DRAFT Introduction to the New York State P-12 Learning Standards for Mathematics (Revised 2017)

In 2015, New York State (NYS) began a process of review and revision of its current mathematics standards adopted in January of 2011. Through numerous phases of public comment, virtual and face to face meetings with committees consisting of NYS educators, teachers of English Language Learners/Multilingual Learners and Students with Disabilities, parents, curriculum specialists, school administrators, college professors, and experts in cognitive research, the New York State P-12 Learning Standards for Mathematics (Revised 2017) were developed. These revised standards reflect the collaborative efforts and expertise of all constituents involved.

The New York State P-12 Learning Standards for Mathematics (Revised 2017) consist of revisions, additions, vertical movement, and clarifications of the current mathematics standards. They are defined as the knowledge, skills and understanding that individuals can and do habitually demonstrate over time because of instruction and experience. These mathematics standards collectively are focused and cohesive in nature, allowing students access to the knowledge and understanding of the mathematical concepts that are necessary to function in a world very dependent upon the application of mathematics, while providing educators the opportunity to devise innovative programs to support this endeavor. Standards need to be rigorous; requiring a balance of conceptual understanding, procedural fluency and application and must represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards, but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English

Language Learners (ELLs)/Multilingual Learners (MLLs) and for Students with Disabilities. However, the department ensured that teachers of English Language Learners (ELLs)/Multilingual Learners (MLLs) and Students with Disabilities participated in the revision of the standards. The New York State Education Department (NYSED) has created two statewide frameworks, the *Blueprint for Improved Results for Students with Disabilities* and the *Blueprint for English Language Leaner Success*, aimed to clarify expectations and to provide guidance for administrators, policymakers, and practitioners to prepare ELLs/MLLs and Students with Disabilities for success. These principles therein the frameworks are intended to enhance programming and improve instruction that would allow for students within these populations to reach the same standards as all students and leave school prepared to successfully transition to post school learning, living and working. No set of grade-specific standards can fully reflect the great variety in abilities, linguistic experiences, needs, learning rates, and achievement levels of students in any given classroom. In designing and delivering mathematics instruction, educators must consider the cultural context and prior academic experiences of all students while bridging prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible. In addition, educators must consider the relationship of language and content, and the vital role that language plays in obtaining and expressing mathematics content knowledge. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate adaptations in order to ensure equitable access and maximum participation of all students.

How to read the Standards for Mathematical Content

The standards are organized by grade level from Prekindergarten through grade eight and by course at the high school level. The high school courses include Algebra I, Geometry, and Algebra II. The Plus (+) standards document is not a course. They are additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics. The Plus (+) standards however, may be dispersed throughout course curriculums, including Algebra I, Algebra II and Geometry to increase rigor and support the needs of individual students and mathematical programs, but are not expected to be addressed on state assessments. The Prekindergarten through grade eight standards are grouped by domains that provide coherence across the grade levels, while the high school courses and Plus (+) standards are grouped similarly by conceptual categories. The conceptual categories include Number and Quantity, Algebra, Functions, Geometry, Statistics and Probability, and Modeling. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Standards define what students should understand and be able to do.
Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.
Conceptual Categories provide coherence across the courses.

Prekindergarten through Grade Eight

High School Courses and Plus (+) Standards

The order in which the standards are presented do not necessitate the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best order and instructional strategies necessary to support the attainment of the grade-level/course expectations and knowledge presented in the standards for their learners. Standards do not dictate curriculum or teaching methods. Learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

The Standards for Mathematical Practice

The Standards for each grade level and course begin with eight Standards for Mathematical Practice. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the

NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents

such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.