. Look at the evidence in your drawing, write an equation, and solve the problem.
5. Write a word sentence.

**For students who require explicit instruction on how to use the RDW process to solve a mathematical problem and complete the *RDW Template*, the following sample *script* is provided to demonstrate one way instruction *might* look like:**

#### **Step 1: Read the problem.**

**T (teacher):** *We are going to use the Read, Draw, Write, or RDW, strategy to help us solve problems. We are going to use the RDW Template to gather the evidence, or important parts, from the word problem that we need to answer the question and solve the problem.*

Display a large version of the *RDW Template* on chart paper or use a document camera to project your work. Hand out student copies and graph paper, and direct students to complete their *RDW Templates* to solve the problem as demonstrated.

Display the word problem:

**Ben has a rectangular area 9 meters long and 6 meters wide. He wants a fence that will go around it as well as grass sod to cover it. How many meters of fence will he need? How many square meters of grass sod will he need to cover the entire area?**

**T:** *The first step is **read**. That means I have to read the problem. What is step 1?*

**S (student):** *Read.*

**T:** *The problem says, “Ben has a rectangular area 9 meters long and 6 meters wide. He wants a fence that will go around it as well as grass sod to cover it. How many meters of fence will he need? How many square meters of grass sod will he need to cover the entire area?” I read the problem, so I can put a check in the box on my RDW Template.*

**Step 2: Draw and label.**

**T:** *Step 2 is **draw and label**. What is step 2?*

**S:** *Draw and label.*

**T:** *I need to draw a picture to help me solve the problem. I need to ask myself, “What do I know?” I know from my reading that the problem says that Ben has a rectangular area, or space, that is 9 meters long and 6 meters wide. What should I draw?*

**S:** *A rectangle.*

**T:** *That’s correct. I am going to draw a rectangle. I also must remember to label the units, so I don’t forget what I’m drawing. Since the problem says the rectangle is 9 meters long and 6 meters wide, I’m going to use graph paper to draw a rectangle with nine columns across and 6 rows down. Each box within the rectangle formed represents one square meter. I have to write “9 meters” next to the long sides and “6 meters” next to the short sides of the rectangle. I wrote “m” next to the 9 and 6 because “m” is the abbreviation for meter.*

**Step 3: Read the problem again.**

**T:** *What is step 3?*

**S:** *Read again.*

**T:** *Let’s read the problem together. [Chorally read the problem with students.] The problem asks how many meters of fence are needed to go around the area, or space, and how many square meters of grass sod are needed to cover the space. We read the problem again, so we can put a check in the box.*

We labeled the outside of the rectangle which will help us with the fence. Where does a fence go?

**S:** Around the edge.

**T:** Right. The distance around the edge of a two-dimensional shape, such as a rectangle, is called the \_\_\_\_\_.

**S:** Perimeter.

**T:** Right again. Let's look at the next question in the problem. Where does the grass sod go?

**S:** Inside the rectangle.

**T:** What is the amount of space inside a two-dimensional shape called?

**S:** The area.

**T:** Excellent job!

#### **Step 4: Write an equation.**

**T:** What is Step 4?

**S:** Write an equation.

**T:** We have to look at our drawings and see if we can use the evidence to write equations. First, we need to figure out how many meters of fence are needed for the perimeter. How do we find the perimeter of the rectangle?

**S:** Add all the sides together. [Write  $9 + 9 + 6 + 6 = 30$  on the RDW Template.]

**T:** Remember, we need to know how many meters of fence are needed, so let's write the answer to the equation as "30m of fence." Did we answer the first question?

**S:** Yes. We found how many meters of fence are needed to go around the space.

**T:** What do we need to find next?

**S:** We need to figure out how many square meters of grass sod are needed to cover the area.

**T:** What formula do I need to use to find the area of a rectangle?

**S:** Area = Length x Width. [Write  $9 \times 6 = 54$  on the RDW Template.]

**T:** *Since we need to know how many square meters of grass sod are needed, let's write the answer to this equation as "54sq m of grass sod." Did we answer the second question in the problem?*

**S:** *Yes. We found how many square meters of grass sod are needed to cover the area.*

**Step 5: Write a word sentence.**

**T:** *What is step 5?*

**S:** **Write a word sentence.**

**T:** *Finally, we have to write two sentences to answer each question. We have to remember to include all the information to tell the whole story. To tell how many meters of fence are needed to go around the perimeter of the rectangle, we will write, "Ben needs \_\_\_\_\_."*

**S:** *30 meters of fence.*

**T:** *Correct. Let's write this sentence. Now, tell me what the sentence needed to answer the second question should say, and write it on your RDW Template.*

**S:** *"Ben needs 54 square meters of grass sod."*

**T:** *Great job! Remember, we are going to use RDW when we need to solve word problems.*

As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Once students demonstrate the ability to use the RDW process with limited prompting, provide multiple, independent practice opportunities to ensure success.

**Student actions:**

Students chorally respond and complete the *RDW Template*. Students may work in pairs or small groups to complete additional practice problems if needed.

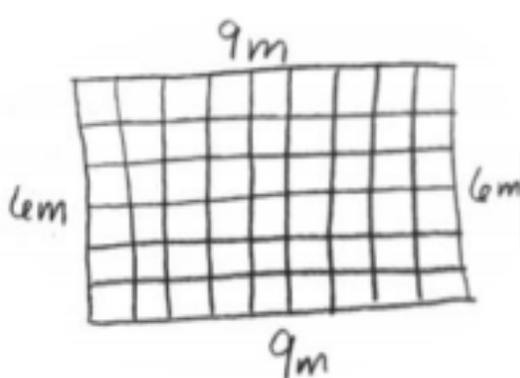
**Student handouts/materials:**

*RDW Template* (found on page 6)

Graph paper

NAME: \_\_\_\_\_

### RDW Template (example)

<b>Read</b>	<p>Make a ✓ after you read the problem.</p> <div style="text-align: center;"><input checked="" type="checkbox"/></div>
<b>Draw and label</b>	<p>Draw a picture and label it.</p> 
<b>Read again</b>	<p>Make a ✓ after you read the problem again.</p> <div style="text-align: center;"><input checked="" type="checkbox"/></div>
<b>Write</b>	<p style="text-align: center;">Write an equation.</p> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="text-align: right;"> <math display="block">\begin{array}{r} 9 \\ 9 \\ 6 \\ +6 \\ \hline \end{array}</math> </div> <div style="text-align: center;"> <math display="block">+ \begin{array}{r} 18 \\ 12 \\ \hline 30 \end{array}</math> <p>30m of fence</p> </div> <div style="text-align: left;"> <p><math>9 \times 6 = 54 \text{sq m of grass sod}</math></p> </div> </div> <p><math>9+9+6+6=30</math></p>
<b>Write</b>	<p>Write a sentence.</p> <p style="font-family: cursive;">Ben needs 30 meters of fence.</p> <p style="font-family: cursive;">Ben needs 54 square meters of grass.</p>

NAME: \_\_\_\_\_

### RDW Template

<b>Read</b>	Make a ✓ after you read the problem. <input data-bbox="863 319 922 373" type="checkbox"/>
<b>Draw and label</b>	Draw a picture and label it.
<b>Read again</b>	Make a ✓ after you read the problem again. <input data-bbox="863 1136 922 1190" type="checkbox"/>
<b>Write</b>	Write an equation.
<b>Write</b>	Write a sentence.

## Concrete-Representational-Abstract (CRA)

### Exemplar from:

[Module 1: Topic B: Lesson 5](#): Concept Development

### Explanation of scaffold:

CRA is a three-part instructional strategy in which the teacher begins by modeling and thinking aloud with **concrete** objects (e.g., blocks, disks, etc.), and then progresses to **representing** the concrete objects with drawings. The final level is the **abstract** level, where only numbers and mathematical symbols are used to complete the algorithm. Although the following exemplar connects to and uses the concept development section in this lesson as an exemplar, CRA is a method that can be used in any lesson when teaching abstract concepts that are difficult for students to understand.

### Teacher actions/instructions:

Provide student partners with place value disks and the *Comparison Place Value Chart (labeled)*. The place value charts should be put in plastic sleeves, so students are able to write on them and erase during lessons. Direct students to follow along as you model how to use these manipulatives. Use a document camera to project your work. As students gain competence in comparing numbers, fade to using drawings on labeled and then unlabeled place value charts, and finally to writing numerals on an unlabeled place value chart.

For students who require explicit instruction on how to use the materials provided to compare numbers, the following sample script (based on the language found in the concept development section of Module 1, Topic B, Lesson 5) is provided to demonstrate one way instruction might look like:

### **Problem 1: Comparing two numbers with the same largest unit.**

#### **Concrete**

Students may need practice using a place value chart and place value disks to represent multi-digit numbers prior to working on comparison problems.

Display: 3,010 ○ 2,040.

**T (teacher):** *Today, we are going to be comparing numbers. First, we will use our place value disks, and then we will use drawings to help us. Look at these two numbers. Let's say the standard form of each one together. [Chorally say each number with the students.]*

*Let's use our place value disks to show these two numbers on our comparison charts. [Place disks on the chart. An exemplar can be found on page 11] Look at the place value chart. What is the name of the largest unit in the number 3,010?*

**S (student):** Thousands.

**T:** That is correct. What is the name of the largest unit in the number 2,040?

**S:** Thousands.

**T:** Right. Let's put a circle around the word "thousands" since this is the unit we are going to use to compare the value of the numbers. If we look at the thousands and count the place value disks, we count 1-2-3 in the top row and 1-2 in the bottom row. On the red lines below these disks, write the number of disks you just counted. [Write down the corresponding numbers.] Which is greater, 3 thousands or 2 thousands? Tell your partner.

**S:** Three thousands.

**T:** There are no disks in the hundreds place, so let's write a 0 here. Let's also write a 0 under the ones place for each number since there are no disks there either. Now, look at the tens place, and count the place value disks. Remember to write the number of disks you count on the red lines. There is 1 in the top row and 1-2-3-4 in the bottom row. Which is greater, 1 ten or 4 tens? Tell your partner.

**S:** Four tens.

**T:** Tell your partner what would happen if we had compared the tens instead of the thousands.

**S:** We would say 2,040 is greater than 3,010.

**T:** That would not be right because \_\_\_\_\_.

**S:** Thousands are greater than tens.

**T:** You got it! Since thousands is the largest common unit, and 3 thousand is greater than 2 thousand [point to the corresponding place value disks], we say that 3,010 is greater than 2,040. Write the comparison statement at the bottom of your place value charts. [Write the comparison statement  $3,010 > 2,040$ .] Tell your partner how to say this comparison statement.

**S:** 3,010 is greater than 2,040.

**T:** We can write another comparison statement for these two numbers,  $2,040 < 3,010$ . Write this comparison statement on your place value charts. Tell your partner how to say this comparison statement.

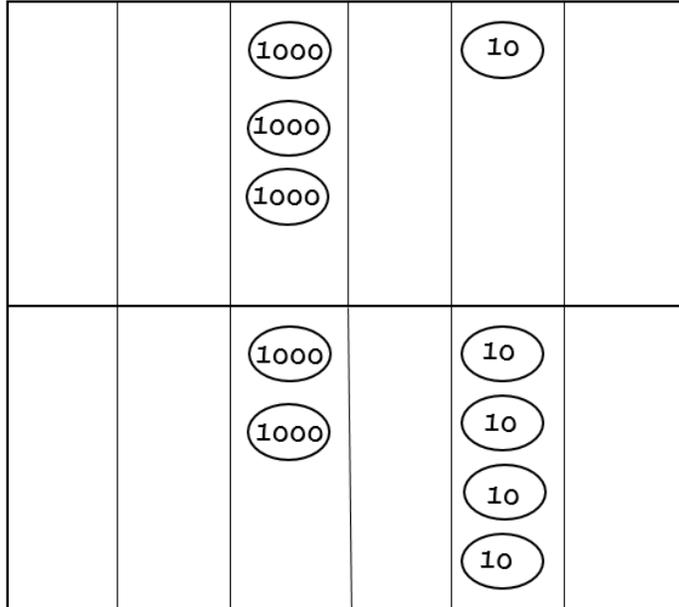
**S:** 2,040 is less than 3,010.

## Representational

A sample script on how to do the previous problem in a representational form can be found in the conceptual development section of this lesson. Students will represent the amounts of units for the two given numbers with either circles or dots. The method of comparison is similar to the previous concrete script. Students may start out with a labeled place value chart, and then progress to an unlabeled place value chart, similar to what is seen in the lesson.

Comparison Place Value Chart (labeled)





### Abstract

Utilize writing numerals in expanded form to help students transition from the pictorial to the abstract. The following sample script is provided to show one way instruction might look like.

**T:** Let's write both of our numbers in expanded form.

$$3,010 = \underline{\hspace{2cm}}$$

$$2,040 = \underline{\hspace{2cm}}$$

**S:** [Using whiteboards]

$$3,010 = 3,000 + 10$$

$$2,040 = 2,000 + 40$$

**T:** Let's circle the digits of the numbers that we are going to use in order to make our comparison.  $3,000 > 2,000$ , so that must mean that  $3,010$  \_\_\_\_  $2,040$ .

**S:**  $3,010 > 2,040$

**T:** Excellent. We can also write  $2,040 < 3,010$  because  $2000 < 3000$ .

Repeat the comparison process using examples provided throughout the lesson, such as 43,021 and 45,302; 2,305 and 2,530; and 970,461 and 907,641. Allow students to transition from the concrete, to the representational, to the abstract for each. As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Have students use an unlabeled hundred thousands place value chart (see page 93 of the module lesson) if appropriate.

**Student actions:**

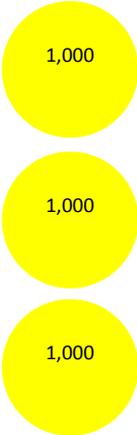
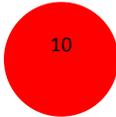
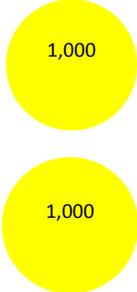
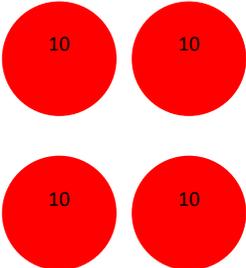
Students chorally respond, use place value disks and drawings to model comparisons on place value charts, and work in pairs to complete problems.

**Student handouts/materials:**

Place value disks

*Comparison Place Value Chart (labeled)* (found on page 13)

Comparison Place Value Chart (labeled) (example)

hundred thousands	ten thousands	thousands	hundreds	tens	ones
		 <u>3</u>	<u>0</u>	 <u>1</u>	<u>0</u>
		 <u>2</u>	<u>0</u>	 <u>4</u>	<u>0</u>

3,010  $>$  2,040 or 2,040  $<$  3,010

### Comparison Place Value Chart (labeled)

hundred thousands	ten thousands	thousands	hundreds	tens	ones

\_\_\_\_\_ ○ \_\_\_\_\_ or \_\_\_\_\_ ○ \_\_\_\_\_

## Worked Problems

### Exemplar from:

[Module 3: Topic B: Lesson 4](#): Homework

### Explanation of scaffold:

Worked problems provide support as students go through the learning stages of acquisition to proficiency to fluency to generalization and can be used to build learners' momentum and self-efficacy. Students are provided models of completed and/or partially completed problems while they work on developing and applying a newly learned skill without corrective feedback from the teacher.

The use of worked problems is based on the *Interleave Worked Solution Strategy (IWSS)* and involves alternating between fully worked and/or partially completed examples that students can use as a reference and practice problems for students to complete independently. A high level of scaffolding can be provided by giving worked examples and practice problems that are very similar in structure and by providing annotations or problem-solving steps alongside worked examples. Support can be faded by providing fewer worked examples or providing practice problems that are less similar to worked examples. Although the homework problems in this lesson are used as an exemplar, worked problems can be used in any lesson to support students who understand the mathematical concept involved but need examples as they practice what they have learned during the day's lesson to complete homework assignments.

### Teacher actions/instructions:

Add worked problems to homework sheets. Provide the adapted sheets as needed to students, and direct them to complete the assigned problems. Tell students that completed problems have been provided as a reference, partially completed problems will help get them started, and uncompleted problems are expected to be done on their own. You may consider additional scaffolding by assigning specific problems on a homework sheet for those students who are likely to benefit from a shorter period of practice in which they are able to complete the task, rather than a longer period of practice in which they are unsuccessful.

### Student actions:

Students complete problems on the adapted sheets as assigned.

### Student handouts/materials:

*Lesson 4 Homework* sheets (found on the following pages)

**\*\*\*Note: Information in or drawings underlined in red were added to the module lesson homework sheets.**

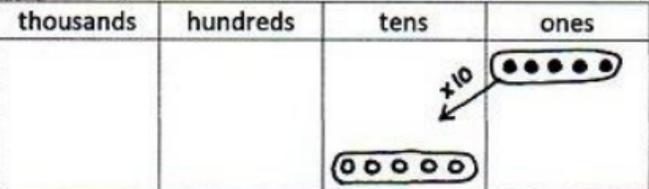
## Lesson 4 Homework

Name \_\_\_\_\_ Date \_\_\_\_\_

Example:

$$5 \times 10 = \underline{50}$$

$$5 \text{ ones} \times 10 = \underline{5} \text{ tens}$$

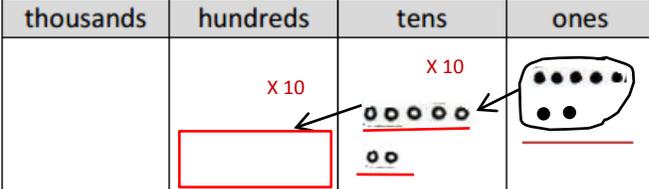
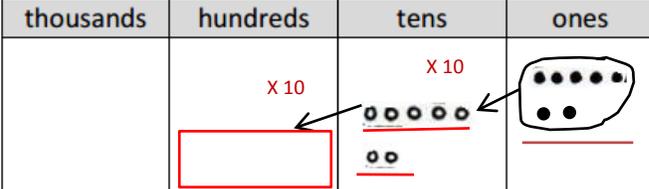
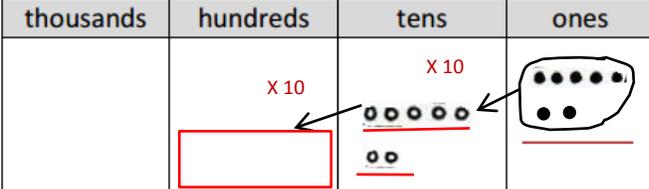
thousands	hundreds	tens	ones
			

Draw place value disks and arrows as shown to represent each product.

1.  $7 \times 100 =$  \_\_\_\_\_

$$7 \times 10 \times 10 =$$
 \_\_\_\_\_

$$7 \text{ ones} \times 100 = \underline{\hspace{2cm}} \text{ 7 hundreds}$$

thousands	hundreds	tens	ones
			

2.  $7 \times 1,000 =$  \_\_\_\_\_

$$7 \times 10 \times 10 \times 10 =$$
 \_\_\_\_\_

$$7 \text{ ones} \times 1,000 =$$
 \_\_\_\_\_

thousands	hundreds	tens	ones

3. Fill in the blanks in the following equations.

a.  $8 \times 10 = \underline{80}$

b.  $\underline{100} \times 8 = 800$

c.  $8,000 = \underline{8} \times 1,000$

d.  $10 \times 3 =$  \_\_\_\_\_

e.  $3 \times$  \_\_\_\_\_  $= 3,000$

f. \_\_\_\_\_  $\times 3 = 300$

g.  $1,000 \times 4 =$  \_\_\_\_\_

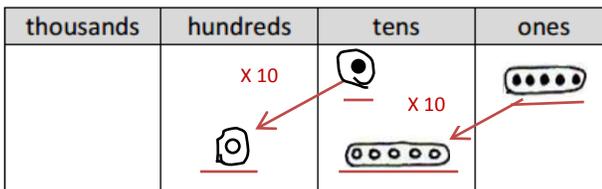
h. \_\_\_\_\_  $= 10 \times 4$

i.  $400 =$  \_\_\_\_\_  $\times 100$

Draw place value disks and arrows to represent each product.

4.  $15 \times 10 = \underline{\hspace{2cm}}$  150

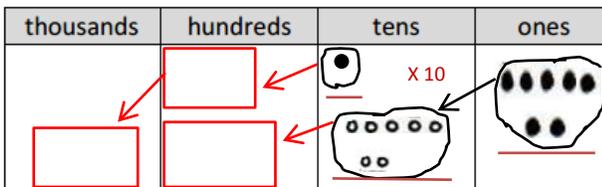
(1 ten 5 ones)  $\times 10 = \underline{\hspace{2cm}}$  15 tens



5.  $17 \times 100 = \underline{\hspace{2cm}}$

$17 \times 10 \times 10 = \underline{\hspace{2cm}}$  1,700

(1 ten 7 ones)  $\times 100 = \underline{\hspace{2cm}}$



6.  $36 \times 1,000 = \underline{\hspace{2cm}}$

$36 \times 10 \times 10 \times 10 = \underline{\hspace{2cm}}$

(3 tens 6 ones)  $\times 1,000 = \underline{\hspace{2cm}}$

ten thousands	thousands	hundreds	tens	ones

Decompose each multiple of 10, 100, or 1000 before multiplying.

7.  $2 \times 80 = 2 \times 8 \times \underline{\hspace{1cm}}$  10  
 $= 16 \times \underline{\hspace{1cm}}$  10  
 $= \underline{\hspace{1cm}}$  160

8.  $2 \times 400 = 2 \times \underline{\hspace{1cm}}$  4  $\times \underline{\hspace{1cm}}$   
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$  100  
 $= \underline{\hspace{1cm}}$

9.  $5 \times 5,000 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$   
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$   
 $= \underline{\hspace{1cm}}$

10.  $7 \times 6,000 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$   
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$   
 $= \underline{\hspace{1cm}}$

## Frayer Model

### Exemplar from:

[Module 3: Topic F: Lesson 22](#): Concept Development

### Explanation of scaffold:

The Frayer model is a four-square graphic organizer that includes a student-friendly definition, a description of important characteristics, examples, and nonexamples. It provides a format to organize information and visual representations of the mathematical term being defined. Developing vocabulary skills is essential for students as they learn to *speak mathematically* and develop their abstract reasoning and problem-solving skills. The following example demonstrates how to provide explicit instruction for those students who need information broken down into smaller, more manageable chunks as well as modeling and guided practice to effectively use this tool to learn new vocabulary words. The term *prime number* is used as an exemplar. However, the Frayer model can be used in any lesson to help students strengthen their conceptual knowledge and develop their understanding of unfamiliar vocabulary.

### Teacher actions/instructions:

Select key mathematical terms. These terms should be limited in number and essential to developing a deeper understanding of the mathematical concepts or skills in the lesson.

Instruct students to complete Frayer models as follows:

1. Write the mathematical term in the middle circle.
2. Define the term, using student-friendly language, in the **Definition** box. Use your own words.
3. Write words to describe the term in the **Characteristics** box. Again, use your own words.
4. List examples of the definition in the **Examples** box. Draw a picture and/or write an equation to help you understand the term if needed.
5. List nonexamples of the definition in the **Nonexamples** box. Again, draw a picture and/or write an equation if needed.
6. Test yourself.

**For students who require explicit instruction on how to use the Frayer model, the following sample script is provided to demonstrate one way instruction might look like:**

#### **Step 1: Write the mathematical term.**

**T (teacher):** *We are going to use a graphic organizer called a Frayer model to help us understand what certain math terms, or vocabulary words, mean. It is very important that we understand what a term means so that we understand what a math problem is asking us to find and so that we can talk about math with others. Understanding vocabulary will make us better mathematicians!*

Display a large version of the Frayer model on chart paper or use a document camera to project your work. Hand out student copies, and direct students to complete their Frayer models as demonstrated.

**T:** We are going to learn about the term **prime number**. What term?

**S (student):** Prime number.

**T:** When we use the Frayer model, the first thing we do is write the vocabulary word in the middle circle. Let's write **prime number** in the circle.

**Step 2: Define the term.**

**T:** You can see there are also 4 boxes. The first box is labeled **Definition**. A definition tells us the meaning of the term. **Prime number** means a number that is greater than 1 that has exactly two different factors, 1 and itself. Let's say that together. [Chorally say the definition with students.] Now, let's write that in the **Definition** box.

**Step 3: Describe the word in terms of its characteristics.**

**T:** The next box is **Characteristics**. This means we want to think of words and pictures and equations that describe **prime number** or that are important to help us understand what it means. [Draw a factor pair table.] If we wanted to list all the factor pairs for the number 7, what would we write?

**S:** 1 and 7.

**T:** What is another factor pair for the number 7?

**S:** There are no other factor pairs for 7.

**T:** That's right. The number 7 only has one factor pair because it has exactly two different factors, 1 and itself. That means 7 is a \_\_\_\_\_.

**S:** Prime number.

**T:** Right again. Now, let's look at the number 23. [Ask students to name all the factor pairs for the number 23. Write down additional information as needed to describe **prime number**.]

**Step 4: List examples.**

**T:** The third box is **Examples**. Let's name some more examples of **prime numbers**. [Write down any reasonable answers and their factors.]

**Step 5: List nonexamples.**

**T:** The last box is **Nonexamples**. This is an important box because it shows we really understand what the word means and what it doesn't mean. We've already written down some examples of **prime numbers**. Now, let's think of some nonexamples. Name a number that is not a **prime number**.

**S:** 10.

**T:** Why isn't 10 a **prime number**?

**S:** Because 10 has more factors besides 1 and 10. It has more than two factors.

**T:** Excellent! You remembered that a **prime number** is a number that has exactly two factors, 1 and itself. It also needs to be a number greater than 1. The number 10 is not a **prime number** because it has more factor pairs besides 1 and 10. Let's write down all the factor pairs for the number 10. Can you think of any other nonexamples of a **prime number**? [Write down any reasonable answers and their factors.]

### Step 6: Test yourself.

The study step is critical to student success in using vocabulary strategies such as the Frayer model. Students need to study the terms to internalize them for later use. Students can quiz each other during "down times," or the models/cards can be used as part of a center activity.

Instruct students to study their Frayer models as follows:

1. Cover each box of the Frayer model with a sticky note. Do not cover the math term in the middle circle.
2. Say the term in the middle and try to say the definition.
3. If you do not know the definition, uncover the **Characteristics** box, and try to say the definition.
4. If you do not know the definition, uncover the **Examples** box, and try to say the definition.
5. If you do not know the definition, uncover the **Nonexamples** box, and try to say the definition.
6. If you do not know the definition, uncover the **Definition** box.

Repeat steps 1-6 for each Frayer model.

### **Student actions:**

Students work either individually or in pairs to make and study Frayer models.

### **Student handouts/materials:**

Frayer Model template (found on page 21)

Sticky notes

NAME: \_\_\_\_\_

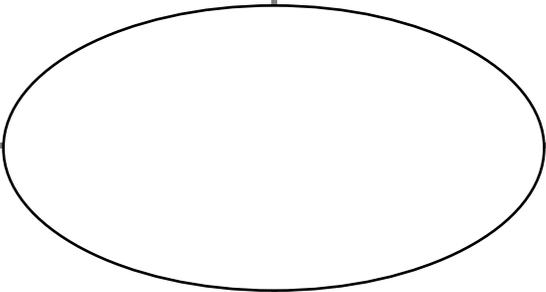
### Frayer Model (example)

<b>Definition</b>  A whole number greater than 1 that has exactly two different factors, 1 and itself.	<b>Characteristics</b> Only two factors, 1 and itself.  <table border="1" data-bbox="1144 462 1549 544"><thead><tr><th colspan="2">Factor Pairs for 7</th></tr></thead><tbody><tr><td>1</td><td>7</td></tr></tbody></table> <p>7 is a prime number</p> <table border="1" data-bbox="1333 625 1738 706"><thead><tr><th colspan="2">Factor Pairs for 23</th></tr></thead><tbody><tr><td>1</td><td>23</td></tr></tbody></table> <p>23 is prime number</p>	Factor Pairs for 7		1	7	Factor Pairs for 23		1	23																						
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NAME: \_\_\_\_\_

### Frayer Model

<b>Definition</b>	<b>Characteristics</b>
<b>Examples</b>	<b>Nonexamples</b>



## Desk Reference Sheet

### **Exemplar from:**

[Module 4: Topics A-C, lessons 1-10](#): Problem Sets and Homework

### **Explanation of scaffold:**

A desk reference sheet provides support to students who have difficulty recalling previously learned facts and information. Although the following desk reference sheet exemplar connects to the module lessons found in topics A-C, a desk reference sheet can be used as a visual support in any lesson until students build fluency remembering key mathematical terms, concepts, processes, and/or skills.

### **Teacher actions/instructions:**

Provide desk reference sheets to students. Explain to students that these sheets can be used as a reference if they forget what a term or word means while working on math problems.

### **Student actions:**

Students work independently to complete various problem set and homework problems using the desk reference sheets as needed.

### **Student handouts/materials:**

*Desk Reference Sheet A* and *Desk Reference Sheet B* (found on the following pages)

## Desk Reference Sheet A Points, Lines, and Rays

**Point:** an exact location in space



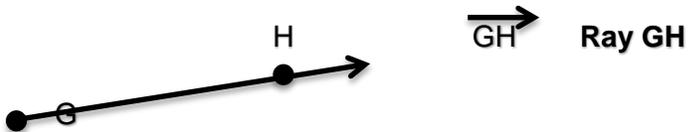
**Line:** an infinite set of points in opposite directions forming a straight path



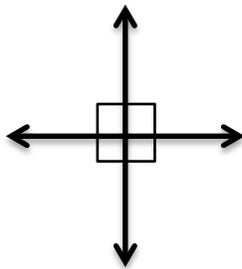
**Line segment:** the set of points on a line consisting of two fixed points and all points between those two fixed points



**Ray:** part of a line that has one endpoint and extends in one direction



**Perpendicular:** two lines, segments, or rays that intersect to form right angles



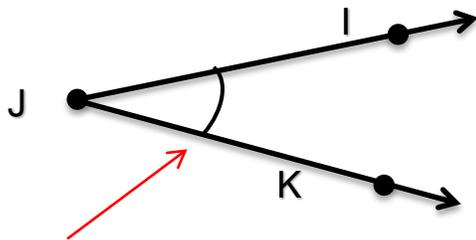
**Parallel lines:** lines in the same plane that never intersect no matter how far they are extended; they are equidistant (equal distance) from each other



## Desk Reference Sheet B

### Angles

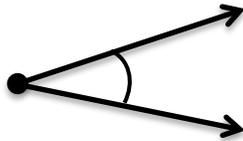
**Angle:** a geometric figure formed by two rays that have a common endpoint called a vertex.



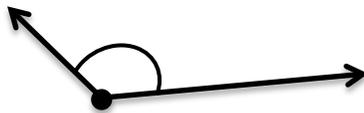
$\angle IJK$  has its vertex at point J

**Arc:** used to identify an angle in a figure

**Acute angle:** an angle whose measure is greater than  $0^\circ$  and less than  $90^\circ$



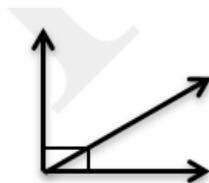
**Obtuse angle:** an angle whose measure is greater than  $90^\circ$  and less than  $180^\circ$



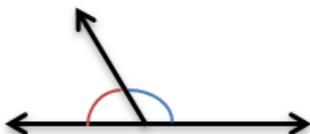
**Straight angle:** an angle that measures  $180^\circ$



**Complementary angles:** two angles whose measures have a sum of  $90^\circ$



**Supplementary angles:** two angles whose measures have a sum of  $180^\circ$



## References

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Archer, A. and Hughes, C. (2011). *Explicit instruction: Effective and efficient teaching*. New York, NY: The Guilford Press.